Inverse of a Multivector

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The problem

A Clifford algebra is generally not a division ring, i.e., every non-zero multivector need not be invertible.

The natural question then is: **How to detect invertible elements?**

This is the question we try to address.

Applications of the results

We will discuss basis free formulas for the inverse of multivectors in Clifford algebras with $n \leq 6$.

1) Solving linear equations: $AX = B$

2) Robotics, computer vision, control theory, stability analysis, model reduction, image and signal processing:

Sylvester equation: $AX + XB = C$

Lyapunov equation: $AX + XA^H = C$

References: Dmitry Shirokov, *Advances in Applied Clifford Algebras, 31 (2021), 70, 19 pp*

The results

$$n = 1, \qquad U^{-1} = \frac{\widehat{U}}{U\widehat{U}}$$

$$n = 2, \qquad U^{-1} = \frac{\widehat{\widetilde{U}}}{U\widehat{\widetilde{U}}}$$

$$n = 3, \qquad U^{-1} = \frac{\widetilde{U}\widehat{U}\widehat{\widetilde{U}}}{U\widetilde{U}\widehat{U}\widehat{\widetilde{U}}}$$

where $U^{\triangle} := \sum_{k=0,1,2,3 \bmod 8} \langle U \rangle_k - \sum_{k=4,5,6,7 \bmod 8} \langle U \rangle_k$

$$n = 4, \qquad U^{-1} = \frac{\widetilde{U}(\widehat{U}\widehat{\widetilde{U}})^{\triangle}}{U\widetilde{U}(\widehat{U}\widehat{\widetilde{U}})^{\triangle}}$$

$$n = 5, \qquad U^{-1} = \frac{\widetilde{U}(\widehat{U}\widehat{\widetilde{U}})^{\triangle}(U\widetilde{U}(\widehat{U}\widehat{\widetilde{U}})^{\triangle})^{\triangle}}{U\widetilde{U}(\widehat{U}\widehat{\widetilde{U}})^{\triangle}(U\widetilde{U}(\widehat{U}\widehat{\widetilde{U}})^{\triangle})^{\triangle}}$$

$$n = 6, \qquad U^{-1} = \frac{\left(\frac{1}{3}\widetilde{U}\widehat{U}\widehat{\widetilde{U}}(\widehat{U}\widehat{\widetilde{U}}U\widetilde{U})^{\triangle} + \frac{2}{3}\widetilde{U}((\widehat{U}\widehat{\widetilde{U}})^{\triangle}((\widehat{U}\widehat{\widetilde{U}})^{\triangle}(U\widetilde{U})^{\triangle})^{\triangle})^{\triangle}\right)}{U\left(\frac{1}{3}\widetilde{U}\widehat{U}\widehat{\widetilde{U}}(\widehat{U}\widehat{\widetilde{U}}U\widetilde{U})^{\triangle} + \frac{2}{3}\widetilde{U}((\widehat{U}\widehat{\widetilde{U}})^{\triangle}((\widehat{U}\widehat{\widetilde{U}})^{\triangle}(U\widetilde{U})^{\triangle})^{\triangle})^{\triangle}\right)}$$

Big picture: Why do the results work?

Inspiration: Structure of inverses in complex numbers, quaternions, split complex numbers,
their representations.

We come up with required notions to generalize the problem to “bigger” Clifford algebras.

Essence: multiply a multivector with its suitably chosen conjugates as to eliminate non-zero grades.
Keep doing that until you eliminate all non-zero grades.

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Thank you for your attention!

See you at my poster!