Exercise 1. Let $R$ be a ring, $I$ an ideal of $R$, and consider the closed immersion $Y = \text{Spec } R/I \hookrightarrow \text{Spec } R = X$. Show that the associated sequence of $\mathcal{O}_X$-modules

$$0 \rightarrow \mathcal{I}_Y \rightarrow \mathcal{O}_X \rightarrow i_*\mathcal{O}_Y \rightarrow 0$$

can be identified with

$$0 \rightarrow \widetilde{I} \rightarrow \widetilde{R} \rightarrow \widetilde{R}/I \rightarrow 0.$$

Exercise 2. Let $R$ be a ring. Let $f$ be a homogeneous element of the graded ring $R[x_0, \ldots, x_n]$ of degree $d$ and let $Y \hookrightarrow \mathbb{P}_R^n$ be the closed immersion corresponding to the homogeneous ideal generated by $f$. Give an isomorphism $\mathcal{I}_Y \cong \mathcal{O}_{\mathbb{P}_R^n}(-d)$.

Exercise 3. Let $\varphi: R \rightarrow S$ be a homomorphism of rings, and let $f: Y = \text{Spec } S \rightarrow X = \text{Spec } R$ be the corresponding morphism of affine schemes. Show that if $f^*: \mathcal{O}_X \rightarrow f_*\mathcal{O}_Y$ is surjective, then $\varphi$ is surjective.

This generalizes [HAG], Exercise II.2.18(d).

Either prove this “by hand”, or directly apply [HAG], Proposition II.5.6 ...