Exercise 1. Let C be a category. A monomorphism in C is a morphism $j\colon U \to X$ in C such that for every object T in C and every pair of morphisms $f, g\colon T \to U$ we have $jf = jg$ if and only if $f = g$.
(a) Assuming that fiber products exist in C, show that the diagonal morphism of a monomorphism is an isomorphism.
(b) Let $j\colon U \to X$ be a morphism of schemes. Assume that (1) j is injective on points, (2) for every $u \in U$, the ring map $j_u^{\#}\colon O_{X,j(u)} \to O_{U,u}$ is surjective. Prove that j is a monomorphism in the category of schemes.
(c) Show that in the category of schemes, open immersions and closed immersions are monomorphisms.
(d) Conclude that open immersions and closed immersions are separated.

Exercise 2. Do [HAG], Exercise II.4.2. In your example under a), describe the equalizer of f, g as closed subscheme of X (cf. TAG 01KM in the Stacks project).

Exercise 3. Let $f\colon X \to \operatorname{Spec} \mathbb{Z}$ be a morphism of schemes. Assume that each pair x, y of points of X that map to a common point s of $\operatorname{Spec} \mathbb{Z}$ are contained in affine opens $x \in U$, $y \in V$ of X such that (1) the intersection $U \cap V$ is affine, (2) the ring map $O_X(U) \otimes_{\mathbb{Z}} O_X(V) \to O_X(U \cap V)$ is surjective. Prove that f is separated. As an application, show that for any graded ring S, the morphism $\operatorname{Proj} S \to \operatorname{Spec} \mathbb{Z}$ is separated.

Exercise 4. Prove (parts of) Corollary II.4.8 in [HAG] without assuming noetherian hypotheses.