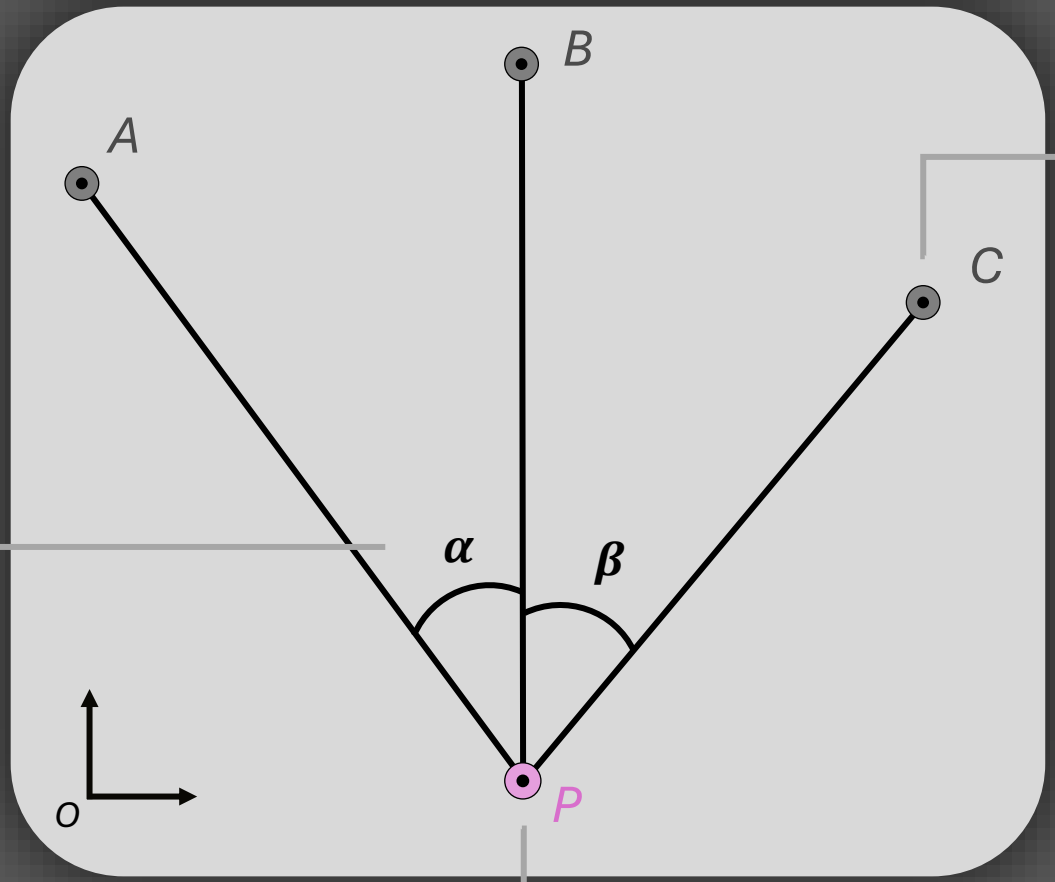
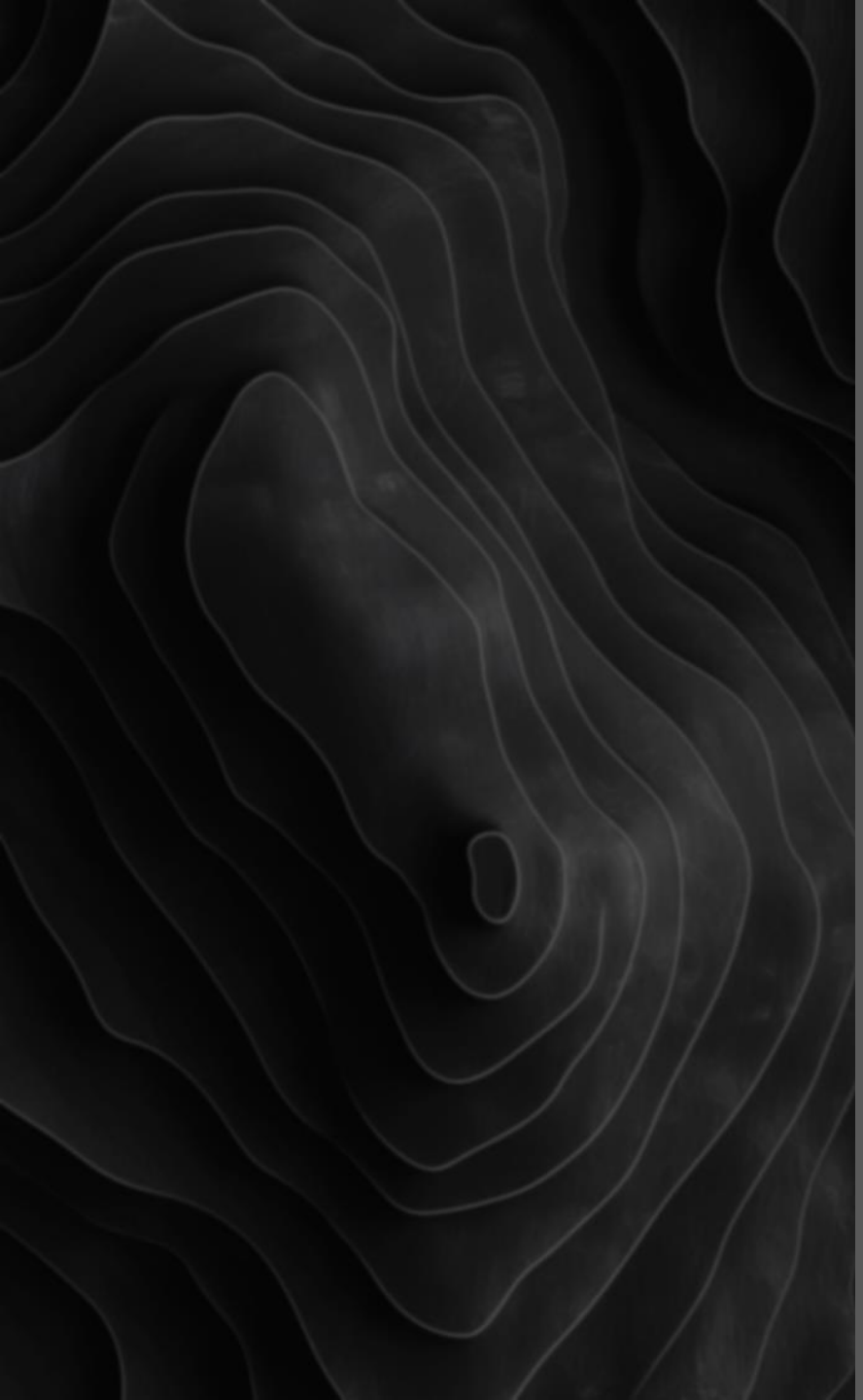
The Resection Problem



Three known points A, B and C (beacons)

Relative angular measurements α and β from P

Unknown point P (station)

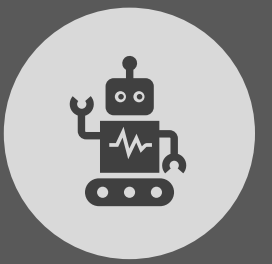


Applications

Geodesy

Navigation

Computer Graphics



Robot Path Planning

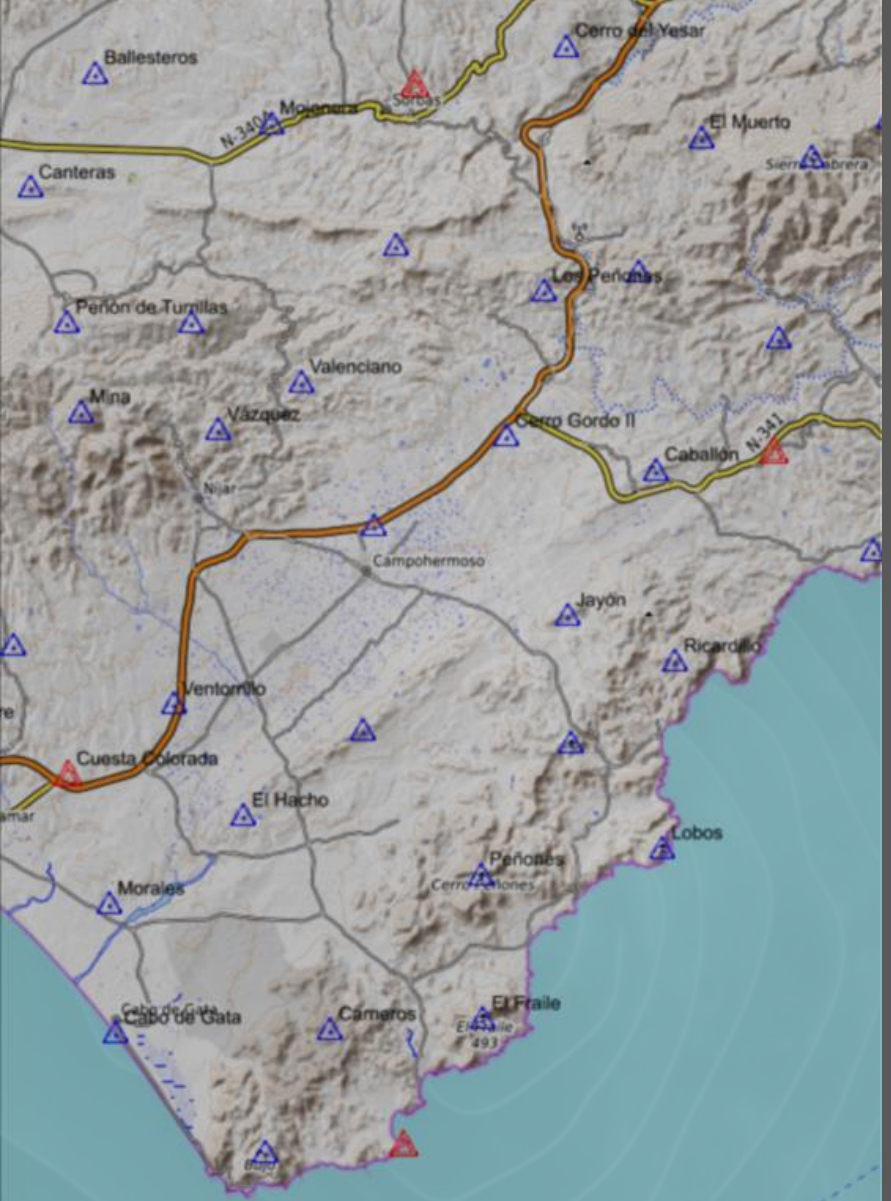
Applications

Geodesy

Navigation

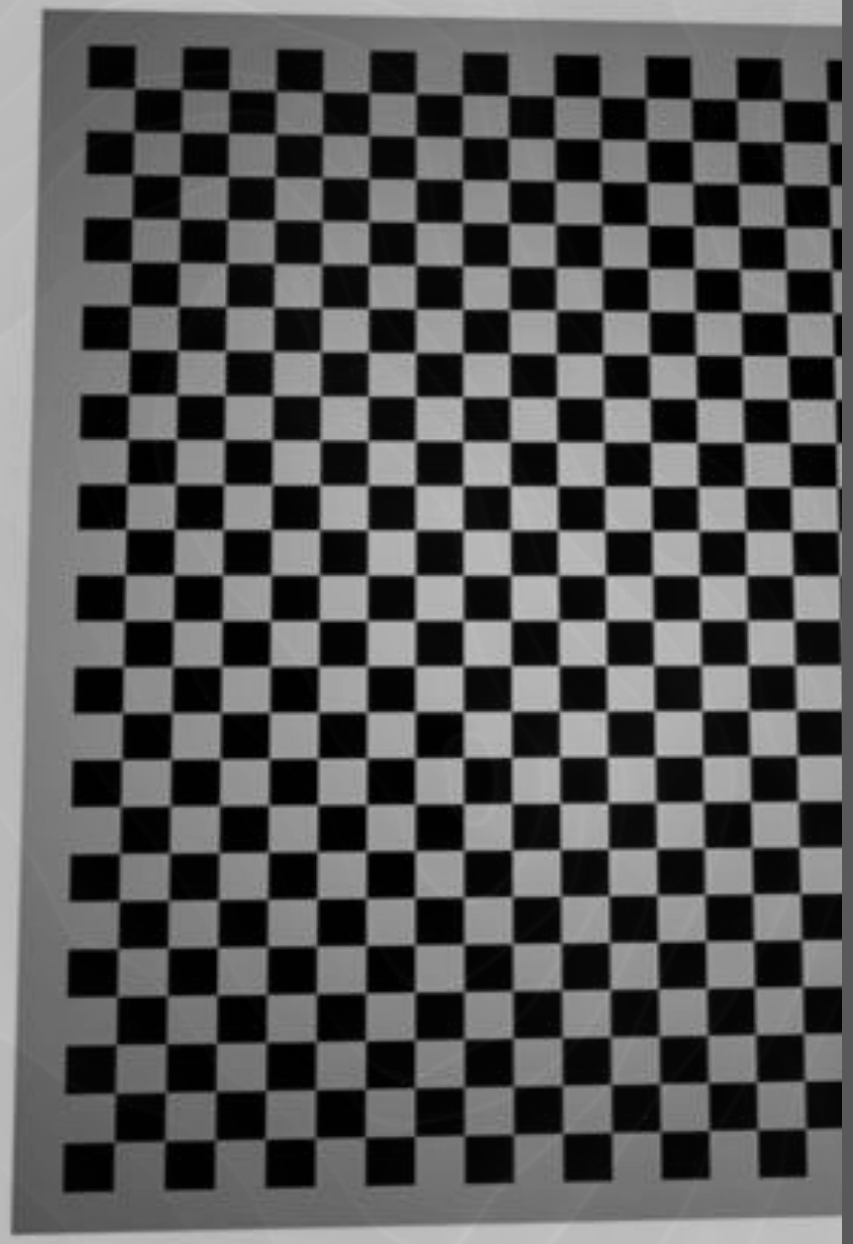
Computer Graphics

Robot Path Planning



Applications

- Geodesy
- Navigation
- Computer Graphics
- Robot Path Planning



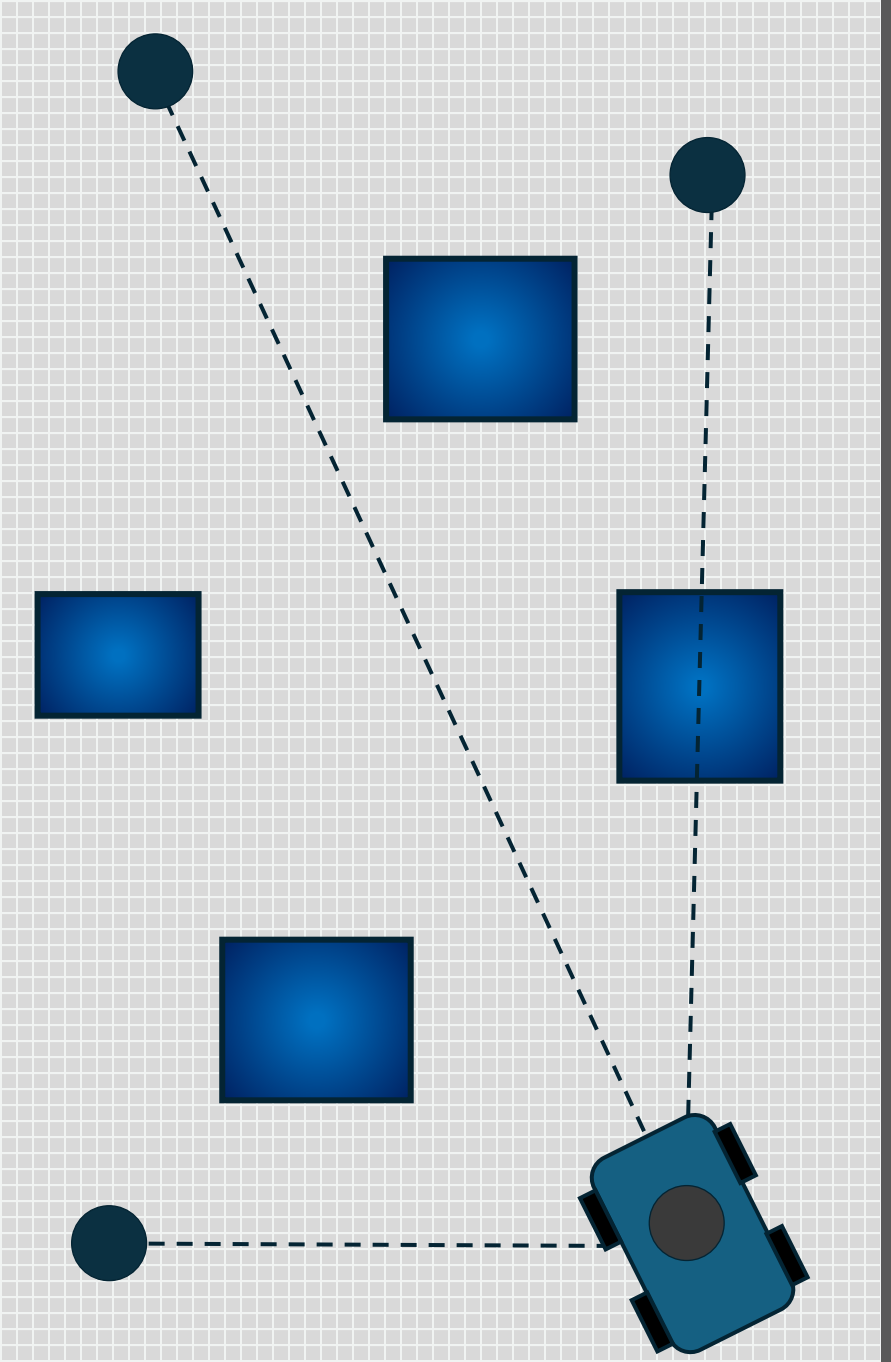
Applications

Geodesy

Navigation

Computer Graphics

Robot Path Planning



Applications

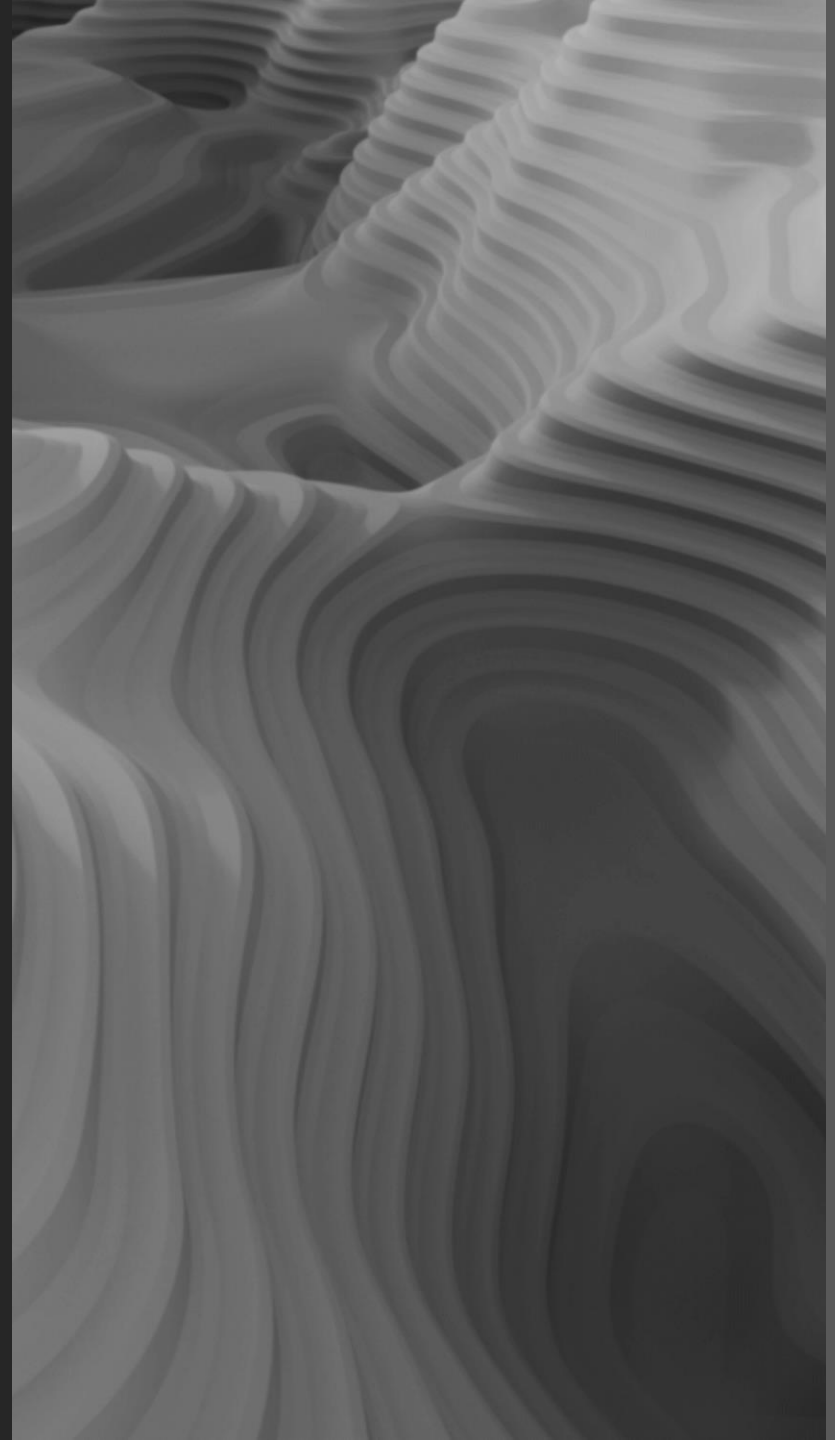
Geodesy

Navigation

Computer Graphics

Robot Path Planning

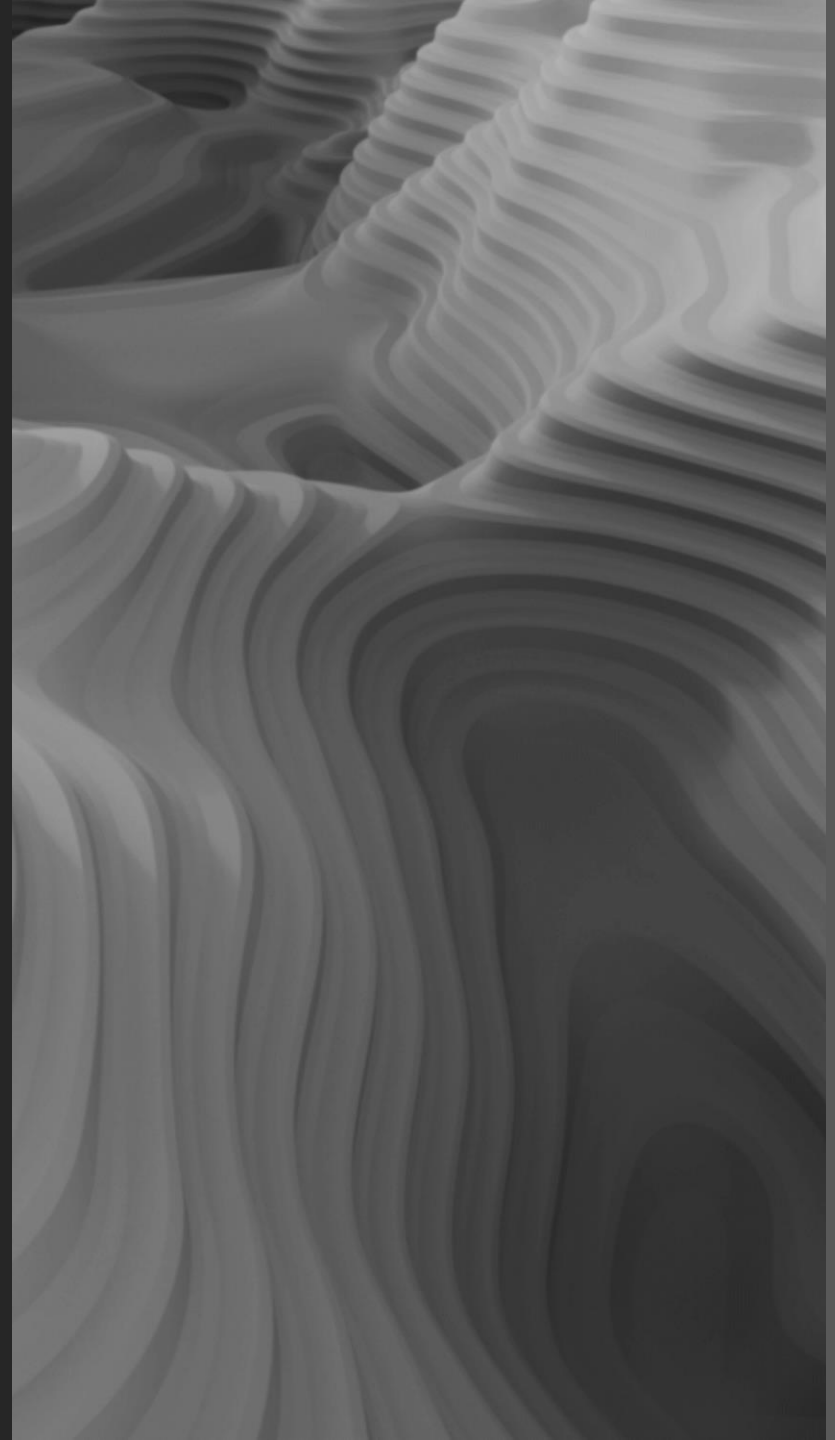
Tradicional Methods



Traditionally, solutions to this problem have relied on heavily algebraically loaded methods, which can be complex and challenging to comprehend.

Trigonometric

Tradicional Methods

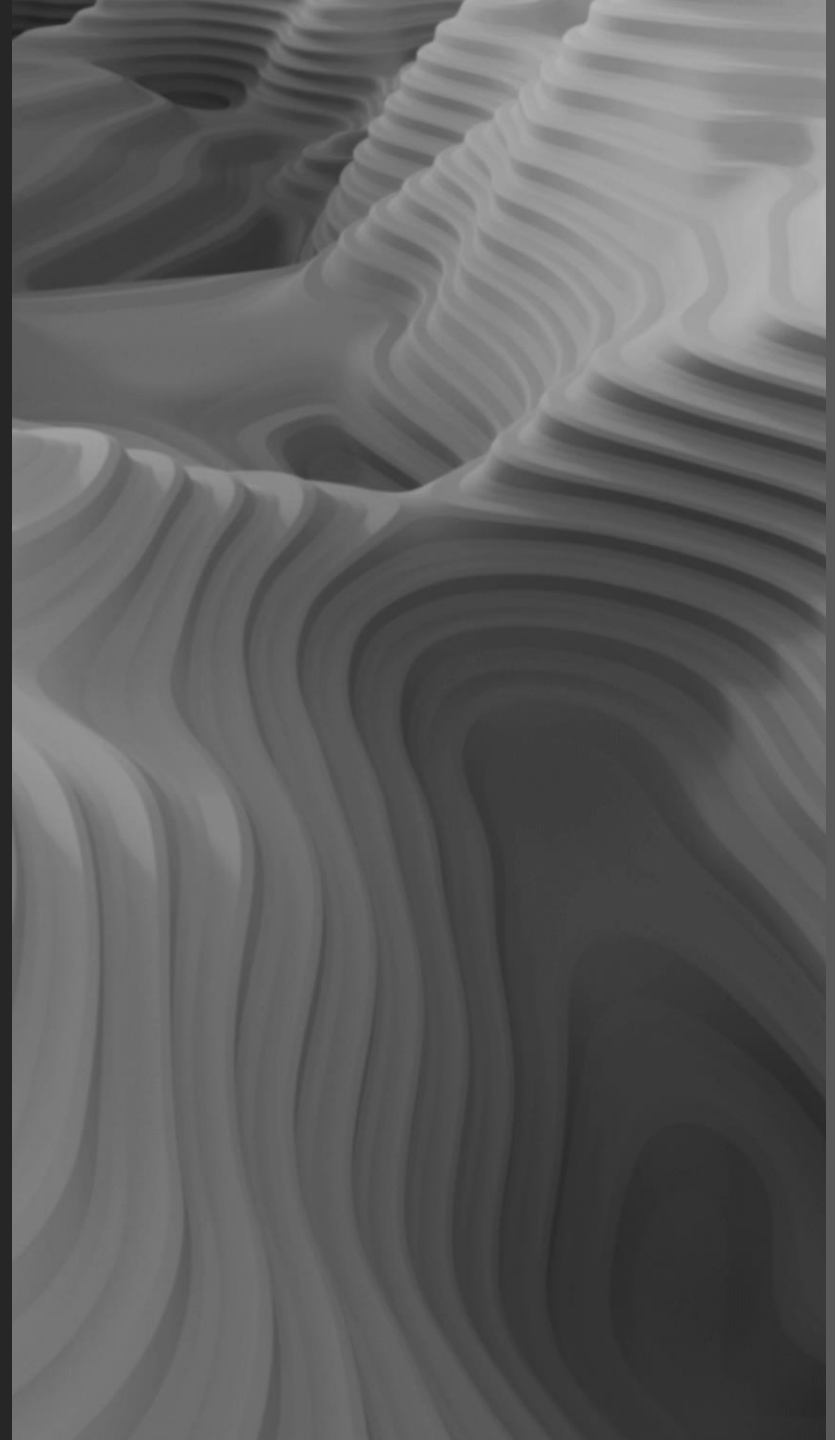


Trigonometric methods are some of the oldest and most famous procedures for solving the 2D resection problem. They are based on the use of trigonometric functions to compute the position of the observer using the angles between the known points and the observer

- Snellius
- Kaestner-Burkhardt
- Madsen and Andersen
- Easton and Cameron
- Cassini

Geometric

Tradicional Methods

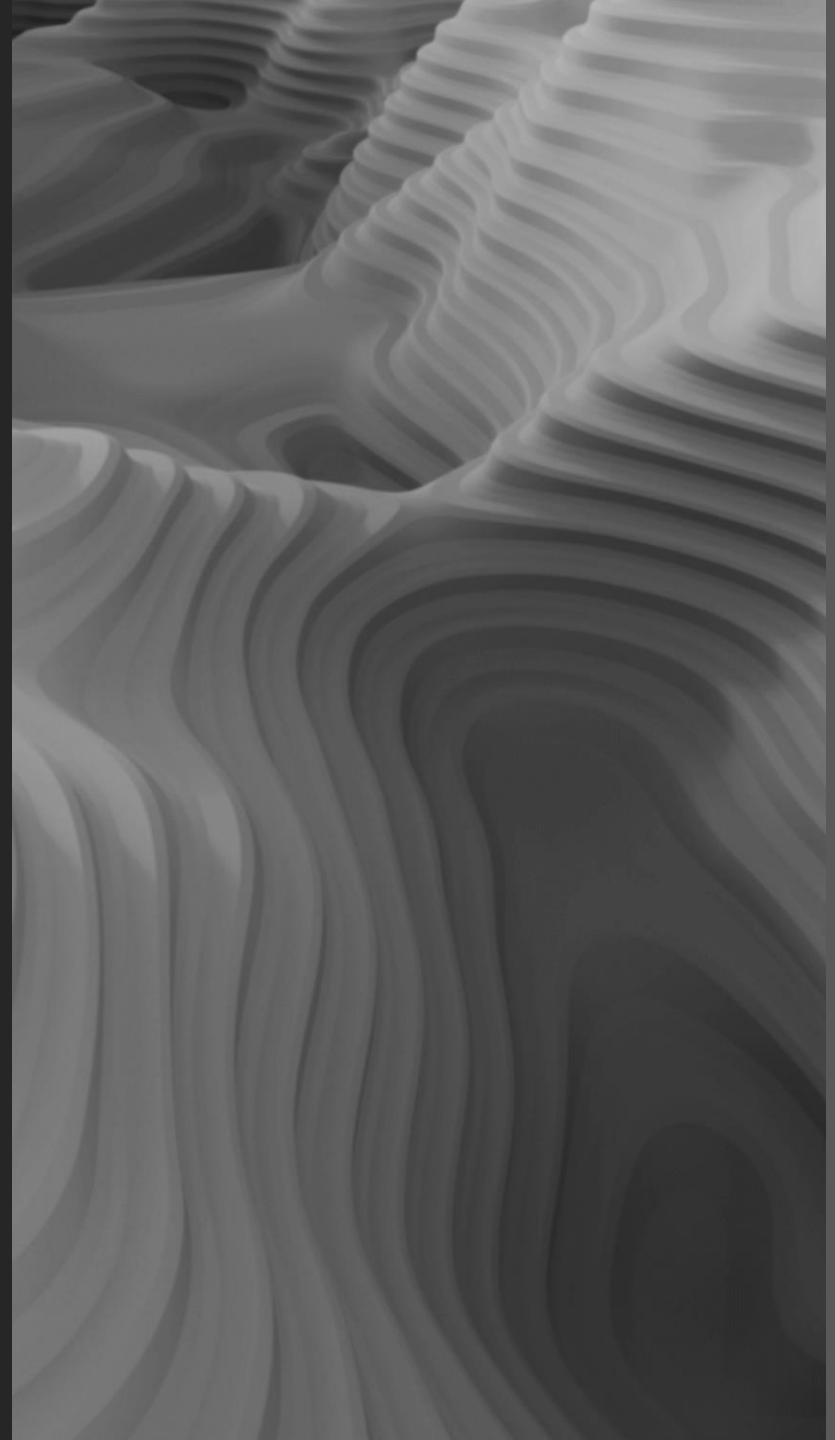


Geometric methods use geometrical constructions and properties of the known points to calculate the position of the observer. These methods are based on the use of geometric and graphic principles to solve the problem.

- Collins
- McGillem
- Cohen and Koss
- Esteves et al.
- Tsukiyama
- Font-Llagunes
- Ligas
- Pierlot et al.
- Cassini

Iterative

Tradicional Methods

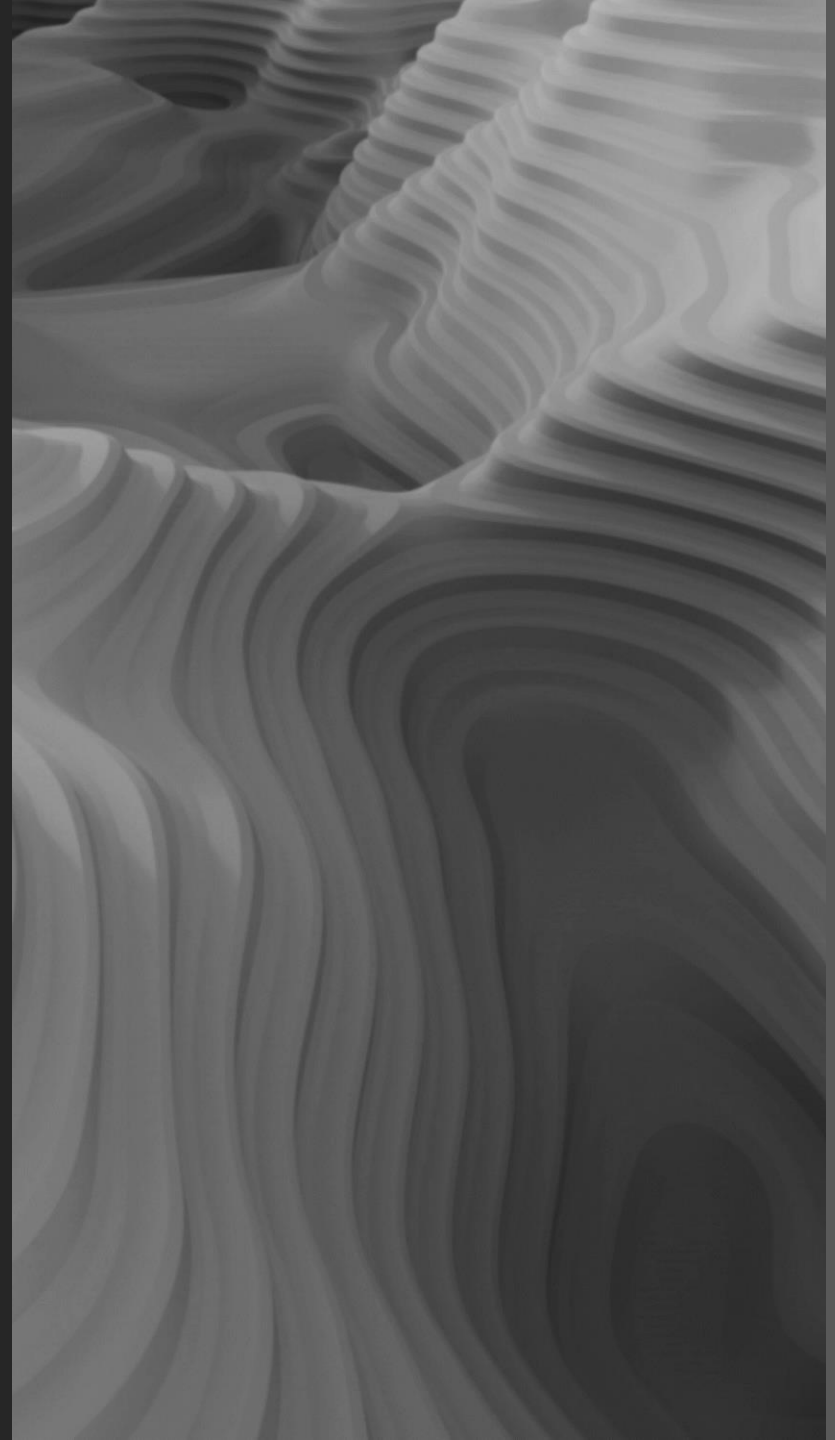


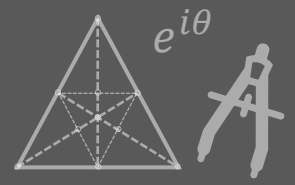
Iterative methods use iterative algorithms to converge to the observer position. These methods are based on the use of an initial estimate of the observer position, which is refined iteratively until convergence is achieved.

- Sanchiz et al.
- Dekov

Others

Tradicional Methods

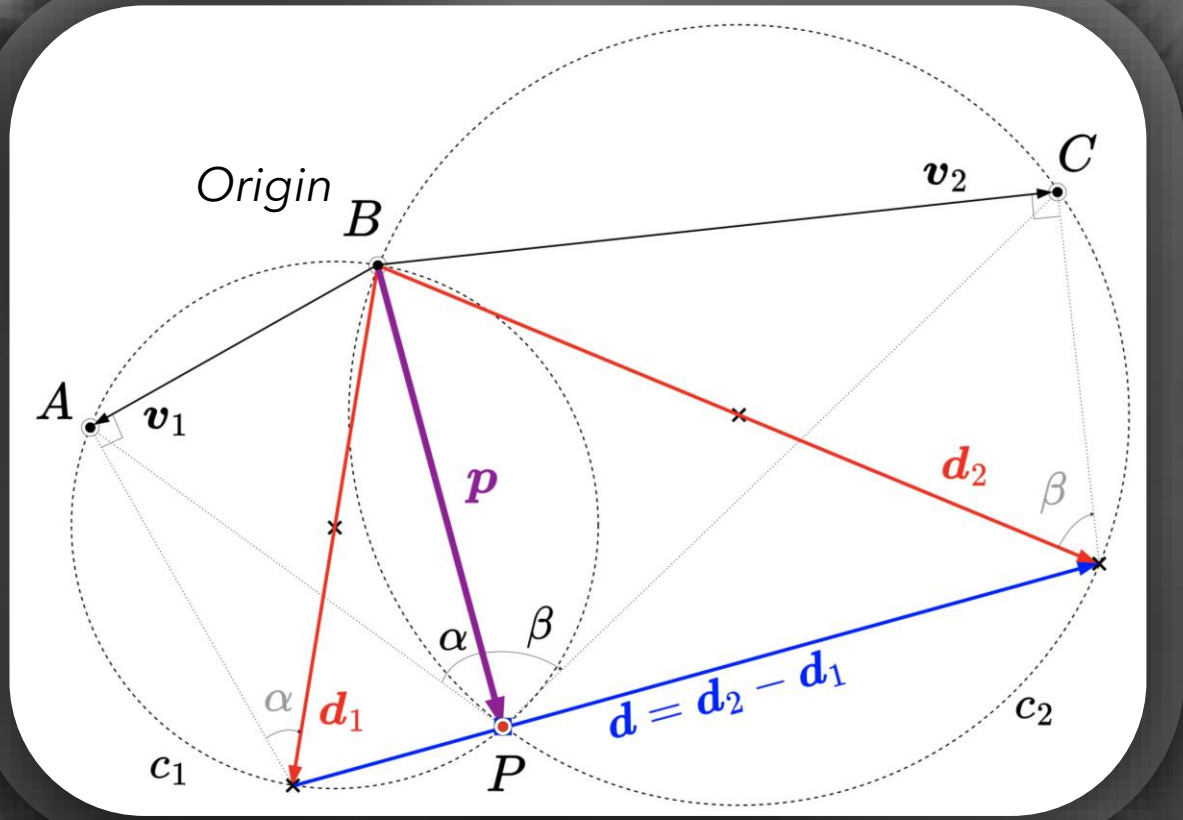




There are other methods that do not fit in with the above. For example, based on barycentric coordinates, complex numbers or graphical methods.

- Collins
- Willerding
- Tienstra

Resection using Geometric Algebra

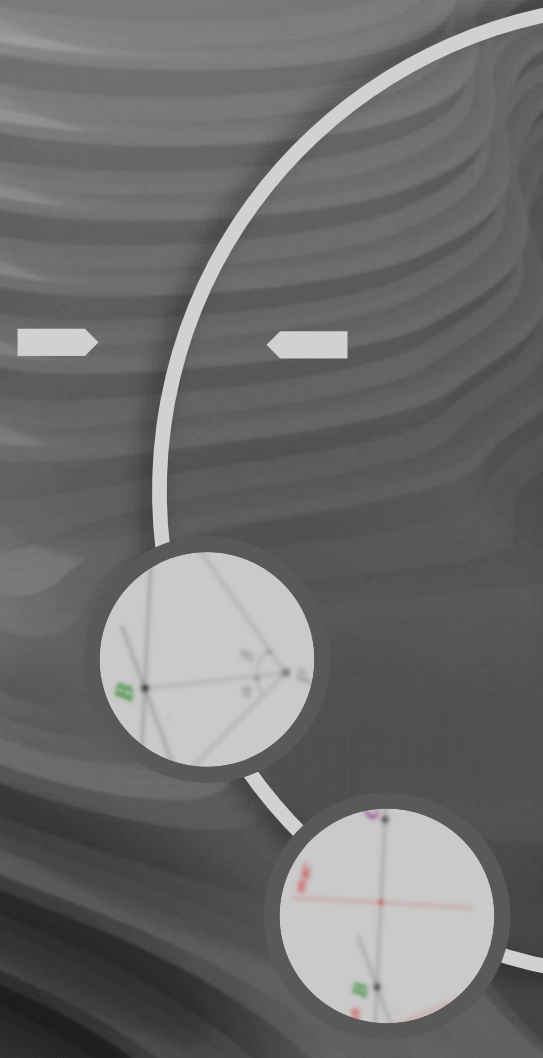


Resection using Geometric Algebra

Compass Ruler Algebra Methods

Cassini Construction

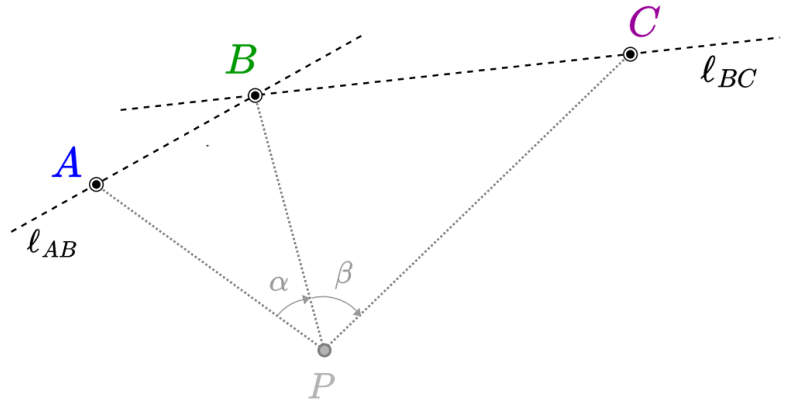
The Cassini method provides a solution to the resection problem by leveraging the inscribed angle theorem.

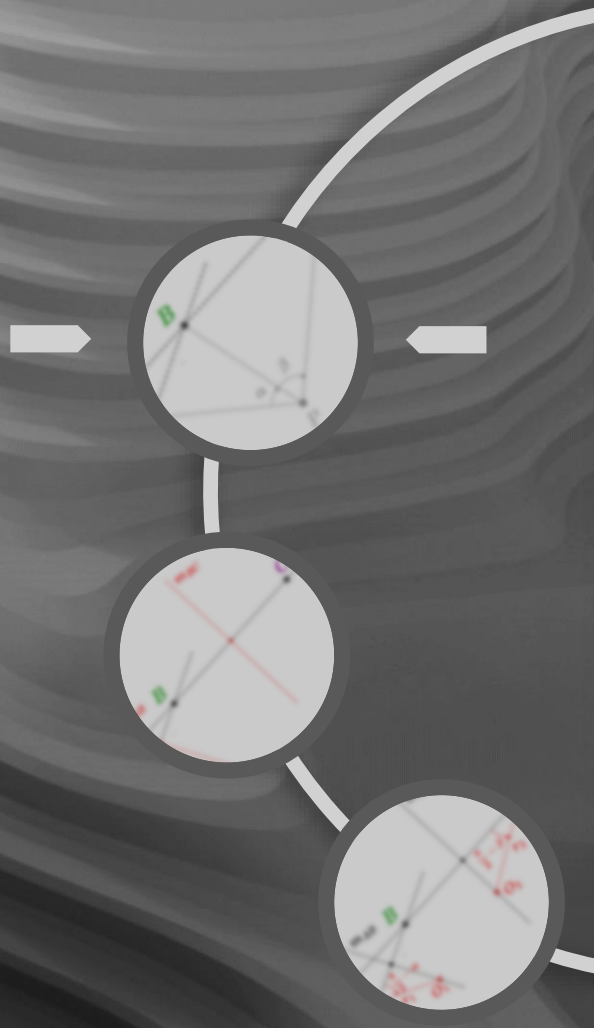


Resection using Geometric Algebra

Compass Ruler Algebra Methods

Cassini Construction

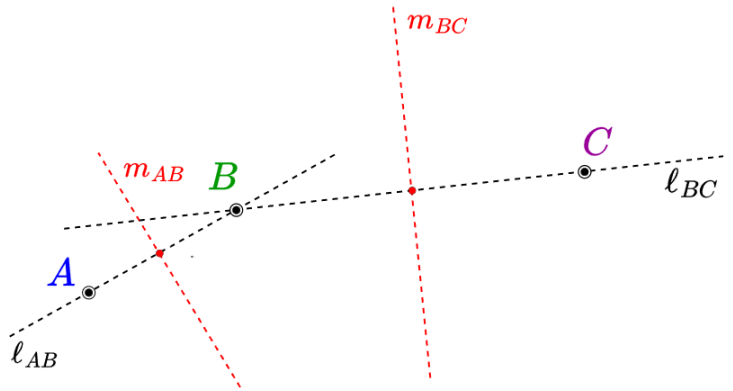


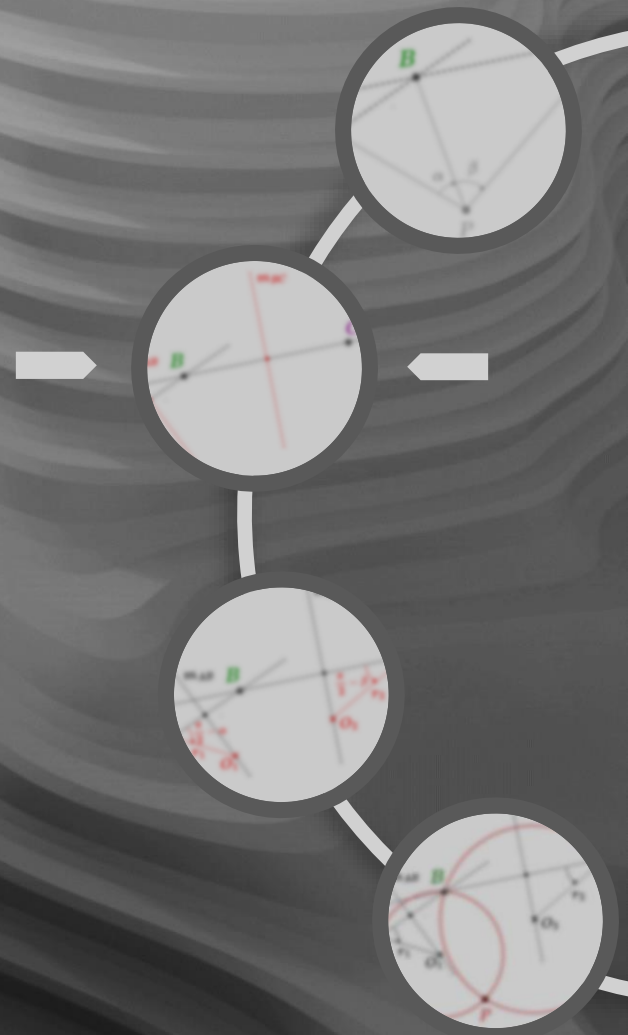


Resection using Geometric Algebra

Compass Ruler Algebra Methods

Cassini Construction

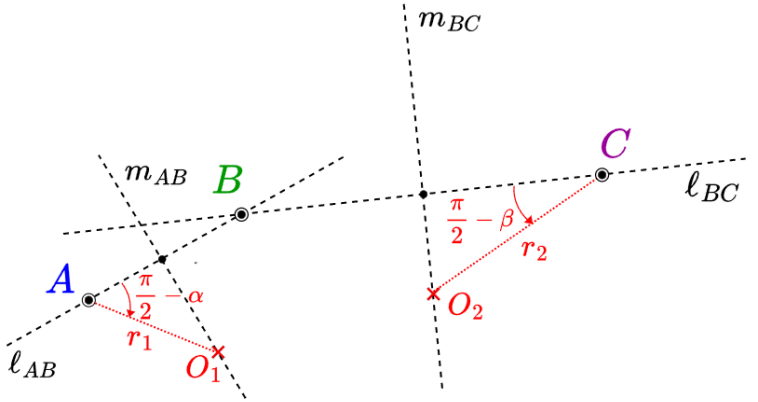


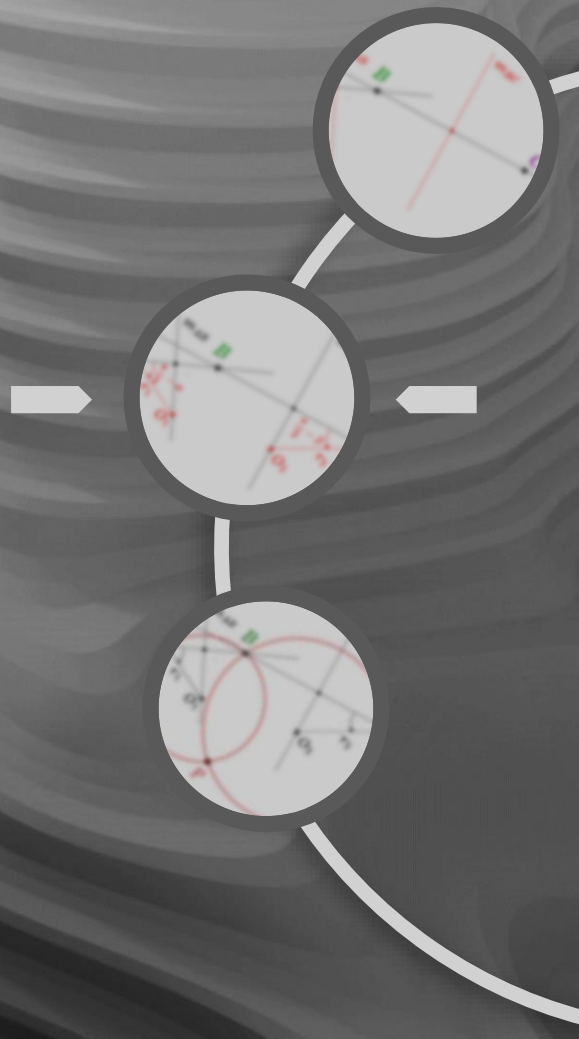


Resection using Geometric Algebra

Compass Ruler Algebra Methods

Cassini Construction

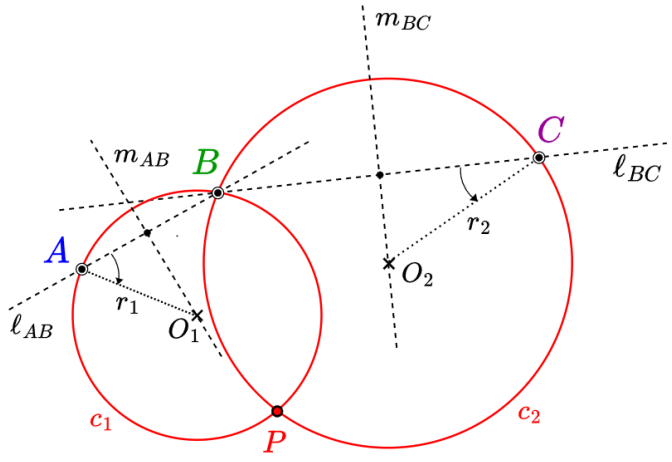


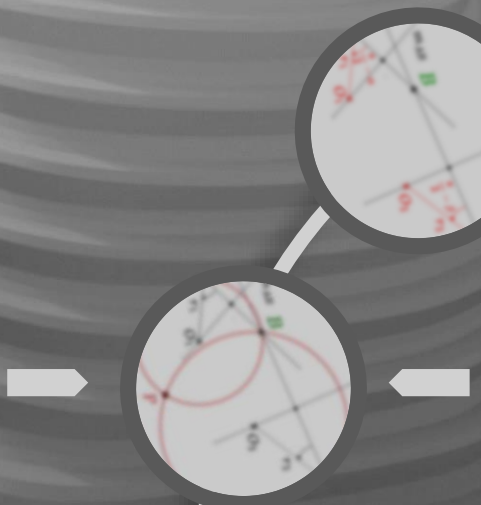


Resection using Geometric Algebra

Compass Ruler Algebra Methods

Cassini Construction





Resection using Geometric Algebra

Compass Ruler Algebra Methods

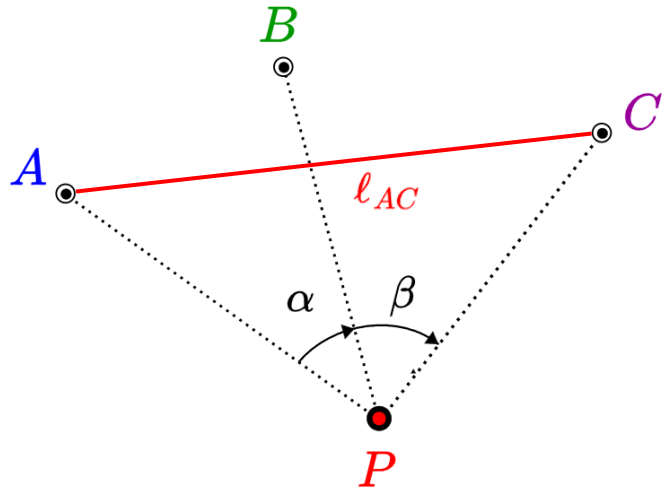
Collins Construction

The graphical method of Collins provides a solution using the intersection of the line passing through the point B and the so-called Collins auxiliary point E with the circle containing the points A, C and E.

Resection using Geometric Algebra

Compass Ruler Algebra Methods

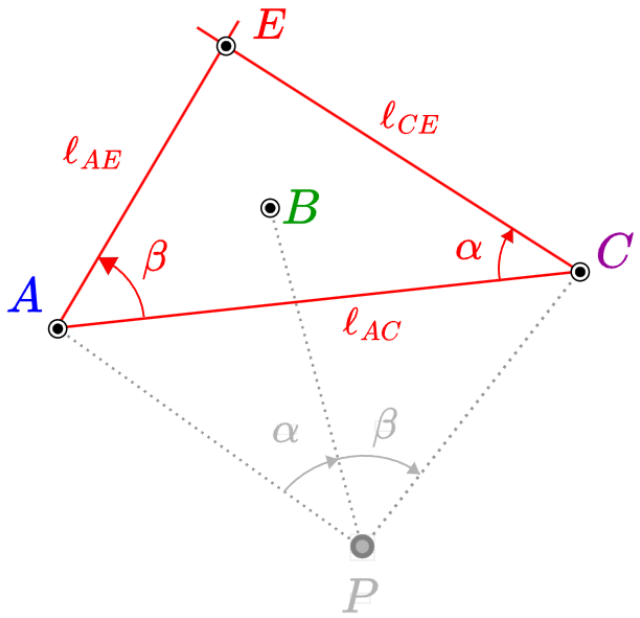
Collins Construction



Resection using Geometric Algebra

Compass Ruler Algebra Methods

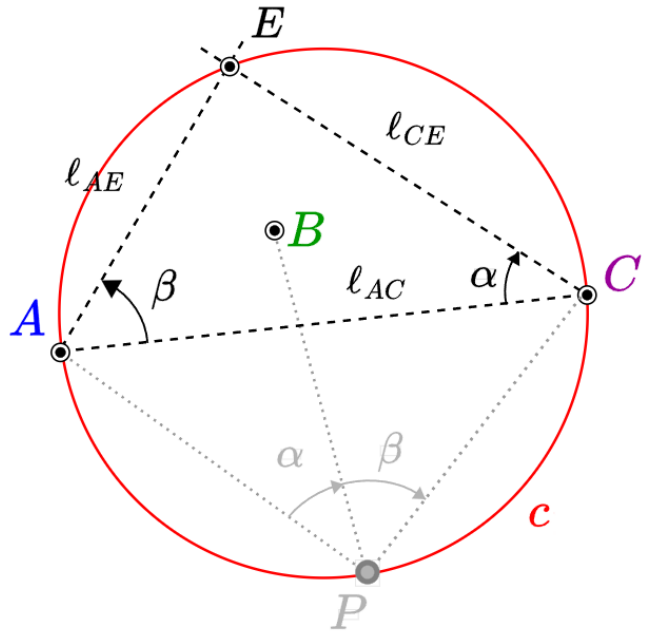
Collins Construction



Resection using Geometric Algebra

Compass Ruler Algebra Methods

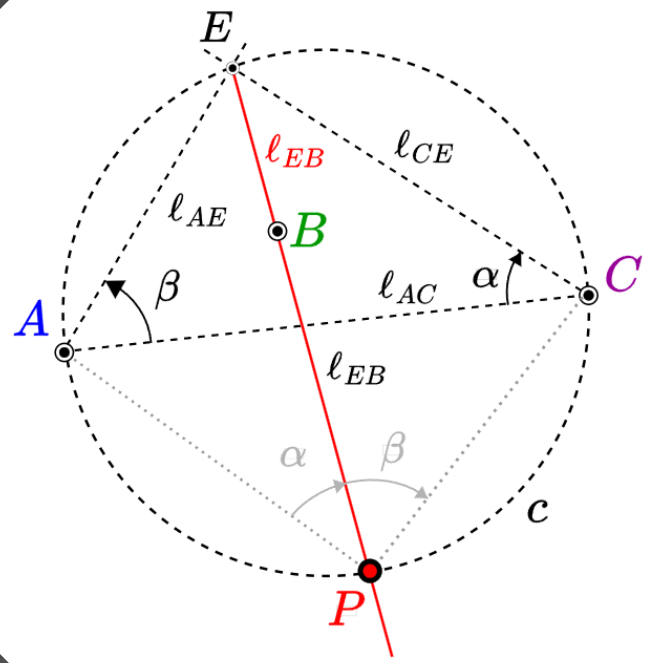
Collins Construction



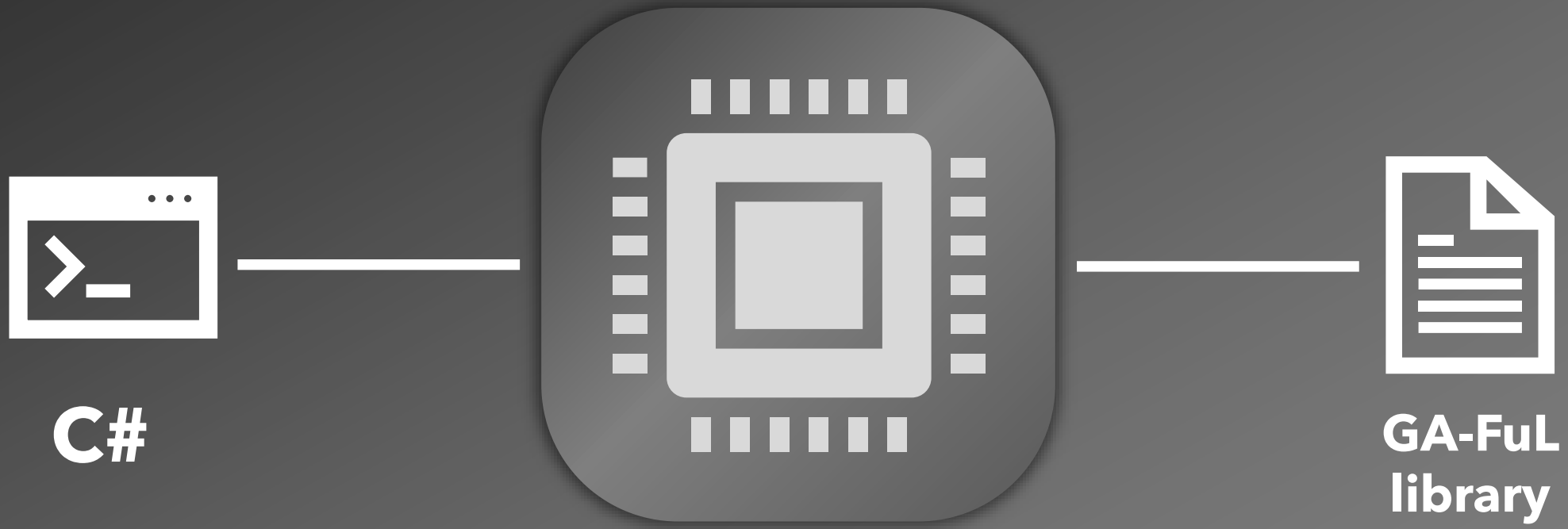
Resection using Geometric Algebra

Compass Ruler Algebra Methods

Collins Construction

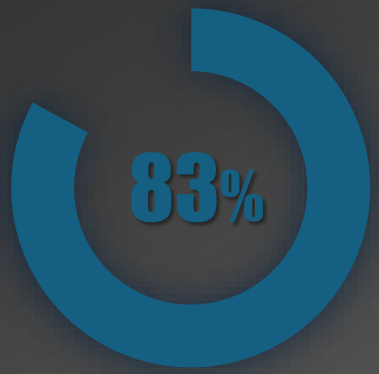


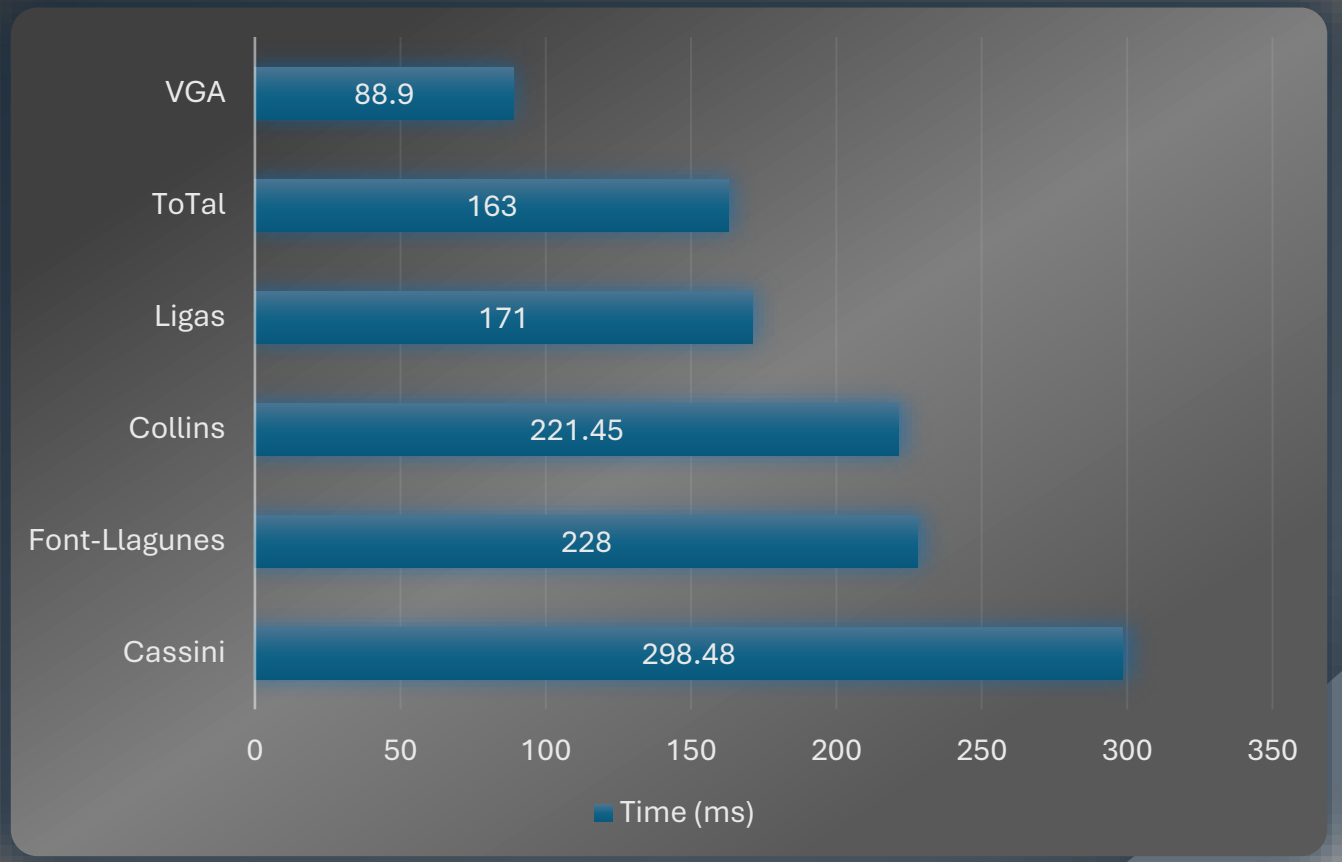
Benchmarks

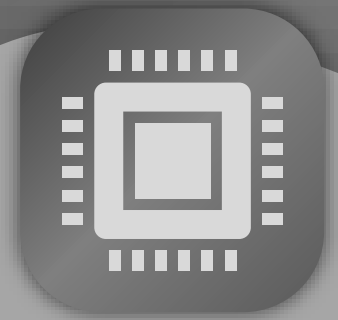


A comprehensive set of benchmarks was conducted, comparing our algorithms against top-efficient. Each algorithm was executed 10^6 times at random locations.

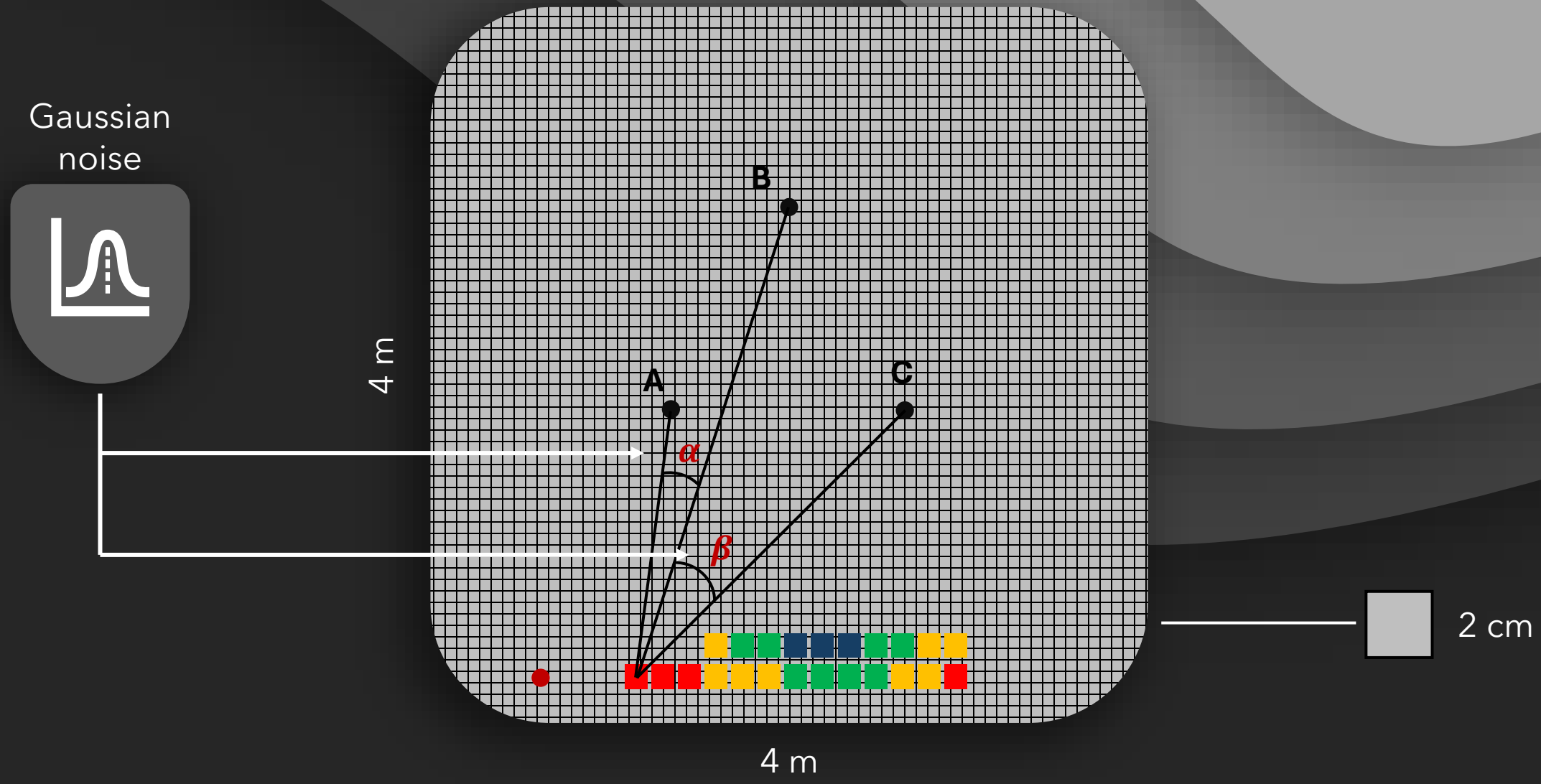
Benchmarks


83%

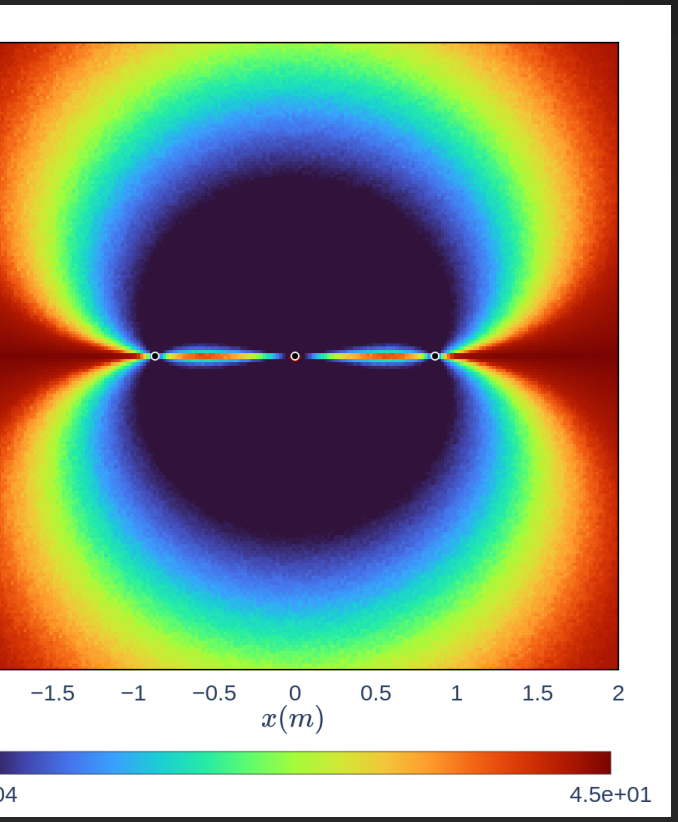

VGA
88.9
ToTal
163
Ligas
171
Collins
221.45
Font-Llagunes
228
Cassini
298.48
0
50
100
150
200
250
300
350
Time (ms)

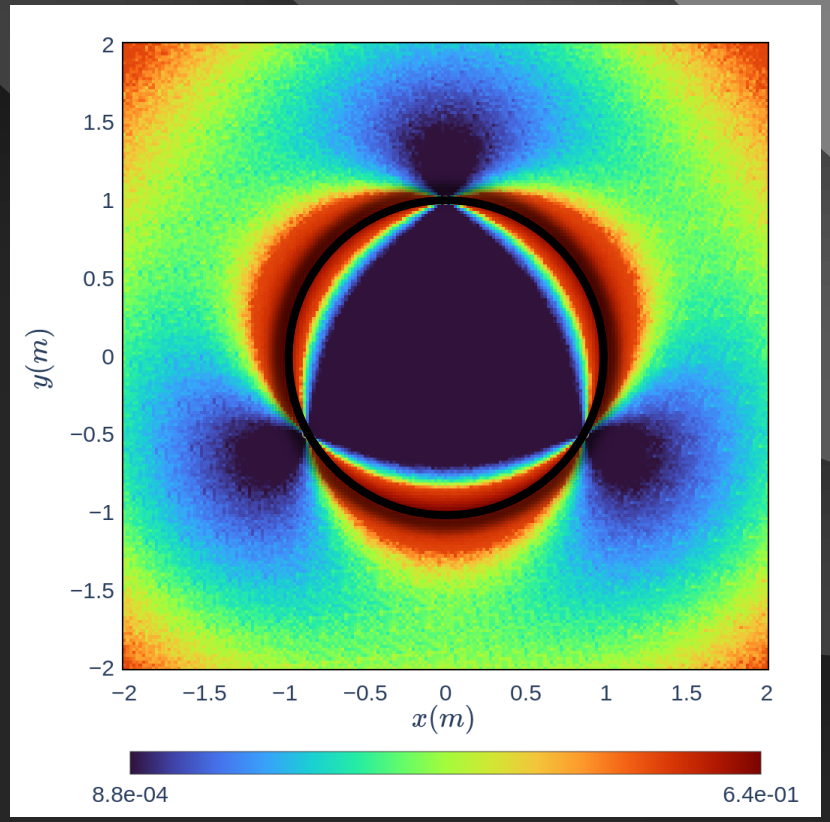


Uncertainty analysis



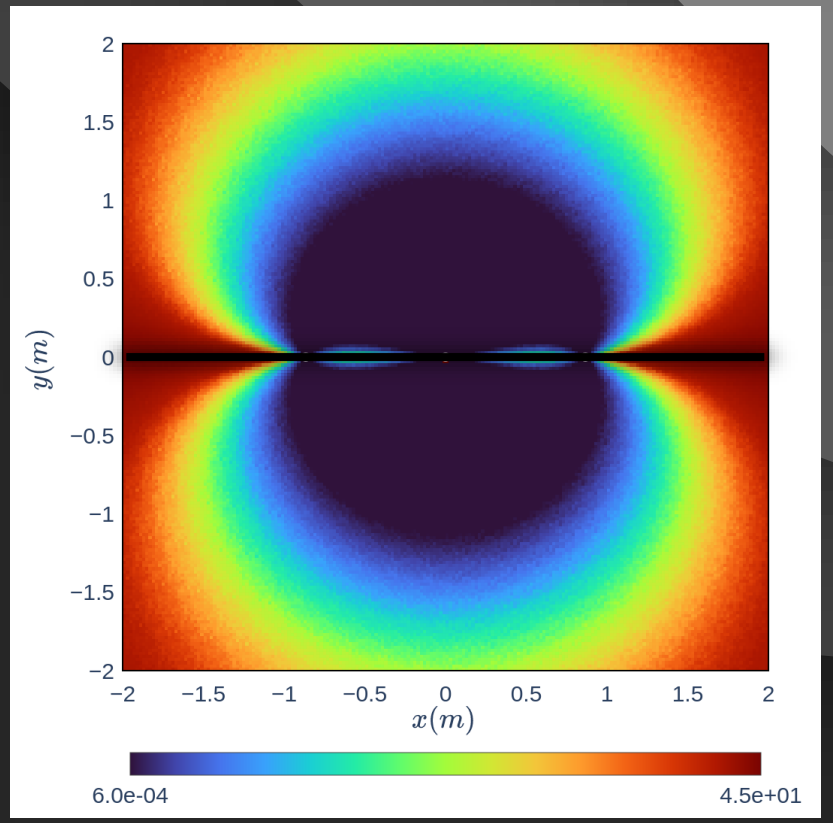
Uncertainty analysis

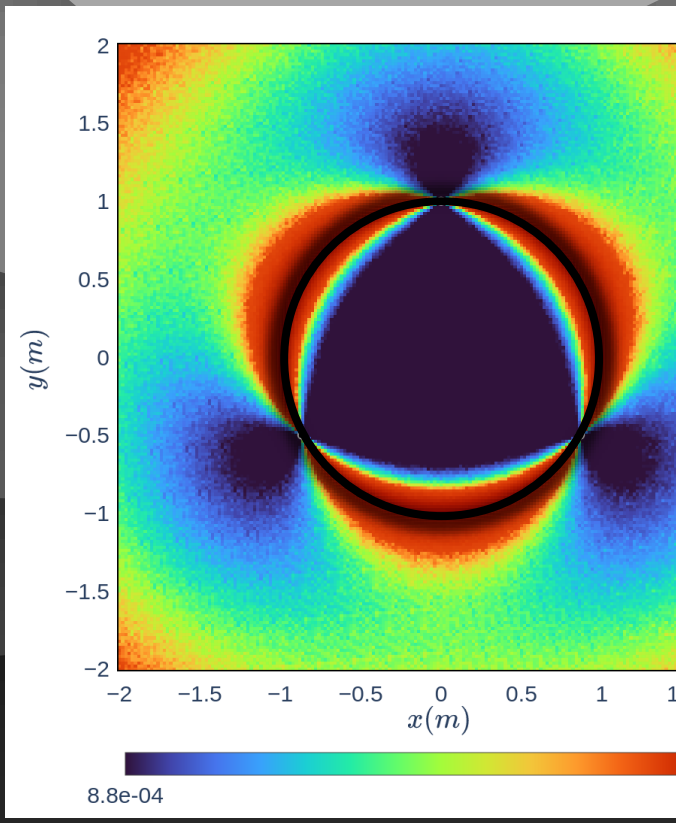




Equilateral Triangle

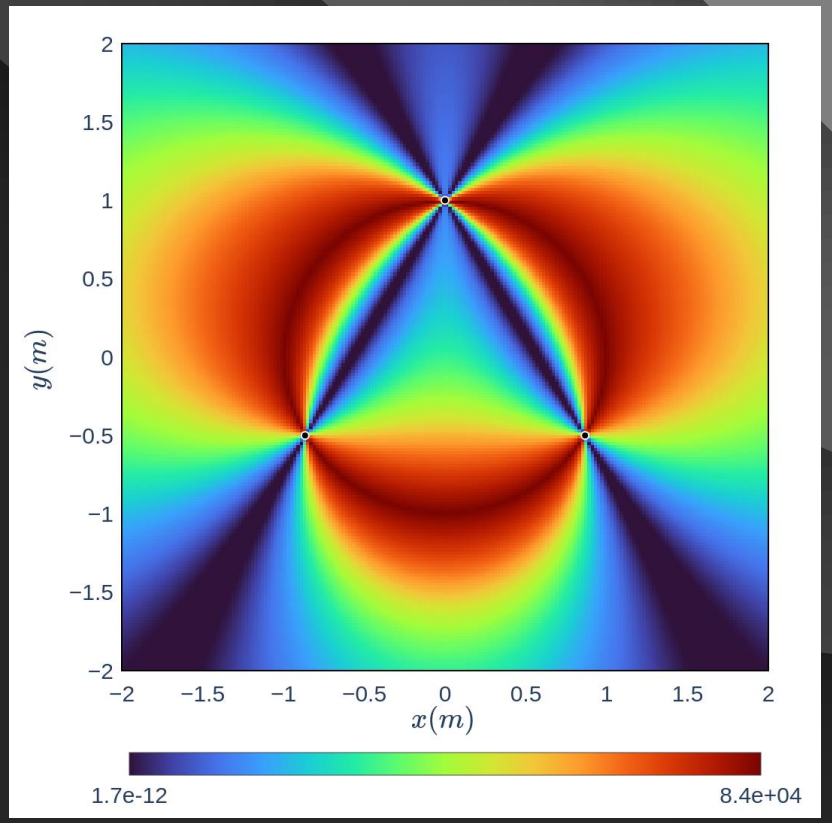
Uncertainty analysis

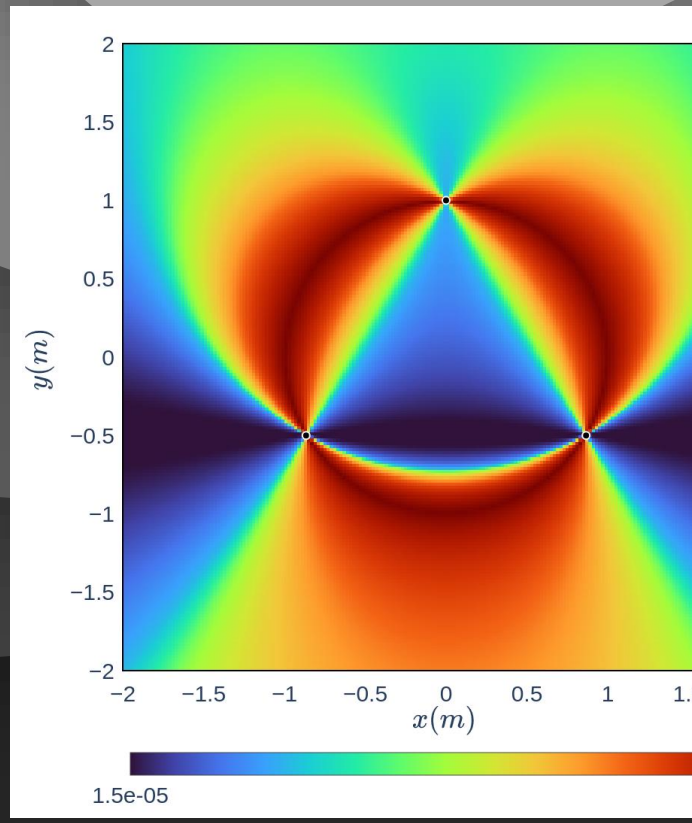




Colinear

Uncertainty analysis

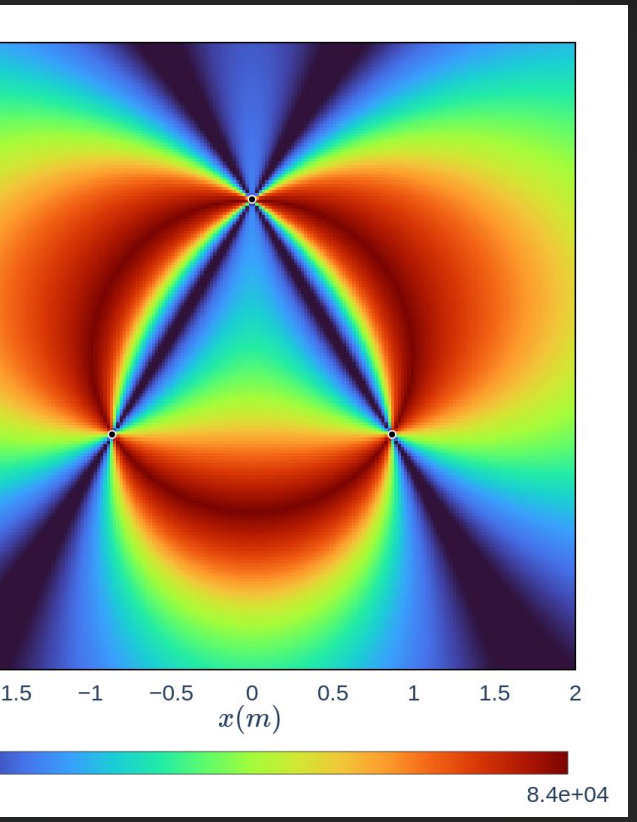


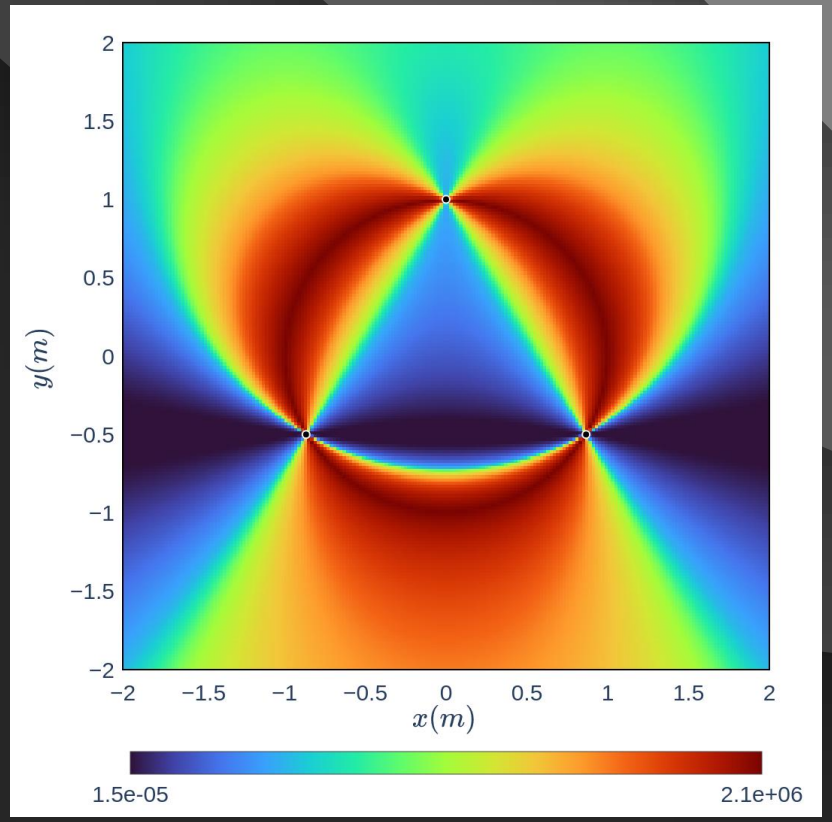


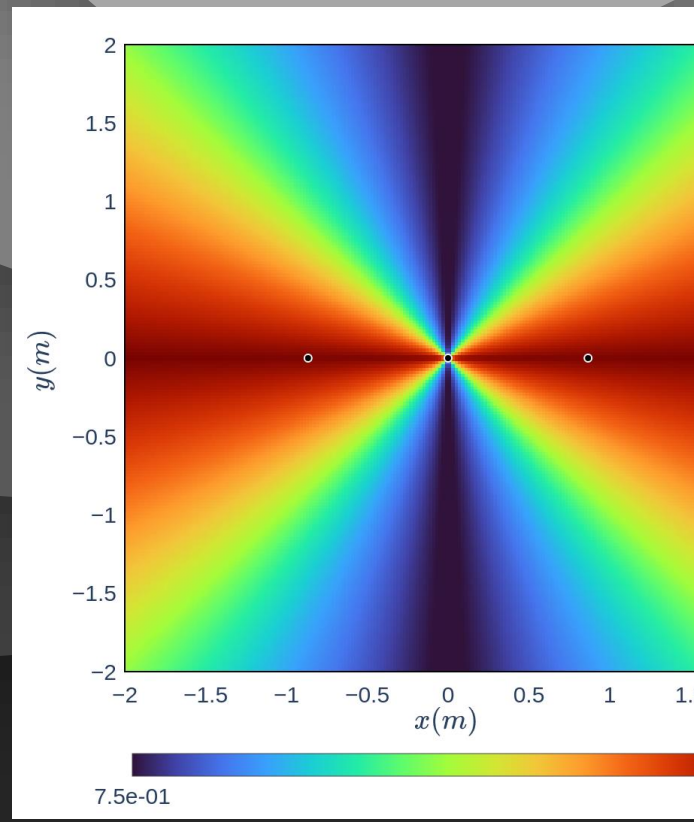
VGA Method

$D = d^2$

Uncertainty analysis



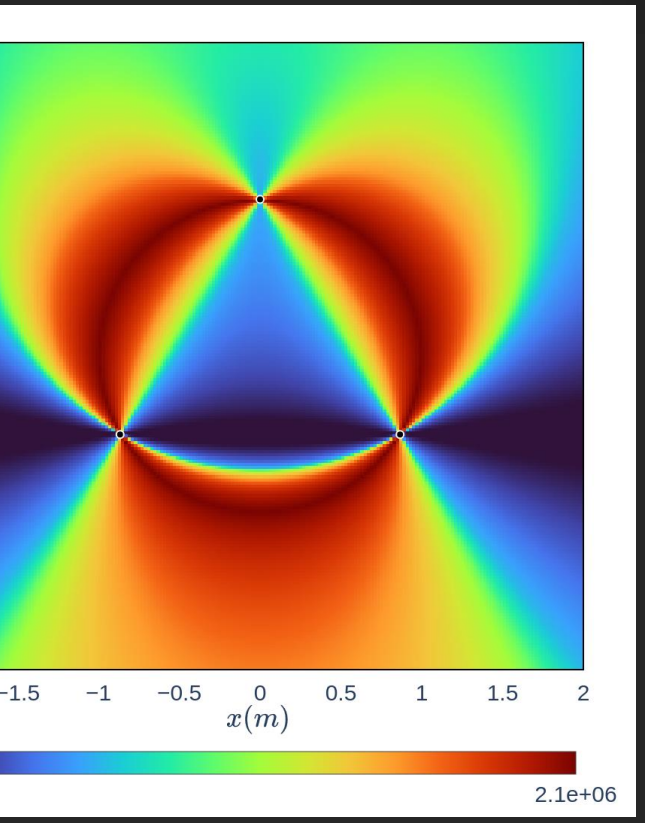


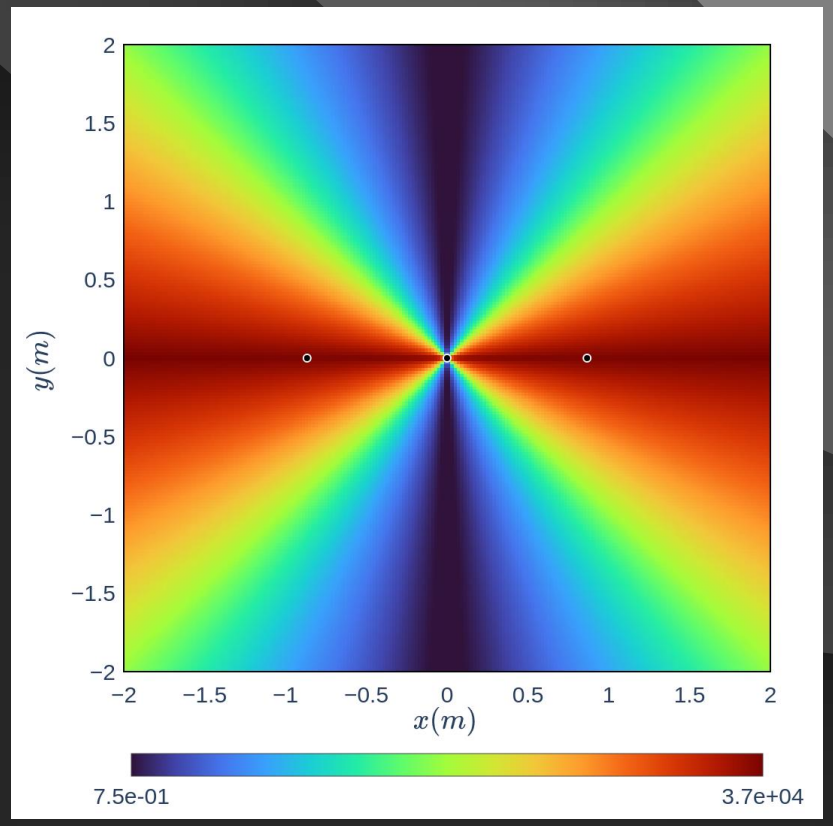


Collins Method

$D = -2(\boldsymbol{e} \cdot \boldsymbol{b})$

Uncertainty analysis





Cassini Method

$D = -2(\boldsymbol{o_1} \cdot \boldsymbol{o_2})$

CONCLUSIONS

This article introduces a novel method for solving the two-dimensional resection problem using conformal geometric algebra (CGA). The CGA approach simplifies the problem by avoiding complex algebraic manipulations and coordinate transformations, offering a more intuitive and efficient solution. Numerical simulations confirm the method's accuracy and efficiency, suggesting its potential applications in fields like surveying, geodesy, computer graphics, robotics, and navigation.

Thank you

Any questions?