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A UNIFYING "ANYCENTRIC" PINHOLE CAMERA MODEL FOR CALIBRATING ENTO-, TELE- AND HYPERCENTRIC LENSES

Simon Hartel, Christian Faber

University of Applied Sciences Landshut

In photogrammetry, camera calibration is often based on the well-known pinhole camera model [1] to describe the camera's object-sided chief rays. However, this model only applies to widely used *entocentric* lenses, whereas optical imaging and metrology systems often use *telecentric* or sometimes even more exotic *hypercentric* lenses. These lenses literally facilitate new perspectives for mastering otherwise complex tasks by differing fundamentally in their chief ray paths – which is why the pinhole model has to be modified or cannot be used at all, depending on the camera lens used. This contribution shows, how the pinhole model can be unified for all these three lens types using *Geometric Algebra* [2]. It contains a smooth transition between the perspectives and is thus capable of directly calibrating telecentricity errors for the important case of telecentric lenses.

1. INTRODUCTION

Camera calibration is a crucial task when working with camera-based optical imaging and metrology systems. The widely used *pinhole camera model* [1] has proven itself useful for this purpose, as it has a comparatively small number of parameters, yet accurately describes the camera's object-sided chief ray path. However, this model only applies to so-called *entocentric* lenses, whose chief rays converge towards the camera (see Figure 2). As a result, objects further away from the camera appear smaller in the camera image (see Figure 1). This lens type is by far the most common.

Nevertheless, other lens types are used especially in optical imaging and metrology to master otherwise quite complex tasks. *Telecentric* lenses provide the same image of an object regardless of the distance from the camera (see Figure 1), making them ideal for size measurement applications. This is achieved by parallel chief rays (see Figure 2). *Hypercentric* lenses provide a diverging chief ray path towards the camera (see Figure 2), resulting in simultaneous imaging of the top and the surrounding sides of an object (see Figure 1).

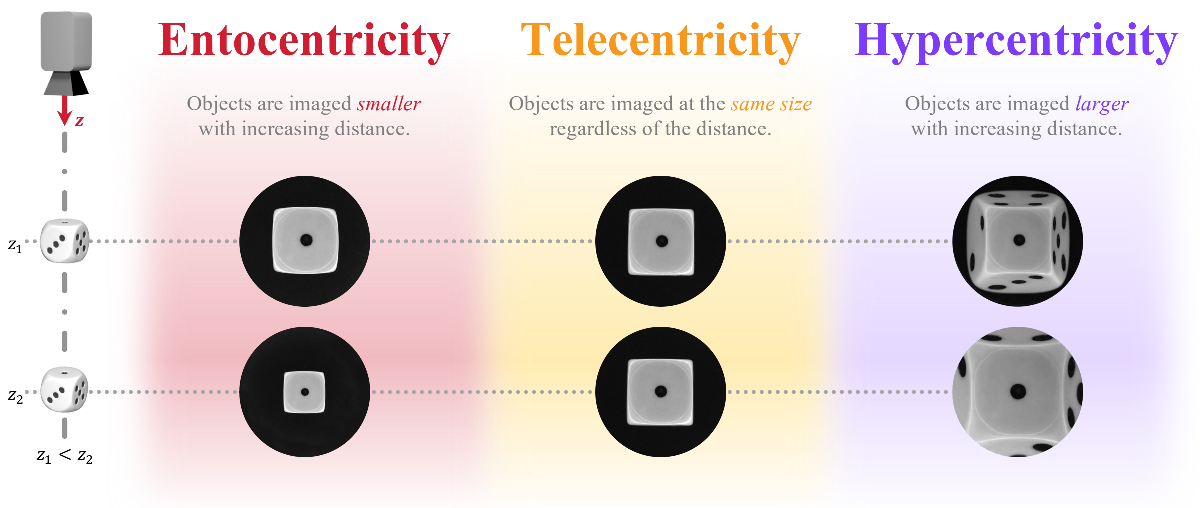

Figure 1: Exemplary images of a dice at different distances from the camera with entocentric, telecentric and hypercentric lenses.

2. STATE OF THE ART

Currently the fundamentally different object-sided chief ray paths of these three lens types cannot be described by one model. Instead, the model must be chosen according to the lens on hand (see Figure 2): The well-known pinhole model is used for entocentric lenses, defining the chief rays by image points in the image plane and a camera center behind the image plane where the chief rays meet (center of the entrance pupil of the optical system). The distance between the camera center and the image plane is called the *camera constant* c. For hypercentric lenses, the pinhole model must be modified by introducing a negative camera constant [3]. For telecentric lenses, no form of the pinhole model can be applied at present because the camera center resp. entrance pupil is at infinity – resulting in an *orthographic* rather than a *perspective projection*.

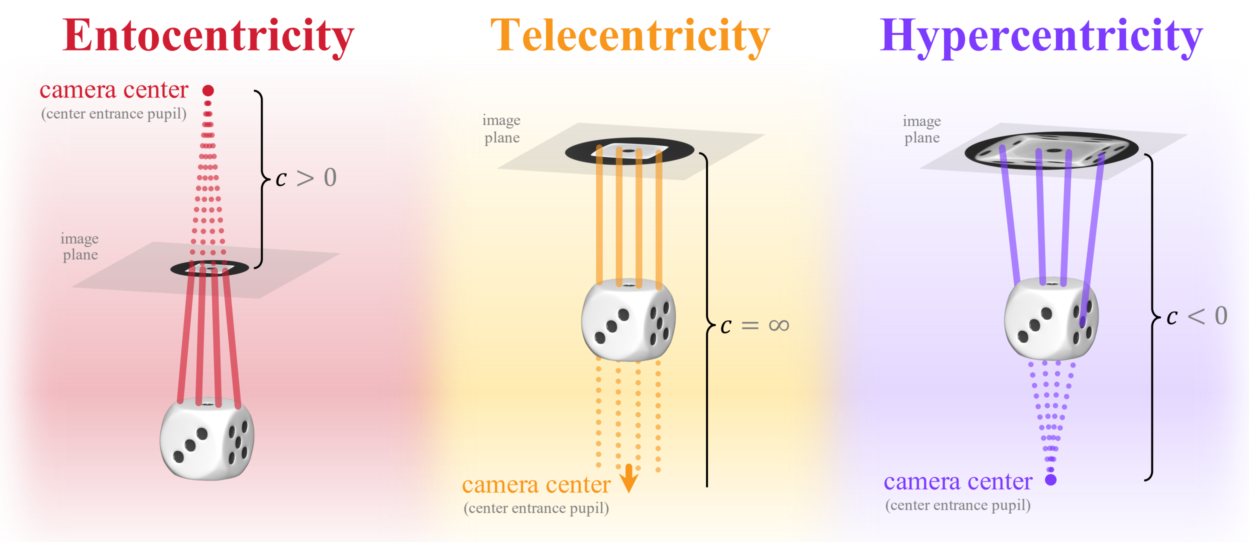

Figure 2: Object-sided chief ray path and camera model with camera constant c for entocentric, telecentric and hypercentric lenses.

This contribution shows how the widely used pinhole camera model can be generalized for *entocentric* (converging chief rays), *hypercentric* (diverging chief rays) and *telecentric* (parallel chief rays) lens types by using the unifying methods of *Plane-based* [4-7] (or similar: *Projective* [8]) *Geometric Algebra (PGA)* $\mathbb{R}^*_{n,0,1}$. By mainly taking advantage of the algebra’s inherently incorporated projective geometry to describe the intrinsic camera behavior, the proposed enhanced *“anycentric”* pinhole model contains a smooth transition between perspective projection and orthographic projection, making it well suited for the optimization process during camera calibration, and resulting in a more complete and more general camera model without greatly increasing its complexity. Furthermore, due to the smooth transition between the perspectives, this camera model provides a highly practical way to directly calibrate telecentricity errors of telecentric lenses.

Note that the following will focus on the projection step of the camera model, but it can easily be extended by additional common calibration parameters like *pixel skew*, *principal point position, distortion* (e.g. [9]) or *extrinsic* camera pose (as *motor*). The model does *not* aim to describe the focusing properties of the imaging system, which can be achieved by implementing the full paraxial ray path for arbitrary rays within the aperture of a given field point – this is done for example in [10] and [11]. It instead describes the (simplified) system behavior of the camera in a concise manner by only modelling the *chief rays* on the object side (assuming an infinite depth of field), as is the standard procedure for camera calibration – but in a unifying manner for ento-, tele- and hypercentric lenses. In [12] the so-called *inversion camera model* was introduced, demonstrating that the classical pinhole model can alternatively be implemented in terms of a circle inversion, making the model amenable to computations within the *Conformal Geometric Algebra* [13]. By then varying the position and size of the inverting circle, it is shown that both a special form of distortion (as in [14]) and the case of a catadioptric camera for the application of omnidirectional viewing (as in [15]) can be incorporated. While the telecentric case can in principle already be represented by these models as well (by deliberately choosing the camera center as an ideal point resp. point at infinity – a possibility that is however not explicitly addressed), they do not contain a smooth transition between the different perspectives. The concept presented below allowing for such a smooth transition with only one calibration parameter may also be adapted accordingly for these models. The formulation of the anycentric camera pinhole model proposed here aims to be minimal, hence PGA was chosen.

3. ANYCENTRIC PINHOLE CAMERA MODEL

The following proposed *"anycentric"* camera model (see Figure 3) is built in *Plane-based Geometric Algebra (PGA)* [4-7] $\mathbb{R}^*_{n,0,1}$ with $e_i^2 = 1$ for $i = 1,2,\dots,n$ and $e_0^2 = 0$, describing the *Euclidean space* with dimension n. Constructing the model can be broken down into the following three steps. The camera center[1] is called $\boldsymbol{c}$, the *Hodge duality operator* [7] is denoted as "$*$".

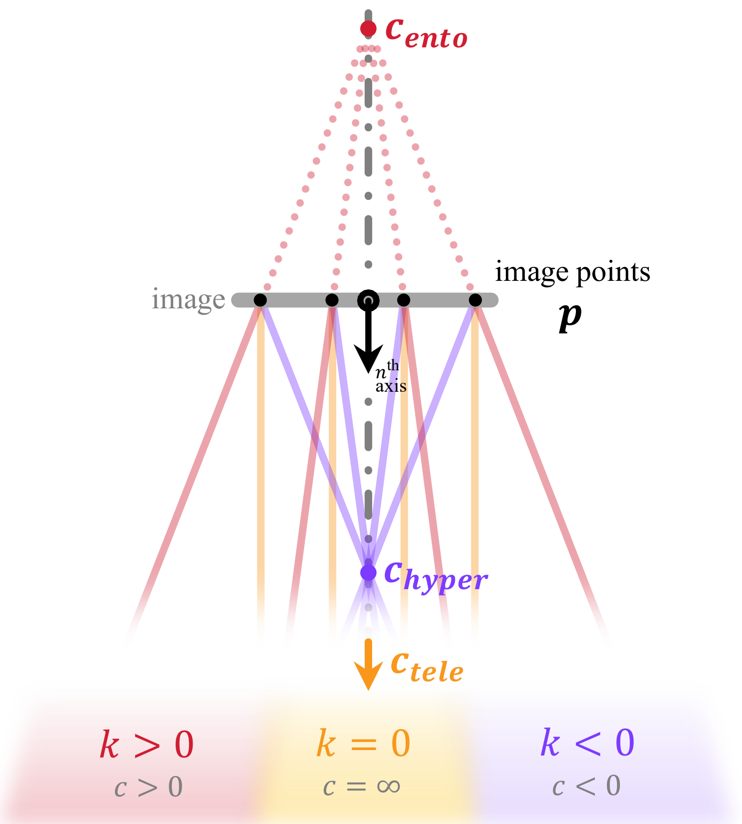

Figure 3: Anycentric camera model with smooth transition between ento-, tele- and hypercentric perspective.

[1] Mind the deliberately subtle difference between the camera center $\boldsymbol{c}$ (multivector, printed in bold) and the classical camera constant c (scalar).

1. *Coordinate system*:
   Although Geometric Algebra is generally coordinate-free, the first step is to introduce a coordinate system to be able to parameterize the model concisely. Classically, the origin of the camera coordinate system is placed in the camera center. However, since it should be possible to move the camera center to infinity (telecentric case), it is preferred to place it into the image with the "last" (n^{th}) axis corresponding to the optical axis of the camera. Of course, for the ento- and hypercentric case, it can be easily traced back to the classical model by the translation $\sqrt{\boldsymbol{c}/\boldsymbol{e}_{\mathbf{0}}^{*}}$.
2. *Camera center* $\boldsymbol{c}$:
   The camera center is described by

$$\boldsymbol{c} = \boldsymbol{e}_{\boldsymbol{n}}^{*} - k\boldsymbol{e}_{\mathbf{0}}^{*} \quad \text{with} \quad k := {}^{1}/_{c} \tag{1}$$

   utilizing the projective geometry incorporated in the algebra. The new calibration parameter k (reciprocal camera constant) replaces the classical camera constant. Due to this formulation, the camera center can *smoothly* be moved (without case differentiation) to infinity. Thus, the model has a smooth transition between perspective and orthographic projection by varying only one calibration parameter (k) – resulting in an entocentric pinhole model for $k > 0$ (resp. $c > 0$ like in [1]), a "*telecentric pinhole model"* for $k = 0$ (resp. $c = \infty$) and a hypercentric pinhole model for $k < 0$ (resp. $c < 0$ like in [3])[2].
3. *Camera chief rays*:
   Given image points $\boldsymbol{p}$ that (of course) lie in the image, the chief rays can finally be constructed by the *regressive product*

$$\boldsymbol{c} \vee \boldsymbol{p}\,. \tag{2}$$

The proposed model is dimension-agnostic: It models a camera in n-dimensional space capturing a $(n-1)$-dimensional image, e.g. a *line camera* for $n = 2$ or the standard areal camera for $n = 3$. In order to be able to distinguish between physical light rays ($\boldsymbol{c} \vee \boldsymbol{p}$) and "lines of sight" ($\boldsymbol{p} \vee \boldsymbol{c}$), a fully oriented version [16] of PGA has been implemented (using both orientation-preserving *undualization* [7] and *sandwich product* [17]), since the camera center $\boldsymbol{c}$ flips its orientation in the entocentric case compared to the tele- and hypercentric cases. It is important to point out that the proposed model has ambiguities (especially with poor starting values). For example, the model in the hypercentric configuration can also be used "entocentrically" if the object is further away than the camera center (which, however, is not even a shortcoming of the model, but mirrors actual physical system behavior), or the camera model can inadvertently be used "backwards" (the object lies in negative optical axis direction, which is physically not possible). However, these ambiguities are more or less a matter of interpretation. If desired, they can easily be detected and reduced to the intuitive case by *multireflections* within the algebra after the calibration.

4. SUMMARY

This contribution demonstrates how the widely used pinhole camera model can be unified in a dimension-agnostic way for ento-, tele- and hypercentric lenses without greatly increasing its complexity using (oriented) PGA. The proposed implementation incorporates a smooth transition between these three perspectives by only varying one calibration parameter and is thus capable of directly calibrating telecentricity errors for the important case of telecentric lenses.

[2] The "missing" case $k = \infty$ ($c = 0$) makes no sense optically, as ports can never coincide with pupils.

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