Paraxial Geometric Optics in 3D through Point-based Geometric Algebra

computer vision lab

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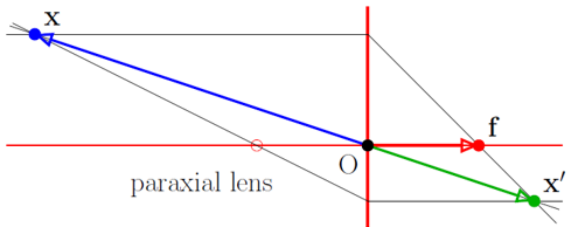
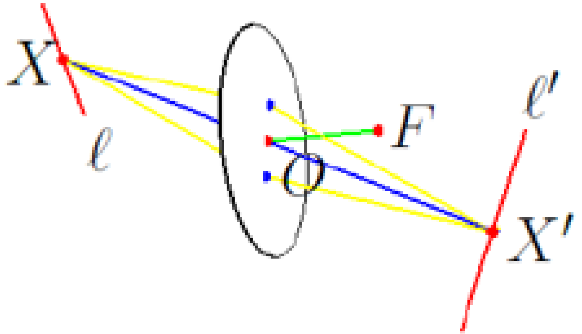
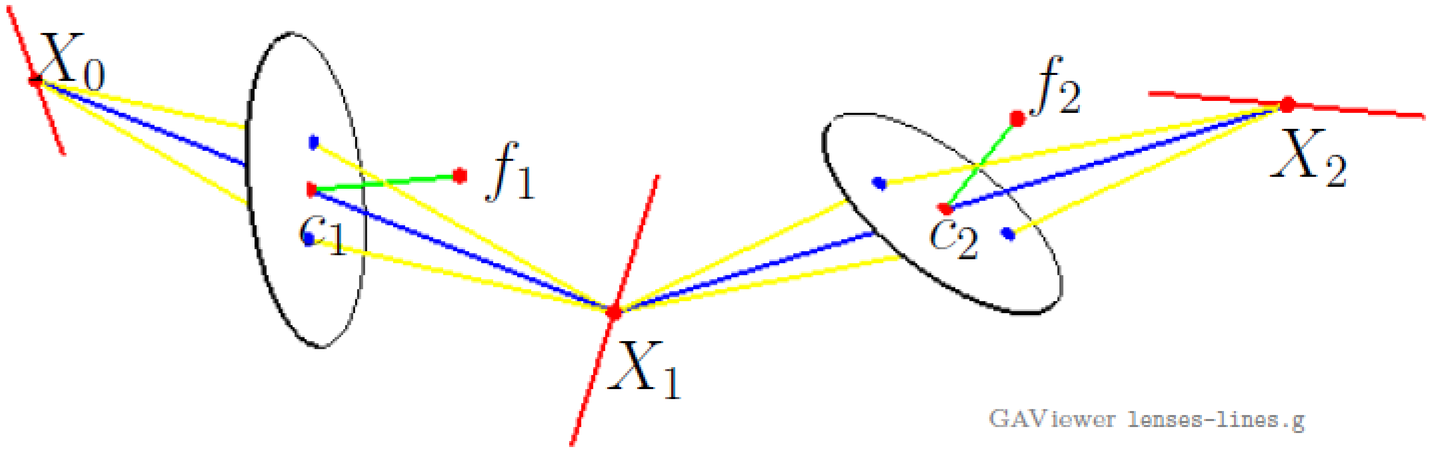
- Usual PGA $\mathbb{R}_{d,0,1}$ is plane-based, has translation versors: Euclidean geometry.
- What about the versors of a point-pased $\mathbb{R}_{d,0,1}$? (What Lengyel promotes...)
- Call it HGA (homogeneous GA), implements homogeneous point coordinates.
- This gives versors for paraxial geometric optics of spherical mirrors and lenses.
- GA bonus: versors! So transforms *any* flat element: points, lines, planes.
- Compact way of generating the ray tracing matrices for an optical element.
- But it only works for the origin as optical center (no translations!)...
- ... since the origin is the only null point $e_0^2 = 0$ in HGA.







- Conformal CGA has null points *everywhere*, and translation versors.
- Embed the HGA at every point of CGA: $e_0 + \mathbf{p} \quad \leftrightarrow n_o + \mathbf{p}$.
- Arbitrary flat X embeds as $X|_c \equiv c \cdot (-n_\infty \wedge X)$ relative to center c.
- Structure-preserving $(X \wedge Y)|_c = X|_c \wedge Y|_c$, 'neopotent': $(X|_{c_1})|_{c_2} = X|_{c_2}$.
- Lens versor is $L_i = \exp(c_i/(c_i \wedge n_\infty \wedge f_i) = \exp(c_i/(c_i \wedge n_\infty \wedge \mathbf{f}_i)$.
- Iterate, each output of a lens is input relative to the next optical center.
- Can be used to generate paraxial ray tracing matrix for total optical system.

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