A Unifying "Anycentric" Pinhole Camera Model for Calibrating Ento-, Tele- and Hypercentric Lenses

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Entocentricity

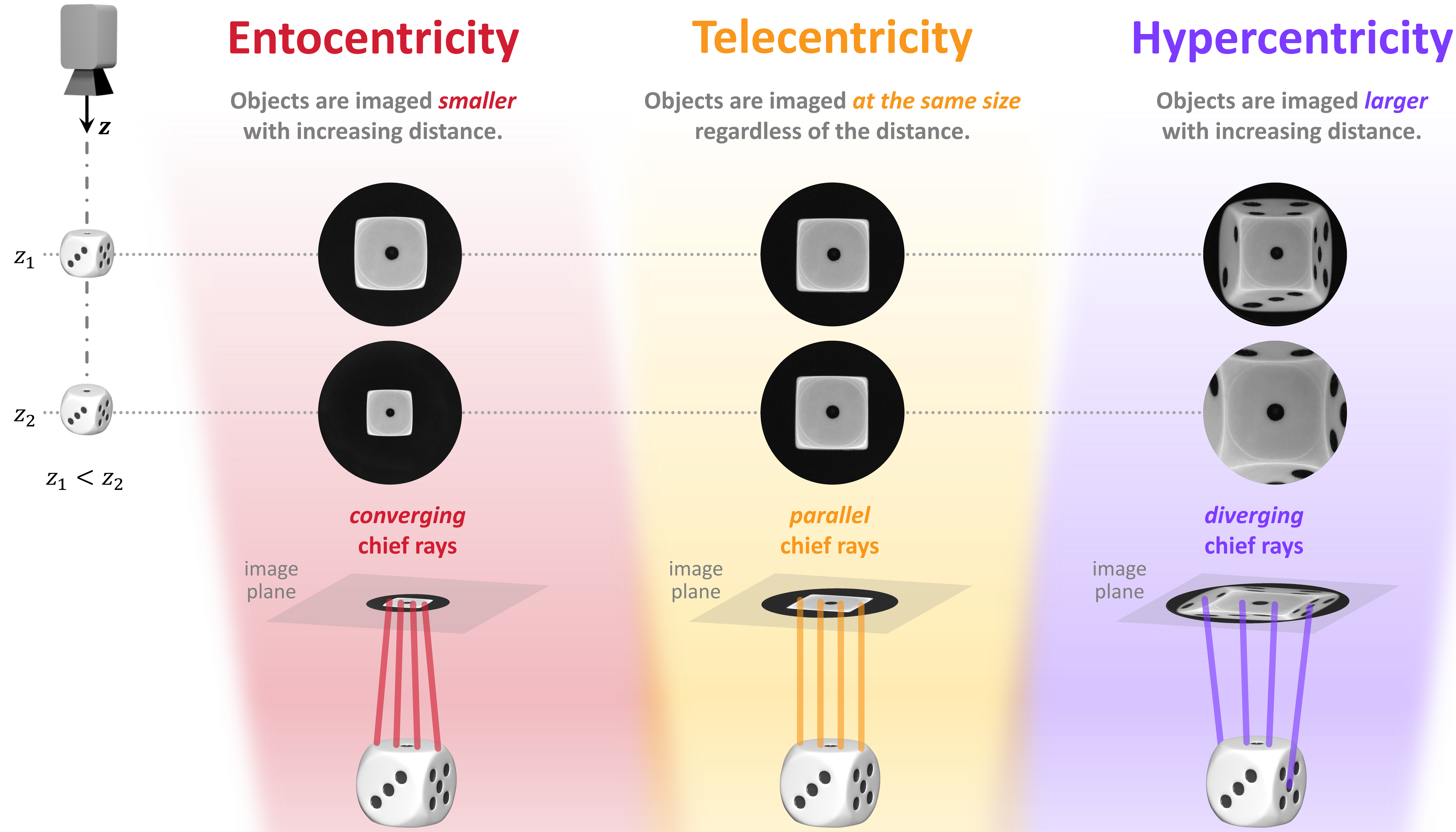
Objects are imaged ***smaller*** with increasing distance.

Telecentricity

Objects are imaged ***at the same size*** regardless of the distance.

Hypercentricity

Objects are imaged ***larger*** with increasing distance.



Anycentric Pinhole Camera Model

Plane-Based Geometric Algebra (PGA) [1-5]

$$\mathbb{R}^*_{n,0,1}$$

$\boldsymbol{e}_i^2 = 1$ for $i = 1,2,\dots,n$ and $\boldsymbol{e}_0^2 = 0$

1. Coordinate System

Located in the image plane at the intersection with the optical (n^{th}) axis. Transition to classical pinhole camera model for ento- [8] and hypercentric [9] case possible with ***motor*** $\sqrt{c/\boldsymbol{e}_0^*}$.

2. Camera Center $\boldsymbol{c}$

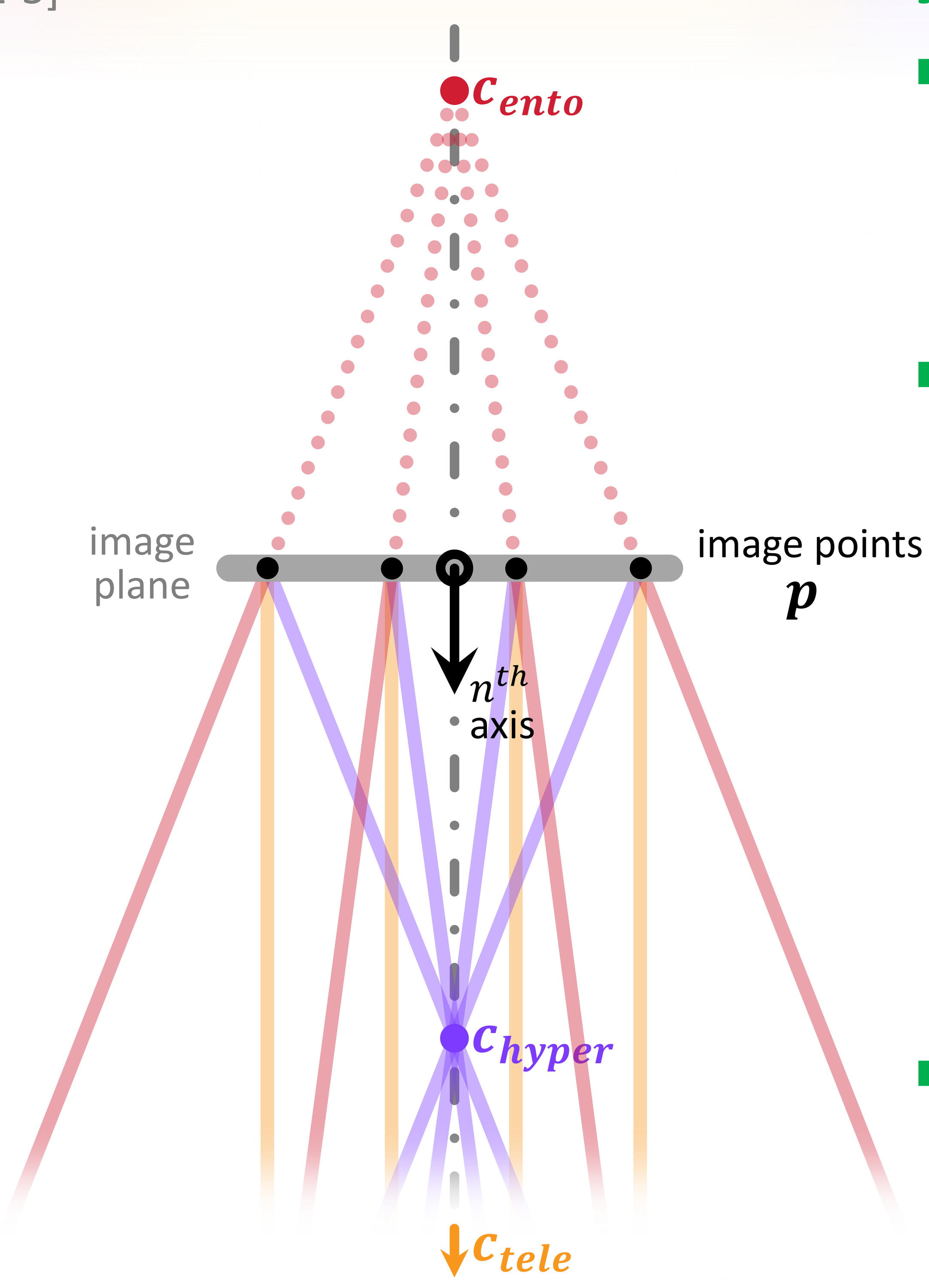
$$\boldsymbol{c} = \boldsymbol{e}_n^* - k\boldsymbol{e}_0^* \quad \text{with} \quad k := {}^1/_c$$

Utilizing the projective geometry included in PGA to model a ***smooth transition*** between perspective and orthographic projection. The (signed) parameter c denotes the classical ***camera constant*** for the ento- [8] and hypercentric [9] case.

3. Chief Rays

$$\boldsymbol{c} \vee \boldsymbol{p}$$

With *oriented* PGA (see right), this describes ***chief*** resp. ***light rays*** towards the camera. ***Sight rays*** "emanating" from the camera are modelled by $\boldsymbol{p} \vee \boldsymbol{c}$.



Further Advantages

- **Dimension-Agnostic Model**
  Model of a camera in n-dimensional space capturing a $(n-1)$-dimensional image, e.g. a *line camera* for $n = 2$ or the standard *areal camera* for $n = 3$.
- **Oriented Model** [6]
  If desired, by using the *orientation-preserving* undual
  $$\boldsymbol{v}^{*^{-1}} = (-1)^{\text{grade}(\boldsymbol{v})n}\, \boldsymbol{v}^* \text{ [4]}$$
  and sandwich product
  $$(-1)^{\text{grade}(\boldsymbol{u})\text{grade}(\boldsymbol{v})}\boldsymbol{uvu}^{-1} \text{ [7]}$$
  with blades $\boldsymbol{u}$ and $\boldsymbol{v}$, ***light*** and ***sight rays*** can be distinguished, as the camera center ***c flips its orientation*** in the entocentric case compared to the tele- and hypercentric cases.
- **Model for Telecentricity Errors**
  Because of the smooth transition, the model can ***directly describe and calibrate telecentricity errors*** of telecentric lenses (which in practice tend to be slightly ento- or hypercentric).

[1] **Gunn, C.; De Keninck, S.**: *Geometric Algebra for Computer Graphics.*
[2] **Dorst, L.; De Keninck, S.:** *May the Forque Be with You.*
[3] **Dorst, L.; Fontijne, D.; Mann, S.:** *Geometric Algebra for Computer Science: An Object-oriented Approach to Geometry.*
[4] **Dorst, L.; De Keninck, S.:** *A Guided Tour to the Plane-Based Geometric Algebra PGA.*
[5] **Lengyel, E.:** *Projective Geometric Algebra Done Right.*
[6] **Stolfi, J.:** *Oriented Projective Geometry.*
[7] **Roelfs, M.; De Keninck, S.:** *Graded symmetry groups: plane and simple.*
[8] **Zhang, Z.:** *A flexible new technique for camera calibration.*
[9] **Ulrich, M.; Steger, C.:** *A camera model for cameras with hypercentric lenses and some example applications.*

$k > 0$	$k = 0$	$k < 0$
(camera constant $c > 0$)	(camera constant $c = \infty$)	(camera constant $c < 0$)

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