

Exercise 1. Let R be a ring, I an ideal of R , and consider the closed immersion $Y = \text{Spec } R/I \hookrightarrow \text{Spec } R = X$. Show that the associated sequence of \mathcal{O}_X -modules

$$0 \longrightarrow \mathcal{I}_Y \longrightarrow \mathcal{O}_X \longrightarrow i_*\mathcal{O}_Y \longrightarrow 0$$

can be identified with

$$0 \longrightarrow \tilde{I} \longrightarrow \tilde{R} \longrightarrow \widetilde{R/I} \longrightarrow 0.$$

Exercise 2. Let R be a ring. Let f be a homogeneous element of the graded ring $R[x_0, \dots, x_n]$ of degree d and let $Y \hookrightarrow \mathbb{P}_R^n$ be the closed immersion corresponding to the homogeneous ideal generated by f . Give an isomorphism $\mathcal{I}_Y \xrightarrow{\sim} \mathcal{O}_{\mathbb{P}_R^n}(-d)$.

Exercise 3. Let $\varphi: R \rightarrow S$ be a homomorphism of rings, and let $f: Y = \text{Spec } S \rightarrow X = \text{Spec } R$ be the corresponding morphism of affine schemes. Show that if $f^\#: \mathcal{O}_X \rightarrow f_*\mathcal{O}_Y$ is surjective, then φ is surjective.

This generalizes [HAG], Exercise II.2.18(d).

Either prove this “by hand”, or directly apply [HAG], Proposition II.5.6 ...