AAG, WEEK 8: EXERCISES

(1) Let $f\colon X \to Y$ be a finite morphism. Assume that Y is locally noetherian. Show that X is locally noetherian. Show that $f_*\mathcal{F}$ is coherent for every coherent $\mathcal{O}_X$-module $\mathcal{F}$.

(2) Let R be a commutative ring, V a free R-module of finite rank and $\psi\colon V \times V \to R$ an R-bilinear form. Consider the functors

$$S \mapsto \mathrm{Aut}_S(S \otimes_R V)$$

and

$$S \mapsto \big\{\alpha \in \mathrm{GL}(V)(S) \mid \forall x, y \in S \otimes_R V, \psi(\alpha(x), \alpha(y)) = \psi(x, y)\big\}$$

from R-algebras to groups.

(a) Show that these are representable by affine group schemes (which we denote by $\mathrm{GL}(V)$ and $\mathrm{O}(V, \psi)$ respectively). Show that the natural map $\mathrm{O}(V, \psi) \to \mathrm{GL}(V)$ is a closed immersion.

(b) Let $R = \mathrm{Spec}\,\mathbf{Z}$, $V = \mathbf{Z}^n$ and ψ the standard inner product. Show that the determinant defines a morphism of group schemes $\mathrm{O}(V, \psi) \to \mu_2$.