Exercise 1. Show that surjective morphisms are stable under base change. That is, let $f\colon X \to S$ and $g\colon Y \to S$ be morphisms of schemes, with f surjective. Show that the projection map $X \times_S Y \to Y$ is surjective.

Exercise 2. Describe the fibers of the morphism $\operatorname{Spec} \mathbb{Z}[X]/(X^2+X+1) \longrightarrow \operatorname{Spec} \mathbb{Z}$. Which fibers are reduced? Which ones are irreducible?
Hint: let $\Phi_d \in \mathbb{Z}[X]$ be the d-th cyclotomic polynomial. Then $X^2 + X + 1 = \Phi_3$.
Let ℓ be a prime. Can you describe the fibers of the morphism $\operatorname{Spec} \mathbb{Z}[X]/(\Phi_\ell) \longrightarrow \operatorname{Spec} \mathbb{Z}$?

Exercise 3. Let $R = \mathbb{Z} \times \mathbb{Z}$ and $S = \mathbb{Z}[X]/(X^2 - 1)$. Describe the fibers of the morphisms $\operatorname{Spec} R \to \operatorname{Spec} \mathbb{Z}$ and $\operatorname{Spec} S \to \operatorname{Spec} \mathbb{Z}$. Show that R, S are not isomorphic as rings.

Exercise 4. Let $\varphi\colon A \to B$ be a ring homomorphism, and S a multiplicative subset of A. Prove that $S^{-1}B = B \otimes_A S^{-1}A$.

Exercise 5. Prove (parts of) Corollary II.4.6 in [HAG] without assuming noetherian hypotheses.

Exercise 6. Do [HAG], Exercise II.4.2. In your example under a), describe the equalizer of f, g as closed subscheme of X (cf. TAG 01KM in the Stacks project).