Gyromagnetics of the electron clock

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Abstract. The Dirac Equation is presented as a complete theory of the electron as a gyromagnetic particle clock with precise physical interpretation for all degrees of freedom. The electron is modeled as a point charge with toroidal **zitter** and variable **zilch**. That is to say, the charge oscillates at the speed of light on a torus centered on a circular orbit around its center of mass $\mathbf{z} = \mathbf{z}(t)$, with an axis at a variable angle $\beta = \beta(\mathbf{z}(t))$ with respect to its spin vector $\mathbf{s} = \mathbf{s}(t)$.

The Dirac Wave Function $\Psi = \Psi(ct + \mathbf{x})$ has a unique factorization $\Psi = (\rho e^{i\beta})^{\frac{1}{2}} U$, where $U = U(ct + \mathbf{x})$ is a spatial rotor with magnetic degrees of freedom, and the quantity $\Psi\widetilde{\Psi} = \rho e^{i\beta}$ specifies an embedding of electron paths in the Vacuum, where the **zilch function** $\beta = \beta(ct + \mathbf{x} - \mathbf{z}(t))$ is a measure of electron energy density. It culminates in a new synthesis of Dirac Electron Theory with Maxwell's Electrodynamics by identifying **zilch** as a common factor that binds them together.

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I. INTRODUCTION

This is a report on a long term research project to incorporate de Broglie's idea of the electron as a particle clock into the interpretation of Dirac's equation and study its implications. The project was started in 1966 by the book *Space Time Algebra* [1], which laid out the essential mathematical insights that have guided robust development to this day. It aims to revitalize de Broglie's idea of an electron clock by giving it a central role in physical interpretation of the Dirac wave function. In particular, it aims for insight into structure of the wave function and fibrations of particle paths it determines. This opens up new questions about physical interpretation.

We begin in Section II with a synopsis of *Spacetime Algebra* (STA), which is an essential tool in all that follows, especially because it provides a geometric interpretation for the role of complex numbers in quantum mechanics.

Section III is the theoretical core of this report. It applies STA in a review of *Real Dirac theory* and presents a new analysis of its physical interpretation in terms of local observables. That provides the context to introduce de Broglie's clock as central to physical interpretation of the Dirac wave function.

Section IV presents a complete formulation and analysis of conservation laws in Dirac theory in terms of local observables, including details that are generally overlooked in literature and the problematic role of the parameter β in the Dirac wave function. That establishes a foundation for a new synthesis of electron theory with electrodynamics developed below.

Section V discusses a new approach to Born's statistical interpretation of the Dirac wave function dubbed *Born-Dirac* theory. It includes a relativistic extension of de Broglie-Bohm Pilot Wave theory to interpret the Dirac wave function as describing a fibration (or ensemble) of possible particle paths. Spin dependence of the so-called *Quantum potential* is made explicit and generalized. More details are given in references cited in this Section, which have been carefully selected for quality and compatibility with the present approach to Dirac theory. They offer a rich store of ideas and results worth exploring both theoretically and experimentally.

Section V closes with a sudden realization that β can be given a clear physical interpretation as a *chirality parameter*. That proves to be a game changer! It enables in Section VI a complete and coherent formulation and physical interpretation for all parameters in the Dirac equation without exception or approximation. Let's call that synthesis the *Zitter Particle Model* (ZPM), because it incorporates Schrödinger's concept of *Zitterbewegung* into a particle model of the electron with *lightlike helical motion called* **Zitter**. We identify it with the physical mechanism in de Broglie's particle clock.

Section VI presents the *Zitter Dirac Equation* as the centerpiece of the ZPM, because it presents a reformulation of the Dirac equation in terms of local observables, which are open to direct physical interpretation and analysis. That provides a unified framework for any application of the Dirac theory. In particular, it shows how the *Canonical momentum* provides quantum numbers for any quantized electron state. And it supports derivation of an *Extended Lorentz Force* which explains how photons are emitted when electrons are accelerated.

Section VII provides the capstone for a *Unified Maxwell-Dirac electron theory* by recognizing that the Zilch angle in the Dirac wave function can be identified with the Zilch parameter in an invariant decomposition of the electromagnetic field.

New elementary solutions of the Dirac equation are identified as zilch signals in the vacuum and proposed as components of Maxwell's famous Displacement current! Then, refining arguments by de Broglie, an alternative solution is proposed to explain how photons are produced by accelerated electrons.

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Section VIII presents the electron as a point singularity with mass stored as energy in the Vacuum. In agreement with Dirac [2], this calls for revitalization of the classical concept of Aether as a substrate for the Vacuum. We identify that substrate as none other than the scalar-valued **Zilch** function that spans all of spacetime, wherein electron (and positron) paths are embedded as singularities along with zilch signals. Details will be discussed at greater length in a subsequent paper.

Section IX reformulates the **Zitter-Dirac equation** as an ordinary differential equation for a point particle embedded in the zilch vacuum field described by Maxwell's equation. This provides a framework for solving many problems in physics, including atomic structure, radiative reaction, electron and photon diffraction, and gravo-magnetic interaction.

We conclude in Section X with a coda arguing for a realist interpretation of particle paths embedded in the Dirac wave function. With a salute to Roget Boudet as a paragon of intellectual probity in the mathematical sciences.

II. SPACETIME ALGEBRA

Spacetime Algebra (STA) plays an essential role in the formulation and analysis of electron theory in this paper. Since thorough expositions of STA are available in many places [1, 3, 4], a brief description will suffice here, mainly to establish notations and define terms.

STA is an associative algebra generated by spacetime vectors with the property that the square of any vector is a (real) scalar. Thus for any vector a we can write

$$a^2 = aa = \varepsilon|a|^2\,, \tag{1}$$

where ε is the *signature* of a and $|a|$ is a positive scalar. As usual, we say that a is *timelike*, *lightlike* or *spacelike* if its signature is positive ($\varepsilon = 1$), null ($\varepsilon = 0$), or negative ($\varepsilon = -1$).

From the *geometric product* ab of two vectors it is convenient to define two other products. The *inner product* $a \cdot b$ is defined by

$$a \cdot b = \tfrac{1}{2}(ab + ba) = b \cdot a\,, \tag{2}$$

while the *outer product* $a \wedge b$ is defined by

$$a \wedge b = \tfrac{1}{2}(ab - ba) = -b \wedge a\,. \tag{3}$$

The three products are therefore related by

$$ab = a \cdot b + a \wedge b\,. \tag{4}$$

This can be regarded as a decomposition of the product ab into symmetric and skewsymmetric parts, or alternatively, into scalar and bivector parts.

For physicists unfamiliar with STA, it will be helpful to note its isomorphism to Dirac algebra over the reals. To that end, let $\{\gamma_\mu;\ 0, 1, 2, 3\}$ be a *right-handed orthonormal frame* of vectors with γ_0 in the forward light cone. The symbols γ_μ have been selected to emphasize direct correspondence with Dirac's γ-matrices. In accordance with (2), the components $\mathrm{g}_{\mu\nu}$ of the metric tensor are given by

$$\mathrm{g}_{\mu\nu} = \gamma_\mu \cdot \gamma_\nu = \tfrac{1}{2}(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu)\,. \tag{5}$$

This will be recognized as isomorphic to a famous formula of Dirac's. Of course, the difference here is that the γ_μ are vectors rather than matrices.

The unit pseudoscalar i for spacetime is related to the frame $\{\gamma_\nu\}$ by the equation

$$i = \gamma_0\gamma_1\gamma_2\gamma_3 = \gamma_0 \wedge \gamma_1 \wedge \gamma_2 \wedge \gamma_3\,. \tag{6}$$

It is readily verified from (6) that $i^2 = -1$, and the geometric product of i with any vector is anticommutative.

By multiplication the γ_μ generate a complete basis of k-vectors for STA, consisting of the $2^4 = 16$ linearly independent elements

$$1, \quad \gamma_\mu, \quad \gamma_\mu \wedge \gamma_\nu, \quad \gamma_\mu i, \quad i\,. \tag{7}$$

Obviously, this set corresponds to 16 base matrices for the Dirac algebra, with the pseudoscalar i corresponding to the Dirac matrix γ_5.

The entire spacetime algebra is obtained from linear combinations of basis k-vectors in (7). A generic element M of the STA, called a *multivector*, can thus be written in the *expanded form*

$$M = \alpha + a + F + bi + \beta i = \sum_{k=0}^{4}\langle M\rangle_k\,, \tag{8}$$

where α and β are scalars, a and b are vectors, and F is a bivector. This is a decomposition of M into its k-vector parts, with $k = 0, 1, 2, 3, 4$, where $\langle\ldots\rangle_k$ means "*k-vector part.*" Of course, $\langle M\rangle_0 = \alpha$, $\langle M\rangle_1 = a$, $\langle M\rangle_2 = F$, $\langle M\rangle_3 = bi$, $\langle M\rangle_4 = \beta i$. It is often convenient to drop the subscript on the scalar part, writing $\langle M\rangle = \langle M\rangle_0$.

We say that a k-vector is even (odd) if the integer k is even (odd). Accordingly, any multivector can be expressed as the sum of even and odd parts. A multivector is said to be "*even*" if its parts are even k-vectors. The even multivectors compose a subalgebra of the STA. We will be using the fact that spinors can be represented as even multivectors.

Computations are facilitated by the operation of *reversion*. For M in the expanded form (8) the *reverse* $\widetilde{M}$ can be defined by

$$\widetilde{M} = \alpha + a - F - bi + \beta i\,. \tag{9}$$

For arbitrary multivectors M and N

$$\widetilde{(MN)} = \widetilde{N}\widetilde{M}\,. \tag{10}$$

It is useful to extend the definitions of inner and outer products to multivectors of higher grade. Thus, for bivector F and vector a we can define inner and outer products

$$F \cdot a = \tfrac{1}{2}(Fa - aF), \tag{11}$$

$$F \wedge a = \tfrac{1}{2}(Fa + aF), \tag{12}$$

so that

$$Fa = F \cdot a + F \wedge a \tag{13}$$

expresses a decomposition of Fa into vector and pseudovector parts. For $F = b \wedge c$ it follows that

$$(b \wedge c) \cdot a = b(c \cdot a) - c(b \cdot a)\,. \tag{14}$$

Many other useful identities can be derived to facilitate coordinate-free computations. They will be introduced as needed throughout the paper.

Any fixed timelike vector such as $\{\gamma_0\}$ defines an inertial frame that determines a unique separation between space and time directions. Algebraically, this can be expressed as the *"spacetime split"* of each vector x designating a spacetime point (or event) into a *time* component $x \cdot \gamma_0 = ct$ and a *spatial position vector* $\mathbf{x} \equiv x \wedge \gamma_0$ as specified by the geometric product

$$x\gamma_0 = ct + \mathbf{x}\,. \tag{15}$$

We call this a γ_0-*split* when it is important to specify the generating vector. The resulting quantity $ct + \mathbf{x}$ is called a *paravector*

This "split" maps a spacetime vector into the STA subalgebra of even multivectors where, by "regrading," the bivector part can be identified as a spatial vector. Accordingly, the even subalgebra is generated by a frame of "spatial vectors" $\{\boldsymbol{\sigma}_k \equiv \gamma_k\gamma_0; k = 1, 2, 3\}$, so that

$$\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2\boldsymbol{\sigma}_3 = \gamma_0\gamma_1\gamma_2\gamma_3 = i. \tag{16}$$

Obviously, this rendition of the STA even subalgebra is isomorphic to the Pauli algebra, though the Pauli algebra is not a subalgebra of the Dirac algebra because the matrix dimensions are different.

We use boldface letters exclusively to denote spatial vectors determined by a spacetime split. Spatial vectors generate a coordinate-free *spatial geometric algebra* with the *geometric product*

$$\mathbf{ab} = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \wedge \mathbf{b} = \mathbf{a} \cdot \mathbf{b} + i\mathbf{a} \times \mathbf{b}, \tag{17}$$

where $\mathbf{a} \times \mathbf{b} = -i(\mathbf{a} \wedge \mathbf{b})$ is the usual vector cross product.

For the even part $\langle M \rangle_+ = Q$ of the multivector M, a spacetime split gives us

$$Q = z + F, \tag{18}$$

where scalar and pseudoscalar parts combine in the form of a complex number

$$z = \alpha + i\beta, \tag{19}$$

and the bivector part splits into the form of a "complex vector"

$$F = \mathbf{E} + i\mathbf{B} = -\tilde{F}. \tag{20}$$

Thus, the even subalgebra in STA has the formal structure of complex quaternions. However, the geometric interpretation of the elements is decidedly different from the usual one assigned to quaternions. Specifically, the bivector $i\mathbf{B}$ corresponds to a "real vector" in the quaternion literature. This difference stems from a failure to distinguish between vectors and bivectors dating back to Hamilton. For complex quaternions, it reduces to failure to identify the imaginary unit i as a pseudoscalar. Geometric interpretation is crucial for application of quaternions in physics.

Reversion in the subalgebra is defined by

$$Q^\dagger \equiv \gamma_0 \widetilde{Q} \gamma_0. \tag{21}$$

This is equivalent to "complex conjugation" of quaternions. In particular,

$$F^\dagger \equiv \gamma_0 \widetilde{F} \gamma_0 = \mathbf{E} - i\mathbf{B}, \tag{22}$$

so that

$$\mathbf{E} = \tfrac{1}{2}(F + F^\dagger), \qquad i\mathbf{B} = \tfrac{1}{2}(F - F^\dagger). \tag{23}$$

Moreover,

$$FF^\dagger = \mathbf{E}^2 + \mathbf{B}^2 + 2\mathbf{E} \times \mathbf{B}, \tag{24}$$

$$F^2 = F \cdot F + F \wedge F = \mathbf{E}^2 - \mathbf{B}^2 + 2i\mathbf{E} \cdot \mathbf{B}, \tag{25}$$

which are familiar expressions from electrodynamics. the bivector F is said to be *simple* if

$$F \wedge F = 0 \quad \Leftrightarrow \quad \mathbf{E} \cdot \mathbf{B} = 0, \tag{26}$$

and is said to be timelike, spacelike or null, respectively, when $F^2 = \mathbf{E}^2 - \mathbf{B}^2$ is positive, negative or zero.

Sometimes it is convenient to decompose the geometric product FG into symmetric and antisymmetric parts

$$FG = F \circ G + F \times G, \tag{27}$$

where the *symmetric product* is defined by

$$F \circ G \equiv \tfrac{1}{2}(FG + GF), \tag{28}$$

and the *commutator product* is defined by

$$F \times G \equiv \tfrac{1}{2}(FG - GF). \tag{29}$$

In particular, for quaternions the symmetric product serves as a "complex inner product," while the commutator product serves as an "outer product for complex vectors." Comparison with (17) shows that for "real vectors"

$$\mathbf{a} \circ \mathbf{b} = \mathbf{a} \cdot \mathbf{b}, \tag{30}$$

and

$$\mathbf{a} \times \mathbf{b} = \mathbf{a} \wedge \mathbf{b} = i(\mathbf{a} \times \mathbf{b}), \tag{31}$$

Note that the cross product on the right is distinguished from the commutator product on the left of this equation by a boldface of the cross product symbol. Also, it should be understood that the equivalence of commutator and outer products in this equation does not generally obtain for arbitrary multivectors.

Concerning the spacetime split of products between even and odd multivectors, for a bivector $F = \mathbf{E} + i\mathbf{B}$ and spacetime vector a with the split $a\gamma_0 = a_0 + \mathbf{a}$, we have

$$(F \cdot a)\gamma_0 = \mathbf{E} \cdot \mathbf{a} + a_0\mathbf{E} + \mathbf{a} \times \mathbf{B}. \tag{32}$$

This may be recognized as the form for a spacetime split of the classical Lorentz force. We will use it as a template for other spacetime splits later on.

Concerning differentiation, the derivative with respect to any multivector variable M is denoted by ∂_M, so the derivative with respect to a vector variable n is denoted by ∂_n. As the derivative with respect to a position vector $\mathbf{x}$ is especially important, we distinguish it with the special symbol

$$\boldsymbol{\nabla} \equiv \partial_{\mathbf{x}} = \boldsymbol{\sigma}_k \partial_k, \tag{33}$$

in agreement with standard vector calculus. Thus, for a relative vector field $\mathbf{A} = \mathbf{A}(\mathbf{x})$ The identity (17) gives us

$$\boldsymbol{\nabla}\mathbf{A} = \boldsymbol{\nabla} \cdot \mathbf{A} + \boldsymbol{\nabla} \wedge \mathbf{A} = \boldsymbol{\nabla} \cdot \mathbf{A} + i\boldsymbol{\nabla} \times \mathbf{A}, \tag{34}$$

which relates the curl to the standard vector cross product.

For field theory, the derivative with respect to a spacetime point must be defined. Though that can be done in a completely coordinate-free way [3], for a rapid survey it is more expedient here to exploit the reader's prior knowledge about partial derivatives.

For each spacetime point x the reciprocal of a standard frame $\{\gamma^\mu\}$ determines a set of "rectangular coordinates" $\{x^\mu\}$ given by

$$x^\mu = \gamma^\mu \cdot x \qquad \text{and} \qquad x = x^\mu \gamma_\mu \,. \tag{35}$$

In terms of these coordinates the derivative with respect to a spacetime point x is an operator $\Box$ defined by

$$\Box \equiv \partial_x = \gamma^\mu \partial_\mu, \tag{36}$$

where ∂_μ is given by

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = \gamma_\mu \cdot \Box \,. \tag{37}$$

The square of $\Box$ is the usual d'Alembertian

$$\Box^2 = g^{\mu\nu}\partial_\mu\partial_\nu \quad \text{where} \quad g^{\mu\nu} = \gamma^\mu \cdot \gamma^\nu. \tag{38}$$

The matrix representation of the *vector derivative* $\Box$ will be recognized as the so-called "Dirac operator," originally discovered by Dirac when seeking a "square root" of the d'Alembertian (38) in order to find a first order "relativistically invariant" wave equation for the electron. In STA however, where the γ^μ are vectors rather than matrices, it is clear that $\Box$ is a vector operator, and we see that it is as significant in Maxwell's equations as in Dirac's.

The symbol $\nabla \equiv \partial_x$ is often used elsewhere [4, 5] instead of $\Box \equiv \partial_x$, but it has the disadvantage of confusability with $\boldsymbol{\nabla} \equiv \partial_{\mathbf{x}}$ in some contexts. Besides, the triangle is suggestive of three dimensions, while the $\Box$ is suggestive of four. That is why the $\Box$ was adopted in the first book on STA [1], and earlier by Sommerfeld [6] and Morse and Feshbach [7].

Note that the symbol ∂_t for the derivative with respect to a scalar variable t denotes the standard partial derivative, though the coordinate index is used as the subscript in (37).

In STA an electromagnetic field is represented by a bivector-valued function $F = F(x)$ on spacetime. Since $\Box$ is a vector operator the expansion (13) applies, so we can write

$$\Box F = \Box \cdot F + \Box \wedge F \,, \tag{39}$$

where $\Box \cdot F$ is the *divergence* of F and $\Box \wedge F$ is the *curl*.

Corresponding to the split of a spacetime point (15), the spacetime split of the vector derivative $\Box = \partial_x$ gives us a paravector derivative

$$\gamma_0 \Box = \partial_0 + \boldsymbol{\nabla}, \tag{40}$$

where $\partial_0 = \gamma_0 \cdot \Box = c^{-1}\partial_t$. Hence, for example, the *d'Alembertian* takes the familiar form

$$\Box^2 = \partial_0^2 - \boldsymbol{\nabla}^2, \tag{41}$$

and the divergence of the vector field $A = (c\varphi + \mathbf{A})\gamma_0$ splits to

$$\Box \cdot A = \partial_0 \varphi + \boldsymbol{\nabla} \cdot \mathbf{A}. \tag{42}$$

Finally, it is worth mentioning that to evaluate vector derivatives without resorting to coordinates, a few basic formulas are needed. For vector n and bivector F, we shall have use for the following derivatives of linear functions:

$$\partial_n n = 4, \quad \partial_n F n = 0, \quad \partial_n (n \cdot F) = 2F. \tag{43}$$

III. ANATOMY OF THE DIRAC WAVE FUNCTION

Considering the central role of Dirac's equation in the spectacular successes of quantum mechanics and QED, it seems indubitable that this compact equation embodies some deep truth about the nature of the electron, and

perhaps elementary particles in general. However, success came with problems that called for action by the doctors of Quantum Mechanics. Soon after the initial success in explaining the hydrogen spectrum and the magical appearance of spin, it was discovered that the electron had an antiparticle twin, the positron, conjoined with it in the Dirac equation. Dirac introduced a surgical procedure called "Hole theory" that suppressed the positron to keep it from interfering with the electron. Eventually, electron and positron were identified with positive and negative energy states and separated by a procedure called "second quantization." That has become the surgical procedure of choice in QED. Here we take a new look at the anatomy of the Dirac equation to see what makes the electron tick. That will lead us to a new unified interpretation of the Dirac equation wherein electron and positron appear as different states of the same object coupled by photons so surgery is unnecessary to separate them.

As first shown in [5, 8], in terms of STA the Dirac equation can be written in the form

$$\gamma^\mu(\partial_\mu\Psi\mathbf{i}\hbar - \frac{e}{c}A_\mu\Psi) = m_e c\Psi\gamma_0\,, \tag{44}$$

where m_e is electron mass and now we use $e = \pm|e|$ for the charge coupling constant, while the $A_\mu = A\cdot\gamma_\mu$ are components of the electromagnetic vector potential. The symbol $\mathbf{i}$ denotes a unit bivector, which can be written in the following equivalent forms:

$$\mathbf{i} \equiv \gamma_2\gamma_1 = i\gamma_3\gamma_0 = i\boldsymbol{\sigma}_3 = \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2 \tag{45}$$

The notation $\mathbf{i}$ emphasizes that it plays the role of the unit imaginary that appears explicitly in matrix versions of the Dirac equation.

Let us refer to (44) as the *real Dirac equation* to distinguish it from the standard matrix version. It is well established that the two versions are mathematically isomorphic [4, 5, 8]. However, the real version reveals geometric structure in the Dirac theory that is so deeply hidden in the matrix version that it remains unrecognized by QED experts to this day. That fact is already evident in the identification of the imaginary unit $\mathbf{i}$ as a bivector. As we see below, this identification couples complex numbers in quantum mechanics inextricably to spin, with profound implications for physical interpretation. It is the first of several insights into geometric structure of Dirac theory that will guide us to a reformulation and new interpretation.

Employing the vector derivative puts the real Dirac equation in the coordinate-free form

$$\Box\Psi\mathbf{i}\hbar - \frac{e}{c}A\Psi = m_e c\Psi\gamma_0\,, \tag{46}$$

where $A = A_\mu\gamma^\mu$ is the electromagnetic vector potential. The spinor "*wave function*" $\Psi = \Psi(x)$ admits of the *Lorentz invariant* decomposition

$$\Psi = \psi e^{i\beta/2} \quad \text{with} \quad \psi(x) = \rho^{\frac{1}{2}}R(x), \tag{47}$$

where $\rho = \rho(x)$ and $\beta = \beta(x)$ are scalar-valued functions, and "*rotor*" $R = R(x)$ is normalized to

$$R\widetilde{R} = \widetilde{R}R = 1. \tag{48}$$

The Lorentz invariant "β-factor" in the general form (47) for a "*Real Dirac spinor*" has been singled out for special consideration. As this factor is so deeply buried in matrix representations for spinors, its existence has not been generally recognized and its physical interpretation has remained problematic to this day. We shall see it as a candidate for corrective surgery on the Dirac wave function.

We shall also be considering singular solutions $\Psi_\pm$ of the Dirac equation (46) called *Majorana states* and defined by

$$\Psi_\pm = \Psi(1 \pm \boldsymbol{\sigma}_2) = \Psi\gamma_\pm\gamma_0, \tag{49}$$

where

$$\gamma_\pm = \gamma_0 \pm \gamma_2. \tag{50}$$

We shall see that STA reveals properties of these states that make them attractive candidates for distinct electron and positron states.

A. Local Observables

We begin physical interpretation of the Dirac wave function with identification of "*local observables*." At each spacetime point x, the rotor $R = R(x)$ determines a Lorentz rotation of a given fixed frame of vectors $\{\gamma_\mu\}$ into a frame $\{e_\mu = e_\mu(x)\}$ given by

$$e_\mu = R\gamma_\mu\widetilde{R}\,. \tag{51}$$

In other words, R determines a unique frame field on spacetime. Whence, the wave function determines four vector fields

$$\Psi\gamma_\mu\widetilde{\Psi} = \psi\gamma_\mu\widetilde{\psi} = \rho e_\mu. \tag{52}$$

Note that the β-factor has cancelled out of these expressions because the pseudoscalar i anticommutes with the vectors γ_μ.

It can be shown [3, 4, 8] that two of the vector fields (51) correspond to well known quantities in matrix Dirac theory. The quantity

$$\psi\gamma_0\widetilde{\psi} = \rho v \quad \text{with} \quad v = R\gamma_0\widetilde{R} = e_0. \tag{53}$$

is the *Dirac current*. The Born interpretation identifies this as a "*probability current*;" whence, ρ is a *probability density*. (We shall consider an alternative interpretation for ρ later on.) The quantity

$$s = \frac{\hbar}{2}R\gamma_3\widetilde{R} = \frac{\hbar}{2}e_3 \tag{54}$$

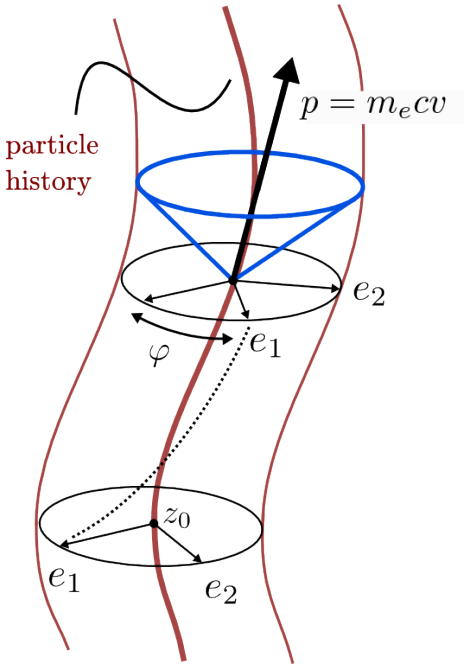

FIG. 1. The "spinning frame" of local observables along an electron path is depicted in a direction orthogonal to the spin vector.

can be identified as the electron "*spin vector*," though it looks rather different than its matrix counterpart. Physical interpretation of e_1 and e_2 is more subtle, as these vectors are not recognized in standard Dirac theory. To clarify the matter, we decompose the rotor R into the product

$$R = Ve^{-\mathbf{i}\varphi}. \tag{55}$$

Then

$$e_1 = R\gamma_1\widetilde{R} = e^{-I\varphi}a_1 e^{I\varphi} = a_1 e^{2I\varphi}, \tag{56}$$

where

$$I \equiv R\,\mathbf{i}\widetilde{R} = V\mathbf{i}\widetilde{V} \quad \text{and} \quad a_1 = V\gamma_1\widetilde{V}, \tag{57}$$

with an analogous equation for e_2. This exhibits the wave function *phase* φ as an angle of rotation in a spacelike plane with tangent bivector $I = I(x)$ at each spacetime point x. Moreover, the direction of that plane is determined by the *spin bivector* defined by

$$S \equiv isv = \frac{\hbar}{2}R\,\mathbf{i}\widetilde{R} = \frac{\hbar}{2}I. \tag{58}$$

Thus, we have a connection between spin and phase with the phase as an angle of rotation in the "*spin plane*."

In general, the Lorentz rotation (51) has a unique decomposition into a spatial rotation followed by a boost, which is generated by the rotor product [3]

$$R = VU \tag{59}$$
$$\text{with} \quad U\gamma_0\widetilde{U} = \gamma_0 \quad \text{and} \quad V = (v\gamma_0)^{1/2}.$$

For simplicity, we often refer to rotors V and U by the same names "boost" and "spatial rotation" used for the Lorentz transformations they generate.

We can further decompose the rotor product into

$$R = U_1V_0\widetilde{U}_1U = U_1V_0U_2 \tag{60}$$

where

$$V_0 = \exp\{\alpha_1\boldsymbol{\sigma}_2\} = \cosh\alpha_1 + \boldsymbol{\sigma}_2\sinh\alpha_1 \tag{61}$$

is a boost in a fixed direction $\boldsymbol{\sigma}_2 = \gamma_2\gamma_0$, while U_1 and U_2 are spatial rotations.

B. Electron clock and chirality

As the notion of an electron clock was central to de Broglie's seminal contribution to quantum mechanics [9], its relevance to interpretation of the Dirac equation deserves thorough investigation. The clock mechanism can be defined by considering a Dirac plane wave solution of the form (55) with momentum p, wherein the phase has the specific form $\varphi = k\cdot x$. Then $\Box\varphi = k$, and the Dirac equation (46) gives us

$$\hbar kRe^{i\beta/2} = m_e cRe^{i\beta/2}\gamma_0, \tag{62}$$

which we solve for

$$k = \frac{m_e c}{\hbar}ve^{-i\beta}. \tag{63}$$

This has two solutions with opposite signs given by $\cos\beta = \pm 1$ and momentum $p = m_e cv = \pm\hbar k$,

Equation $v\cdot x = c\tau$ describes a propagating hyperplane with unit normal v, so (63) gives

$$p\cdot x = m_e c^2\tau. \tag{64}$$

Accordingly, the vector e_1 in (56) rotates in (or on) the hyperplane with frequency

$$\omega_e \equiv \frac{2m_e c^2}{\hbar} = \pm 2\frac{d\varphi}{d\tau}. \tag{65}$$

The handedness is opposite for the two solutions This will be recognized as the *zitterbewegung* frequency of Schrödinger. It is precisely twice the *de Broglie frequency* because the wave function phase angle is precisely half the rotation angle of the observables in (56). The sign of the phase specifies the sense of rotation, which is opposite for electron and positron.

We can now give the vector e_1 a picturesque physical interpretation as *the hand on de Broglie's electron clock*, with its rotation given by (56). The *face of the clock* is the bivector I in (57), and the reference point for an initial time on the clock face is given by the vector a_1. This description of the electron clock is completely general, as the equations hold for an arbitrary electron wave function. Indeed, equation (57) shows that the electron clock

can be described as an "*inertial clock*," because it retains the mark of initial time even as interactions change the rotor R and hence spin direction and the *attitude* of the clock in spacetime.

Of course, interactions can change the clock frequency by changing the phase φ. Nevertheless, the free electron frequency remains as a reference standard for the electron clock. This suggests that we define the free electron clock period τ_e as the fundamental unit of electron time. Its empirical value, which I propose to call the *zit*, is

$$1 \text{ zit} = \tau_e = \frac{2\pi}{\omega_e} = \frac{h}{2m_e c^2} = 4.0466 \times 10^{-21}\text{sec} \quad (66)$$

Approximately: 1 zit $\approx$ 4 zepto-sec; 1 sec $\approx 1/4$ zetta-zit. Remarkably, direct measurement of the "zit" may be possible with electron channeling experiments [10].

The two signs in (65) indicate clocks with opposite "handedness" or *chirality*, as we shall say. We identify the negative sign with an *electron clock* and the the positive sign with a *positron clock*. Indeed, in standard theory the two signs are interpretated as states with opposite energy and the negative energy state is identified with the positron. However, we have seen that the sign is actually determined by $\cos\beta = \pm 1$ without reference to a concept of energy. This suggests that we interpret β as a "chirality parameter." Be that as it may, we can see that the vector e_2 specifies the clock-face direction of motion for the clock hand e_1. Hence "antiparticle conjugation" should be defined to reverse the direction of e_2 while keeping the direction of e_1 unchanged.

Finally, we note that the Born probability density has been set to $\rho = 1$ on the propagating hyperplane, thus implying that all points on the hyperplane are equally probable positions x_0 for the electron at initial time τ_0. However, for any initial position x_0, the velocity $v = \dot{x}$ integrates to a unique position

$$x(\tau) = v\tau + x_0. \quad (67)$$

Thus, the plane wave solution consists of an ensemble of equally probable particle paths composing a congruence (or *fibration*) of non-intersecting, timelike paths that sweep out (*fibrate*) a region of spacetime.

C. Electron clock with zitter

There is another plane wave solution that has been largely overlooked in the literature. In this case the parameter β plays no role. We simply switch (55) into the form (with $\rho = 1$)

$$\psi = e^{-\mathbf{i}\varphi} V_0, \quad (68)$$

which is of type (60) with constant V_0 given by (61). This solves the Dirac equation with $\varphi = p\cdot x/\hbar$ and $p = m_e c\gamma_0$. To verify that:

$$\Box\psi\mathbf{i}\hbar = -m_e c\gamma_0 i\boldsymbol{\sigma}_3\psi i\boldsymbol{\sigma}_3 = m_e c\psi\gamma_0. \quad (69)$$

Note that $\gamma_0 i\boldsymbol{\sigma}_3 = i\gamma_3$ commutes with ψ, whereas γ_0 and $i\boldsymbol{\sigma}_3$ do not. Generalization to a solution for arbitrary constant $p = m_e c V\gamma_0\widetilde{V}$ is obviously given by a boost to $\psi' = V\psi$.

Now, using (61) we can express the wave function (68) as the sum of positive and negative energy solutions:

$$\psi = \cosh\alpha_1 e^{-\mathbf{i}k\cdot x} + \boldsymbol{\sigma}_2 \sinh\alpha_1 e^{+\mathbf{i}k\cdot x} \equiv \psi_+ + \psi_-. \quad (70)$$

The analog of hermitian conjugate in standard matrix Dirac algebra is defined by $\psi^\dagger = \gamma_0\widetilde{\psi}\gamma_0$. Whence, the velocity is given by

$$\begin{aligned} v &= \psi\gamma_0\widetilde{\psi} = \psi\widetilde{\psi}^\dagger\gamma_0 \\ &= \{|\psi_+|^2 + |\psi_-|^2 + \psi_+\psi_-^\dagger + \psi_-\psi_+^\dagger\}\gamma_0 \\ &= |\psi|^2\gamma_0 + 2 < \psi_-\psi_+^\dagger > \gamma_2 e^{\mathbf{i}2\phi}. \end{aligned} \quad (71)$$

In agreement with [11], this exhibits zitterbewegung as arising from interference between positive and negative energy states, as originally formulated by Schrödinger. However, it also exhibits zitterbewegung as circulation of electron velocity in the spin plane. I have coined the term *zitter* to distinguish this interpretation of zitterbewegung from other alternatives in the literature.

This result settles a long-standing controversy about the interpretation of zitterbewegung. To this day, studies of Dirac wave packets (e.g. [12]) fail to recognize the connection of zitterbewegung to spin. Instead, it is identified as a high frequency interference effect, often attributed to interaction with the vacuum with a negative energy component ψ_- presumed to express presence of positrons. On the contrary, in the zitter model here the "negative energy" term has nothing to do with positrons. Instead, it is a structural feature of electron motion involving electron spin and phase.

We can associate our zitter plane wave with particle motion in the same way for the plane wave in the preceding subsection. Without loss of generality, we can write $p = m_e c\gamma_0$, so $\varphi = p\cdot x/\hbar = \omega_e\tau/2$ defines a plane propagating in the direction of p with proper time τ. Then (68) and (71) gives us a parametric equation for the particle velocity:

$$v(\tau) = e^{-\frac{1}{2}\mathbf{i}\omega_e\tau} v_0\, e^{\frac{1}{2}\mathbf{i}\omega_e\tau} = a\gamma_0 + b\,\gamma_2\, e^{\mathbf{i}\omega_e\tau}, \quad (72)$$

where a and b are constants, while ω_e is the free particle *zitter frequency*. For $v = \dot{x}$, this integrates to

$$x(\tau) = \gamma_0 ac\tau + b\lambda_e\, e_1 + x_0, \quad (73)$$

where $\lambda_e = c/\omega_e$ and

$$e_1(\tau) = \gamma_1\, e^{\mathbf{i}\omega_e\tau}, \quad (74)$$

is the electron clock vector.

The particle path $x(\tau)$ specified by (73) is a timelike helix with pitch $b\lambda_e/a$. Thus, the zitter plane wave solution consists of an ensemble of equally probable particle

paths that fibrate a region of spacetime with a congruence of non-intersecting, timelike helices.

Though the circular frequency ω_e is constant, the circular speed increases with radius $b\lambda_e$ without reaching the limiting case $\lambda_e\omega_e = c$ at the speed of light. In that limit, $V_0 \to 1+\boldsymbol{\sigma}_2$ in (60), and we get the Majorana wave function Ψ_+ defined in (49), so the velocity vector (72) becomes a null vector

$$\begin{aligned} u(\tau) &= \Psi_+\gamma_0\widetilde{\Psi}_+/\rho \\ &= e^{-\frac{1}{2}\mathbf{i}\omega_e\tau}\gamma_+ e^{\frac{1}{2}\mathbf{i}\omega_e\tau} = \gamma_0 + \gamma_2\, e^{\mathbf{i}\omega_e\tau}. \end{aligned} \tag{75}$$

In this case, zitter with the electron clock is intrinsic to electron motion, whereas in the previous case described by (72) the zitter can vanish with $b = 0$.

Thus, we have three distinct kinds of free particle (plane wave) states: *Kind A*, given by (55), with no zitter; *Kind B*, given by (68) and (72), with zitter velocity ranging between zero and the speed of light; and *Kind C*, given by (75), with zitter velocity $\lambda_e\omega_e = c$.

Kind B is related to *Kind A* by a unitary transformation. For example, (68) is related to (55) by

$$\gamma_1(e^{-\mathbf{i}\varphi}V_0)\gamma_1^\dagger = V_0 e^{-\mathbf{i}^\dagger\varphi}, \tag{76}$$

where $\gamma_1^\dagger = \gamma_1^* = -\gamma_1$ and the right side is interpreted as a positive energy factor with $\mathbf{i}$ replaced by $\mathbf{i}^\dagger$. It can be generated by the continuous unitary transformation

$$\psi \to W\psi W^\dagger, \quad \text{where} \quad W = e^{\gamma_1\,\alpha_0}, \tag{77}$$

which may be recognized as a Foldy-Wouthuysen (FW) transformation [13].

The FW transformation is commonly used to eliminate negative energy components in electron wave functions, often because they are regarded as "unphysical." Without going into arguments supporting this practice, the point here is that it suppresses the role of zitter in describing electron motion.

To ascertain what the Dirac equation can tell us about the physical significance of zitter, the parameter β and the electron clock, we study the properties of local observables thoroughly in the next section. This will help us address such questions as: Is zitter an objectively real physical property of the electron? Should electron phase (de Broglie's clock) be regarded as a feature of electron zitter? What is the role of zitter in quantization? Of course, the answers will lead to more questions and speculation.

IV. FLOW OF LOCAL OBSERVABLES

We turn now to a general analysis of conservation laws implied by the Dirac equation as a foundation for physical interpretation. To facilitate comparison with conventional Dirac theory, we first express the conservation laws in terms of the wave function. Then we peel them apart to reveal their structure in terms of local observables.

A conservation law for the Dirac current $\Psi\gamma_0\tilde{\Psi} = \rho v$ is easily derived from the Dirac equation (46) and takes the form

$$\Box \cdot (\rho v) = 0. \tag{78}$$

This can be interpreted as flow of a fluid with proper density ρ. Precisely what kind of fluid depends on the interpretation of other local observables, in particular, observables describing the flow of energy, momentum, charge and electromagnetic potential. Following a systematic approach in defining these observables within the Dirac theory, we shall discover hidden structure that has been generally overlooked.

The original formulation of the Dirac equation was based on interpreting

$$\underline{p}_\mu = \underline{i}\hbar\,\partial_\mu - \frac{e}{c}A_\mu \tag{79}$$

as a gauge invariant energymomentum operator. The underbar notation here designates a linear operator. Specifically, the operator $\underline{i}$ designates multiplication by the unit imaginary in the matrix version of Dirac theory, and right multiplication by the unit bivector $\mathbf{i} = i\gamma_3\gamma_0$ in the STA version, as specified in

$$\underline{p}_\mu\Psi = \hbar\,\partial_\mu\Psi i\gamma_3\gamma_0 - \frac{e}{c}A_\mu\Psi. \tag{80}$$

Equivalence of operators in the matrix version to expressions in the present STA version is discussed in [14].

The energymomentum operator also led to the definition of an *energy momentum tensor* $\underline{T}(n)$ with components

$$T^{\mu\nu} = T^\mu \cdot \gamma^\nu = \langle\gamma_0\widetilde{\Psi}\gamma^\mu\underline{p}^\nu\Psi\rangle, \tag{81}$$

where

$$T^\mu = \underline{T}(\gamma^\mu). \tag{82}$$

The *stress tensor* $\underline{T}(n)$ *is defined physically* as a vector-valued tensor field specifying, at each spacetime point, the energymomentum flux through a hypersurface with unit normal n. Its adjoint $\overline{T}(n)$ can be defined by

$$\gamma^\mu \cdot \overline{T}(\gamma^\nu) = \underline{T}(\gamma^\mu) \cdot \gamma^\nu = T^{\mu\nu}. \tag{83}$$

Note the overbar notation $\overline{T}(n)$ to indicate the adjoint of a linear function $\underline{T}(n)$ specified by an underbar. In this case the linear functions are vector-valued, but the same notation is used for bivector-valued linear functions below.

The Dirac equation (46) can be derived from the Lagrangian

$$\mathcal{L} = \left\langle \hbar\,\Box\Psi i\gamma_3\widetilde{\Psi} - \frac{e}{c}A\Psi\gamma_0\widetilde{\Psi} - m_e c\Psi\widetilde{\Psi}\right\rangle. \tag{84}$$

As is well known, a major advantage of this approach is that conservation laws consistent with the equations

of motion can be derived from symmetries of the Lagrangian. The most elegant and efficient way to do this is the method of multivector differentiation introduced by Lasenby, Doran and Gull in [15]. In particular, from translation invariance of the Lagrangian they derived the stress tensor

$$\begin{aligned}\underline{T}(n) &= \gamma_\mu \left\langle (\underline{p}^\mu \Psi)\gamma_0 \widetilde{\Psi} n \right\rangle \\ &= \gamma^\mu \left\langle (\hbar\partial_\mu \Psi i\gamma_3\gamma_0)\gamma_0 \widetilde{\Psi} n \right\rangle - \frac{e}{c} A\rho\,(v\cdot n). \end{aligned} \quad (85)$$

This is equivalent to the stress tensor most commonly employed in Dirac theory.

However, when Lasenby, Doran and Gull generalized their method in a ground breaking paper on Gauge Theory Gravity [16], translation invariance gave instead the adjoint stress tensor

$$\overline{T}(n) = \left\langle \hbar(n\cdot\Box\Psi) i\gamma_3 \widetilde{\Psi} \right\rangle_1 - \frac{e}{c}(A\cdot n)\rho\, v. \quad (86)$$

This raises a question as to which stress tensor is correct for the electron: $\underline{T}(n)$ or $\overline{T}(n)$? We will leave that question open for the time being while we examine both and compare their properties. The first derivation of this tensor from the Dirac equation was made by Tetrode [17], so it is fair to call it the *Tetrode tensor*.

The dynamics of flow is determined by the divergence of the stress tensor:

$$\begin{aligned}\grave{\underline{T}}(\grave{\Box}) &= \partial_\mu \underline{T}(\gamma^\mu) = \partial_\mu T^\mu \\ &= \left\langle \hbar(\Box^2\Psi) i\gamma_3 \widetilde{\Psi} \right\rangle_1 - \frac{e}{c}\partial_\mu(\rho\, v A^\mu). \end{aligned} \quad (87)$$

We need the Dirac equation (46) to evaluate this. Since

$$\left\langle \partial_\mu \Psi i\gamma_3\, \partial^\mu \widetilde{\Psi} \right\rangle_1 = 0, \quad (88)$$

we have

$$\begin{aligned}\left\langle \hbar(\Box^2\Psi) i\gamma_3 \widetilde{\Psi} \right\rangle_1 &= \frac{\hbar}{2}[\Box^2\Psi i\gamma_3\widetilde{\Psi} - \Psi i\gamma_3 \Box^2 \widetilde{\Psi}] \\ &= \rho\frac{e}{c}(\Box\wedge A)\cdot v + \frac{e}{c}\partial_\mu(\rho\, v A^\mu). \end{aligned} \quad (89)$$

Whence

$$\grave{\underline{T}}(\grave{\Box}) = \partial_\mu T^\mu = \frac{e}{c} F\cdot(\rho v) \equiv \rho f, \quad (90)$$

where $F = \Box\wedge A$ is an external electromagnetic field. This has precisely the form for the Lorentz force on a classical charged fluid, and it supports the interpretation of the Dirac current $e\rho v$ as a charge current.

A conservation law for angular momentum can be derived from invariance of the Lagrangian (84) under Lorentz rotations [15], but we derive it directly from properties of the stress tensor, as it makes structure more explicit. From (90) we get

$$\partial_\mu(T^\mu \wedge x) = T^\mu \wedge \gamma_\mu + \rho f \wedge x. \quad (91)$$

To see how this equation gives us angular momentum conservation, we need to analyze the first term on the right. In doing so we find other interesting results as byproducts.

First, note that

$$\gamma^\mu \left\langle \hbar(\partial_\mu\Psi) i\gamma_3 \widetilde{\Psi} \right\rangle_1 = \hbar(\Box\Psi) i\gamma_3 \widetilde{\Psi} + \Box(i\rho s). \quad (92)$$

Then, combine this with the Dirac equation (46) in the form

$$\hbar(\Box\Psi) i\gamma_3 \widetilde{\Psi} = m_e c\rho e^{i\beta} + \frac{e}{c} A\rho v \quad (93)$$

to get

$$\partial_n \overline{T}(n) = \gamma_\mu \overline{T}(\gamma^\mu) = \Box(i\rho s) + m_e c\rho e^{i\beta}. \quad (94)$$

The scalar part of this equation gives us the trace of the stress tensor:

$$Tr(\overline{T}) = \partial_n \cdot \overline{T}(n) = m_e c\rho\cos\beta, \quad (95)$$

and the pseudoscalar part gives us

$$\Box\cdot(\rho s) = m_e c\rho \sin\beta. \quad (96)$$

This displays a peculiar relation of β to mass and spin of questionable physical significance. However, β plays no role in the bivector part of (94), which gives us

$$\gamma_\mu \wedge \overline{T}(\gamma^\mu) = \underline{T}(\gamma^\mu)\wedge\gamma^\mu = \Box\cdot(i\rho s) = \partial_\mu S^\mu, \quad (97)$$

where

$$S^\mu = \underline{S}(\gamma^\mu) = \gamma^\mu\cdot(i\rho s) = \rho i(s\wedge\gamma^\mu). \quad (98)$$

is identified as a bivector-valued spin flux tensor.

Equation (97) gives us an explicit relation between the stress tensor and its adjoint:

$$\overline{T}(n) - \underline{T}(n) = n\cdot(\gamma_\mu\wedge T^\mu) = (n\wedge\Box)\cdot(i\rho s). \quad (99)$$

And inserting this into (90) with $n = \Box$, we find that the divergence of the stress tensor is equal to the divergence of its adjoint:

$$\partial_\mu \overline{T}(\gamma^\mu) = \partial_\mu \underline{T}(\gamma^\mu) = \frac{e}{c} F\cdot(\rho v) = \rho f. \quad (100)$$

This equivalent divergence of the stress tensor and its adjoint has been overlooked in the literature. Let's compare these two tensors more closely.

The flux of momentum along the Dirac current is especially significant, because that is the direction of particle flow. Accordingly, we define a momentum density ρp along this flow by

$$\rho p \equiv \underline{T}(v). \quad (101)$$

The adjoint determines a "*conjugate momentum*" density ρp_c defined by

$$\rho p_c \equiv \overline{T}(v). \quad (102)$$

We will be looking to ascertain the physical difference between these two kinds of momenta. First we note a small difference in angular momentum.

Returning now to the question of angular momentum conservation, inserting (97) into (91), we get the desired conservation law:

$$\grave{\underline{J}}(\grave{\Box}) = \partial_\mu J^\mu = \rho f \wedge x. \tag{103}$$

where the total angular momentum tensor flux is a bivector-valued tensor with orbital and spin parts defined by

$$J^\mu = \underline{J}(\gamma^\mu) = T^\mu \wedge x + S^\mu. \tag{104}$$

Accordingly, the angular momentum flux along the Dirac current is given by

$$\underline{J}(v) = \rho(p \wedge x + S), \tag{105}$$

where $\underline{S}(v) = \rho S$ confirms our earlier identification of $S = isv$ as a spin bivector.

Alternatively, we can define a "conjugate" angular momentum tensor

$$J^\mu_c = \overline{T}(\gamma^\mu) \wedge x - S^\mu, \tag{106}$$

which by the same argument yields the conservation law

$$\partial_\mu J^\mu_c = \rho f \wedge x, \tag{107}$$

But the conjugate angular momentum flow has a spin of opposite sign:

$$\overline{J}_c(v) = \rho(p_c \wedge x - S). \tag{108}$$

This sign difference can be interpreted geometrically as an opposite orientation of spin S to velocity v or momenta p and p_c. To probe the difference between the momenta p and p_c more deeply, we express them as explicit functions of local observables.

The dynamics of the local observables $e_\mu = R\gamma_\mu\widetilde{R}$ is determined by the linear bivector-valued function

$$\Omega_\mu = \underline{\Omega}(\gamma_\mu) \equiv 2(\partial_\mu R)\tilde{R}. \tag{109}$$

Thus,

$$\partial_\nu e_\mu = \Omega_\nu \cdot e_\mu. \tag{110}$$

In particular, the derivatives of the velocity and spin vectors are

$$\partial_\nu v = \Omega_\nu \cdot v \quad \text{and} \quad \partial_\nu s = \Omega_\nu \cdot s, \tag{111}$$

while the derivative of the spin bivector $S = isv$ is given by the commutator product:

$$\partial_\mu S = \Omega_\mu \times S. \tag{112}$$

Now, with the wave function in the form

$$\Psi = \psi e^{i\beta/2} = Re^{(\alpha+i\beta)/2}, \tag{113}$$

its derivatives can be related to observables by

$$\hbar(\partial_\mu \Psi)i\gamma_3\widetilde{\Psi} = \rho[(i\partial_\mu\alpha + \partial_\mu\beta)s + \Omega_\mu S v], \tag{114}$$

with the product expansion

$$\Omega_\mu S = P_\mu + \partial_\mu S + iq_\mu \tag{115}$$

where we have identified components $P_\mu = \gamma_\mu \cdot P$ of the *"canonical momentum vector"* P defined by

$$P_\mu = \Omega_\mu \cdot S = \frac{\hbar}{2} e_1 \cdot \partial_\mu e_2 = -\frac{\hbar}{2} e_2 \cdot \partial_\mu e_1, \tag{116}$$

and the pseudoscalar part is given by

$$iq_\mu = \Omega_\mu \wedge S = i\Omega_\mu \cdot (sv) = i(\partial_\mu s) \cdot v. \tag{117}$$

Finally, by inserting (114) into (86) we get the components of the stress tensor in the transparent form

$$T_{\mu\nu} = \rho[v_\mu(P_\nu - \frac{e}{c}A_\nu) + (v \wedge \gamma_\mu) \cdot \partial_\nu S - s_\mu \partial_\nu \beta]. \tag{118}$$

This gives us an informative expression for the conjugate momentum:

$$\overline{T}(v) = \rho\{[(P - \frac{e}{c}A) \cdot v]v + \dot{S} \cdot v - s\dot{\beta}\} = \rho p_c. \tag{119}$$

The first two terms in this expression for momentum flow along a streamline of the Dirac current make perfect physical sense. Note that the factor $(P - \frac{e}{c}A) \cdot v$ serves as a gauge invariant variable mass determined by the frequency of the electron clock, which is specified by

$$P \cdot v = \Omega_v \cdot S = \frac{\hbar}{2} e_1 \cdot \dot{e}_2 = -\frac{\hbar}{2} e_2 \cdot \dot{e}_1, \tag{120}$$

where

$$\dot{e}_\mu = v \cdot \Box\, e_\mu = \Omega_v \cdot e_\mu \quad \text{and} \quad \Omega_v = \underline{\Omega}(v). \tag{121}$$

The second term $\dot{S} \cdot v = \dot{v} \cdot S$ in (119) specifies a contribution of spin to linear momentum due to acceleration. However, a physical interpretation for the last term involving the directional derivative $\dot{\beta} = v \cdot \Box\beta$ remains problematic. Resolving that problem will be a major goal in the rest of this paper.

From the stress energy components (118), we also get a remarkably simple expression for the momentum density:

$$\underline{T}(v) = \rho(P - \frac{e}{c}A) = \rho p. \tag{122}$$

And for flux in the spin direction we get:

$$\overline{T}(s) = -\rho s\dot{\beta}. \tag{123}$$

Combining (122) with (119) we get

$$p_c = (p \cdot v)v + \dot{S} \cdot v - s\dot{\beta}. \tag{124}$$

As we shall see, it is especially important to note that in these equations both p and p_c are defined independently

of ρ, and physical interpretation of the strange parameter β appears to be tied up with spin.

Having thus identified the canonical momentum P as a local observable, we can express the Dirac equation as a constitutive equation relating observables. Thus, from (114) and (115) we derive the expression

$$\hbar(\Box\Psi)i\gamma_3\widetilde{\Psi} = [\rho P + [\Box(\rho e^{i\beta}S)]e^{-i\beta}]v, \tag{125}$$

which we insert into the Dirac equation (93) to get it in the form

$$\rho(P - \frac{e}{c}A)e^{i\beta} = m_e c\rho v - \Box(\rho e^{i\beta}S) \tag{126}$$

Its vector part is a constitutive equation involving the Dirac current:

$$\rho(P - \frac{e}{c}A)\cos\beta = m_e c\rho v - \Box\cdot(\rho e^{i\beta}S). \tag{127}$$

The right side of this equation has vanishing divergence, and we identify it as the well known *Gordon current*. Unlike the vector part, the trivector part of (126) does not have any evident physical meaning, though it does serve as a constraint among the variables.

This completes our exact reformulation of Dirac Theory in terms of local observables. We have found clear physical interpretations for all components of the Dirac wave function except the parameter β. The strangeness of β is most explicit in equation (127) for the Gordon current, where the factor $e^{i\beta}$ generates a duality rotation without obvious physical significance. And that equation implies the conservation law

$$\Box\cdot[\rho(P - \frac{e}{c}A)\cos\beta] = \Box\cdot(\rho p\cos\beta) = 0, \tag{128}$$

where again the role of β is problematic.

The Gordon current can be regarded as a reformulation of the Dirac equation in terms of local observables, as our derivation of (127) shows. For this reason, it plays a fundamental role in our analysis of alternative physical interpretations in subsequent sections. But first we try to make some sense of β.

A. Problems with β

We begin our study of β by reformulating the Dirac Lagrangian (84) with $\Psi = \psi e^{i\beta/2}$ to make the role of β explicit and then to relate it to the explicit role of other observables:

$$\begin{aligned}\mathcal{L} &= \left\langle \hbar\Box\psi i\gamma_3\widetilde{\psi} - \frac{e}{c}A\psi\gamma_0\widetilde{\psi} - m_e c\rho\cos\beta - \rho s\Box\beta \right\rangle \\ &= \rho(P - \frac{e}{c}A)\cdot v + (v\wedge\Box)\cdot(\rho S) \\ &\quad - m_e c\rho\cos\beta - \rho s\cdot\Box\beta. \end{aligned}\tag{129}$$

The mass term $< m_e c\Psi\widetilde{\Psi} >= m_e c\rho\cos\beta$ has always been problematic in QED. Indeed, it has been eliminated from the Standard Model, which aims to derive the mass from fundamental theory. We shall see that is a major mistake, amounting to "throwing out the baby with the bathwater!"

When the β-factor $e^{i\beta}$ is constant, (47) can be used to factor it out of the Dirac equation (46) to exhibit its role explicitly:

$$\{\hbar\Box\psi i\gamma_3\gamma_0 - \frac{e}{c}A\psi\}\widetilde{\psi}e^{i\beta} = m_e c\psi\gamma_0\widetilde{\psi} = m_e c\rho v. \tag{130}$$

Note that setting $e^{i\beta} = -1$ amounts to reversing orientation of the bivector $\mathbf{i} = i\gamma_3\gamma_0$ that generates rotations in the phase plane along with reversing the sign of the charge, as required for antiparticle conjugation according to the chirality hypothesis. Accordingly, the Dirac equation is resolved into separate equations for electron and positron.

We have seen how plane wave solutions of the Dirac equation suggest that the β distinguishes particle from antiparticle states. Let me call that suggestion the "*chirality hypothesis*." Some credence to this hypothesis is given by the fact that unitary spinors R and Ri are distinct spin representations of the Lorentz group, so it is natural to associate them with distinct particles. However, the Dirac spinor $Re^{i\beta/2}$ is a continuous connection between both representations, suggesting that β parametrizes an admixture of particle/antiparticle states.

After I discovered that $\cos\beta = \pm 1$ solves the problem of negative energies for plane waves and thereby separates electron and positron plane wave state, I set about studying the physical significance of β in the general case. I got great help from my graduate student Richard Gurtler, who thoroughly examined the behavior of β in the *Darwin solutions* of the Dirac equation for Hydrogen [18]. The results do not seem to support the chirality hypothesis, for the parameter β varies with position in peculiar ways. The values $\cos\beta = \pm 1$ appear only in the azimuthal plane, which suggests that 2d solutions might satisfy the chirality hypothesis, but the chirality jumps in sign across nodes in the plane in an unphysical way. At about the same time I began a systematic study of local observables in Dirac theory [19] and their roles in Pauli and Schrödinger theories [20–22] , but I was unable to make sense of the peculiar behavior I found for β.

This problem of interpreting β has never been recognized in standard QED. Indeed, it is commonly claimed that second quantization solves Dirac's problem of negative energies. My suspicion is that $\cos\beta = \pm 1$ has been tacitly assumed in QED when it begins by quantizing plane wave states. Consistency of that procedure with the Darwin solutions has never been proved to my knowledge. It seems that a perceived need for such a proof is avoided by claiming that the Darwin case is concerned with one-particle quantum mechanics, whereas QED is a many-particle theory.

The next Section makes the essential role of β in standard Dirac theory explicit in preparation for subsequent identification its physical significance.

V. PILOT WAVE THEORY WITH THE DIRAC EQUATION

Let me coin the name *Born–Dirac* for standard Dirac theory with the *Born rule* for interpreting the Dirac wave function as a probability amplitude.

The Born rule was initially adopted for Schrödinger theory and subsequently extended to Dirac theory without much discussion — in fact, without even establishing the correct relation between Dirac and Schrödinger wave functions. The latter is supposed to describe a particle without spin. However, a correct derivation from the Dirac equation [21, 22] implies instead that the Schrödinger equation describes an electron in a spin eigenstate, and its imaginary unit must be identified with the spin bivector $\mathbf{i}\hbar = 2i\mathbf{s}$.

Subsequently, physical interpretation of Schrödinger theory has been hotly debated, while, ironically, relevant implications of the more precise Dirac theory have been overlooked. To correct this deficiency, our first task here is to update Born-Dirac theory with recent insights on interpretation of Schrödinger theory. Then we can consider enhancements from our study of local observables in Dirac theory.

After decades of debate and clarifications, it seems safe to declare that de Broglie–Bohm "*Pilot Wave*" theory is well established as a viable interpretation of quantum mechanics, though that may still be a minority opinion among physicists. Current accounts suitable for our purposes are given in [23, 24]. The point to be emphasized here can be regarded as a refinement of the Born rule, which says the wave function for a single electron specifies its probable position at a given time. The *Pilot Wave rule* extends that to regarding the wave function as specifying an ensemble of possible particle paths, with the electron traversing exactly one of those paths, but with a certain probability for each path. So to speak, the wave function serves to guide the electron along a definite path, but with a specified probability. Hence the name *"pilot wave"* for the wave function. In his "theory of the double solution," de Broglie argued for a physical mechanism to select precisely one of those paths, but that alternative is not available in conventional Pilot Wave theory. Instead, path selection is said to require an act of observation, which continues to be a subject of contentious debate and will not be discussed here.

Strictly speaking the *Pilot-Wave rule* requires only an assignment of particle paths to interpret the wave function; whence, $\rho(\mathbf{x}, t)$ can be interpreted as a *density of paths.* However, for agreement with the Born rule it allows assignment of probabilities to the wave function in its initial conditions, which then propagate to probabilities at any subsequent time. Accordingly, these probabilities should not be interpreted as expressions of randomness inherent in Nature as commonly claimed for Schrödinger theory. Rather, consistent with its realist perspective, Pilot Wave theory regards probabilities in quantum mechanics as expressing limitations in knowledge of specific particle states (or paths). This viewpoint is best described by Bayesian probability theory, as most trenchantly expounded by Jaynes [25]. Accordingly, we regard the Born-Dirac wave function as specifying Bayesian conditional probabilities for electron paths.

The Schrödinger wave function in Pilot Wave theory is a many particle wave function. Here we confine attention to the single particle theory, and we review some well known specifics [24] to focus on crucial points.

With wave function

$$\psi = \rho^{1/2} e^{S/\mathbf{i}\hbar}, \tag{131}$$

Schrödinger's equation can be split into a pair of coupled equations for real functions $\rho = \rho(\mathbf{x}, t)$ and $S = S(\mathbf{x}, t)$ with scalar potential $V = V(\mathbf{x})$:

$$\partial_t S + \frac{(\boldsymbol{\nabla} S)^2}{2m} - \frac{\hbar^2}{2m}\frac{\boldsymbol{\nabla}^2 \rho^{1/2}}{\rho^{1/2}} + V = 0, \tag{132}$$

$$\partial_t \rho + \boldsymbol{\nabla} \cdot \left(\frac{\rho \boldsymbol{\nabla} S}{2m}\right) = 0. \tag{133}$$

Equation (132) can be written

$$(\partial_t + \frac{1}{m}(\boldsymbol{\nabla} S) \cdot \boldsymbol{\nabla})\boldsymbol{\nabla} S = -\boldsymbol{\nabla}(V + Q), \tag{134}$$

where

$$Q = Q(\mathbf{x}, t) = \frac{\hbar^2}{2m}\frac{\boldsymbol{\nabla}^2 \rho^{1/2}}{\rho^{1/2}}. \tag{135}$$

Identifying

$$m^{-1}\boldsymbol{\nabla} S = \mathbf{v} = \dot{\mathbf{x}} \tag{136}$$

as the velocity of a curve $\mathbf{x}(t)$ normal to surfaces of constant S, from (134) we get an equation of motion for the curve:

$$(\partial_t + \frac{1}{m}\dot{\mathbf{x}} \cdot \boldsymbol{\nabla})m\dot{\mathbf{x}} = m\ddot{\mathbf{x}} = -\boldsymbol{\nabla}(V + Q). \tag{137}$$

This has the form of a classical equation of motion, but with the classical potential V augmented by the quantity Q, commonly called the *Quantum Potential* to emphasize its distinctive origin.

A striking fact about Q is its influence on electron motion even in the absence of external forces. Its noteworthy use in [26] to compute particle paths in electron diffraction stimulated a resurgence of interest in Pilot Wave theory. That computation supported interpretation of Q as a "causal agent" in diffraction, but identification of a plausible "physical mechanism" to explain it has remained elusive. So interpretation of Q as an intrinsic property of the wave function that does not require further explanation has remained the default position in Pilot Wave theory.

The Pauli equation has been used to analyze the effect of spin on electron paths in 2-slit diffraction [27]. The

authors identify the correct generalization of the Pilot Wave *guidance law* (136) as

$$\dot{\mathbf{x}} = \boldsymbol{\nabla} S + \rho^{-1} \boldsymbol{\nabla} \times (\rho \mathbf{s}) \tag{138}$$

However, they failed to note the more fundamental fact that, even in Schrödinger theory, the "quantum force" is spin dependent, though that was spelled out in one of their references [28]. Indeed that reference derived the equation of motion

$$\rho m \ddot{\mathbf{x}} = \rho \mathbf{f} + \grave{\mathbf{T}}(\grave{\boldsymbol{\nabla}}), \tag{139}$$

where the accent indicates differentiation of the stress tensor $\mathbf{T}(\mathbf{n})$, and the applied force has the general form

$$\mathbf{f} = e[\mathbf{E} + \mathbf{v} \times \mathbf{B}/c] + \frac{e}{mc} \grave{\boldsymbol{\nabla}} \grave{\mathbf{B}} \cdot \mathbf{s}, \tag{140}$$

while components of the stress tensor are

$$\boldsymbol{\sigma}_i \cdot \mathbf{T}(\boldsymbol{\sigma}_j) = \frac{\rho}{m} \mathbf{s} \cdot [\partial_i \partial_j \mathbf{s} + \mathbf{s}\, \partial_i \partial_j \ln \rho] = T_{ji}. \tag{141}$$

When the spin vector $\mathbf{s}$ is constant, the stress tensor term in (139) reduces to the "Quantum force" $-\boldsymbol{\nabla} Q$ in Schrödinger theory. Thus we see that the $\hbar^2$ factor in Q comes from squaring the spin vector, and the Quantum force is actually a momentum flux. All this puts the diffraction problem in new light. Indeed, we shall see that spin dependence of the quantum force is even more obvious in Dirac theory.

Derivation of Pauli and Schrödinger equations as nonrelativistic approximations to the Dirac equation in [21] also traces corresponding changes in local observables. That brings to light many inconsistencies and omissions in standard treatments of those approximations. The most egregious error is failure to recognize that the Schrödinger equation describes the electron in an eigenstate of spin. Implications of that fact are discussed at length in [22].

Another surprising result from [21, 22] is proof that β makes an indisputable contribution to the energy in Pauli-Schrödinger theory, even though it has been banished from the wave function. It arises from the spin density divergence (96), which in the non-relativistic approximation takes the form

$$m_e c \rho \beta = -\boldsymbol{\nabla} \cdot (\rho \mathbf{s}). \tag{142}$$

This deepens the mystery of β. More clues come from solutions to the Dirac equation.

A. Pilot Waves in Dirac Theory

Extension of the Pilot Wave interpretation for nonrelativistic wave functions [23] to Dirac theory with STA has been critically examined at length in [29], where it is demonstrated with many examples that calculations and analysis with the Real Dirac equation is no more complicated than with the Pauli equation. Indeed, the first order form of the Dirac equation makes some of it decidedly easier. The treatment of scattering at potential steps is generalized to include both spin and oblique incidence, with STA simplifications not to be found elsewhere. The analysis of evanescent waves exhibits the flow of Dirac streamlines (without commitment to their interpretation as particle paths). The study of tunneling times shows how part of the wave packet passes through the barrier while part slows down and turns back. No notion of wave function collapse is needed to interpret observations. It is also shown that the distribution of tunneling times observed experimentally can be attributed entirely to structure of the initial wave packet, thus making it clear that, contrary to claims in the literature, no superluminal effects are involved. The general conclusion is that interpretation of Dirac streamlines as particle paths is consistent with the Dirac equation and helpful in physical interpretation.

Indeed, the fundamental equation for momentum balance in Dirac theory gives us a complete and straightforward relativistic generalization of Pilot Wave theory that seems not to have been recognized heretofore. One needs only to apply it to a single streamline $z = z(\tau)$ with proper velocity $\dot{z}$ and spin bivector $S = S(z(\tau))$. Then the equation can be put in the form of a generalized *Pilot Wave guidance equation*:

$$\Box \Phi = m_e c \dot{z} + S \cdot \Box \ln \rho + \dot{S} \cdot \dot{z}, \tag{143}$$

where

$$\Box \Phi = P - \frac{e}{c} A. \tag{144}$$

is the gradient of a generalized electron phase expressed in action units. This gradient expression may have important implications for electron diffraction. For a free particle, the generalized momentum P is necessarily a phase gradient. However, electron motion in diffraction might also be influenced through a vector potential generated by material in the guiding slits. Since the curl $\Box \wedge A$ must vanish in the vacuum near the slits, the vector potential is necessarily a gradient, so it can be combined with P as in (144). This possibility has been overlooked in the literature on diffraction. It may be crucial for explaining how the slits transfer momentum to each electron in diffraction.

The remaining piece of Pilot Wave theory is given by the conservation law for the Dirac current (78). Evaluated on the particle path it gives us

$$\Box^2 \Phi = -m_e c \dot{z} \cdot \Box \ln \rho, \tag{145}$$

which describes the evolution of path density.

The relativistic guidance law (143) not only combines the the two basic equations (132) and (138) of nonrelativistic theory into one, it generalizes the scalar *Quantum Potential* into a vector $S \cdot \Box \ln \rho$ and makes its spin dependence explicit.

To compare the two versions, we perform a spacetime split of (143), taking due account of their different notations. For the velocity split we have

$$\dot{z}\gamma_0 = \gamma + \dot{\mathbf{r}} \qquad \text{with} \qquad \gamma = c\dot{t}. \tag{146}$$

And for the spin bivector $S = isv$ we have the split

$$S = \mathbf{s} \times \dot{\mathbf{r}} + i\mathbf{s}_\perp. \tag{147}$$

where $\mathbf{s}_\perp = \gamma\mathbf{s} - s_0\dot{\mathbf{r}}$. Writing $a = \Box \ln \rho$ with the split $a\gamma_0 = a_0 + \mathbf{a}$ and using (32), we get the split of the "Quantum Vector Potential:"

$$(S \cdot a)\gamma_0 = (\mathbf{s} \times \dot{\mathbf{r}}) \cdot \mathbf{a} + a_0 \mathbf{s} \times \dot{\mathbf{r}} + \mathbf{a} \times \mathbf{s}_\perp. \tag{148}$$

Putting it all together, for the split of the guidance law (143), we get the generalization of (132) and (138):

$$c^{-1}\partial_t \Phi = m_e c\gamma + (\mathbf{s} \times \dot{\mathbf{r}}) \cdot \boldsymbol{\nabla} \ln \rho, \tag{149}$$

$$\boldsymbol{\nabla}\Phi = m_e c\,\dot{\mathbf{r}} + (c^{-1}\partial_t \ln \rho)\mathbf{s} \times \dot{\mathbf{r}} - \mathbf{s}_\perp \times \boldsymbol{\nabla} \ln \rho. \tag{150}$$

A detailed proof that the term $(\mathbf{s} \times \dot{\mathbf{r}}) \cdot \boldsymbol{\nabla} \ln \rho$ does indeed reduce to Bohm's quantum potential in the nonrelativistic limit is not needed here. Suffice it to say that both have been derived from Dirac's equation. The term $\dot{S} \cdot \dot{z}$ has been ignored in these equations, because it has no analogue in the nonrelativistic theory. It's implications are studied in the following Sections.

B. Cauchy problem with β

To solve the Cauchy problem for an electron, we need to project the Dirac equation to a spacelike hyperplane. Accordingly, we perform a spacetime split of operators in the Dirac equation (46) to put it in the form

$$(\partial_t + c\boldsymbol{\nabla})\Psi \mathbf{i}\hbar = m_e c^2 \Psi^* + e(A_0 - \mathbf{A})\Psi, \tag{151}$$

where the mysterious parameter β is hidden in the mass term with

$$\Psi^* = \gamma_0 \Psi \gamma_0, \tag{152}$$

with implications to be discussed later. The definition of electron energy in Dirac theory differs from the definition in the nonrelativistic theories by including the rest energy. As explained in [21], we can remove the rest energy while retaining the definition of energy in terms of the wave function by transforming the wave equation with

$$\Psi \to \Psi e^{-i\boldsymbol{\sigma}_3 m_e c^2 t/\hbar}. \tag{153}$$

to the equivalent form

$$(\partial_t + c\boldsymbol{\nabla})\Psi \mathbf{i}\hbar = m_e c^2(\Psi^* - \Psi) + e(A_0 - \mathbf{A})\Psi, \tag{154}$$

This equation is readily re-expressed in standard Hamiltonian form

$$\partial_t \Psi \mathbf{i}\hbar = \underline{\mathrm{H}}\Psi, \tag{155}$$

though the structure of the Hamiltonian operator $\underline{\mathrm{H}}$ may look unfamiliar at first..

Boudet has applied this approach to a thorough treatment of the Darwin solutions for Hydrogen and their application to basic state transitions [30]. (See also [29] for a somewhat different STA treatment.) For a stationary state with constant energy E and central potential $V(r)$, the wave function has the form

$$\Psi(\mathbf{r}, t) = \psi(\mathbf{r}) e^{-i\boldsymbol{\sigma}_3 E t/\hbar}. \tag{156}$$

And Boudet puts equation (151) in the form

$$\boldsymbol{\nabla}\psi = \frac{1}{\hbar c}[-E_0\psi^* + (E + V)\psi] i\boldsymbol{\sigma}_3, \tag{157}$$

where $E_0 = m_e c^2$. He then splits the wave function into even and odd parts defined by

$$\psi = \psi_e + i\psi_o \qquad \psi^* = \psi_e - i\psi_o \tag{158}$$

to split (157) into a pair of coupled equations for quaternionic spinors:

$$\boldsymbol{\nabla}\psi_e = \frac{1}{\hbar c}[-E_0 - E - V)\psi_o \boldsymbol{\sigma}_3, \tag{159}$$

$$\boldsymbol{\nabla}\psi_o = \frac{1}{\hbar c}[-E_0 + E + V)\psi_e \boldsymbol{\sigma}_3. \tag{160}$$

These he solves to get the Darwin solutions.

The same even-odd split was used in [21] to get nonrelativistic approximations to the Dirac equation. The split there mixes β and boost factors in a peculiar way with no obvious meaning. Indeed, the peculiar behavior of β and local velocity in the Darwin solutions defies any obvious physical interpretation in terms of local observables, with nodes separating positive and negative energy components in strange ways [18]. These facts are not even recognized in the standard literature, let alone regarded as problematic. Nevertheless, they pose a challenge to associating particle properties with the wave function. A capstone for this challenge is the following *virial theorem* for electron energy derived in [21]:

$$\langle E \rangle = m_e c^2 \left\langle \frac{\cos\beta}{v_0} \right\rangle = m_e c^2 \int d^3x \cos\beta, \tag{161}$$

where $v_0 = v(x) \cdot \gamma_0$ is the time component of the electron's velocity field. A straightforward interpretation of this result is that $\cos\beta(x)$ is a measure of energy density in the field of an electron. We will find confirming evidence for this interpretation later in Section VII.

C. Scattering and QED with zitter

The link between standard quantum mechanics (QM) and quantum electrodynamics (QED) passes through the Dirac equation. It is commonly claimed that the link requires second quantization with quantum field theory (QFT). But Feynman vehemently denied that claim. When the issue arose in a QED course I attended, I recall him dramatically remonstrating that, if anyone dares to defend axioms of QFT, "I will defeat him. I will CUT HIS FEET OFF!" (with a violent cutting gesture for emphasis). Indeed, the famous formula $[p, q] = i\hbar$, which Born proposed as a foundation of QM (and had engraved on his tombstone), cannot be as general as he thought. For there is no explanation why Planck's constant here is related to electron spin or the Dirac equation. Also, one can argue that QFT commutation relations for particle creation and annihilation operators are merely bookkeeping devices for multiparticle physics without introducing new physics. Let's look at how Feynman got along without it.

A reformulation of Feynman's approach to QED with STA is laid out in [31, 32], with explicit demonstrations of its advantages in Coulomb and Compton scattering calculations. For example, the S-matrix is replaced by a scattering operator S_{fi} that rotates and dilates the initial state to the final state, as expressed by

$$\psi_f = S_{fi}\psi_i \tag{162}$$

with

$$S_{fi} = \rho_{fi}^{1/2} R_{fi}, \tag{163}$$

where R_{fi} is a rotor determining the change in direction of spin as well as momentum, while $\rho_{fi} = |S_{fi}|^2$ is a scalar dilation factor determining the cross section.

Feynman linked QM to QED by reformulating the Dirac equation as an integral equation coupled to Maxwell theory through the vector potential:

$$\psi(x) = \psi_i(x) - e\int d^4x' S_F(x - x')A(x')\psi(x'). \tag{164}$$

This solves the Dirac equation (46) with $p_0 = m_e c\gamma_0$ if the Green's function $S_F(x - x')$ satisfies the equation

$$\begin{aligned}\Box S_F(x - x')M(x')\mathbf{i} - S_F(x - x')M(x')p_0 \\ = \varphi^4(x - x')M(x'),\end{aligned} \tag{165}$$

where $M = M(x)$ is an arbitrary multivector valued function of x. It has the *causal solution*

$$\begin{aligned}S_F(x - x')M\mathbf{i} = &-\frac{\Theta(t - t')}{(2\pi)^3}\int \frac{d^3p}{2E}(pM + Mp_0)\mathbf{i}e^{-\mathbf{i}p\cdot(x-x')} \\ &+ \frac{\Theta(t - t')}{(2\pi)^3}\int \frac{d^3p}{2E}(pM + Mp_0)\mathbf{i}e^{\mathbf{i}p\cdot(x-x')},\end{aligned} \tag{166}$$

where $E = p \cdot \gamma_0 > 0$. Note that $S_F(x - x')$ is a linear operator on M here. In general M does not commute with p, p_0, or the bivector $\mathbf{i} = i\boldsymbol{\sigma}_3$, so it cannot be pulled from under the integral.

We can draw several important conclusions from the present approach to QED. One advantage of the integral form (164) for the Dirac equation is that the causal boundary condition (166) explicitly enforces the association of electron/positron states with positive/negative energy states respectively. As noted in Section IVB, these states can be switched by multiplication with the pseudoscalar i.

At this point, permit me to insert a relevant anecdote that I heard Feynman tell on himself. One day, when he was demonstrating his spectacular prowess at complex QED calculations, a brave student objected: "You can't normalize negative energy states to plus one, you must use negative one." "O yes I can!" retorted Feynman with the confidence of one who had won a Nobel prize with his calculations and demonstrated them repeatedly over more than a decade in QED courses and lectures. Then he proceeded to prove that the student was right! Sure enough, check out eqn. (62) to see that the minus sign comes from squaring the unit pseudoscalar (which, of course, Feynman never did learn)!

Returning to the main point, we note that the absence of a β-factor $e^{i\beta}$ in the scattering operator (163) shows that positive and negative energy states are not mixed in scattering. Indeed, the question of a β-factor never arises in QED, because all calculations are based on plane waves without it, and it is not generated by conventional wave packet construction.

Of course, the Born rule is not an intrinsic feature of the Dirac equation, but is imposed only for purposes of interpretation. It is important, therefore, to recognize that results of plane wave scattering have a straight forward geometric interpretation without appeal to probability: Indeed, the Dirac equation generates a unique spacetime path for each point on an initial plane wave. The conservation law for the Dirac current implies that these paths do not intersect, though they may converge or separate. Accordingly, if we assign uniform density to paths beginning on the initial plane wave, then the scattering operator determines the density of particle paths intersecting a surface surrounding the scattering center. In other words. *the squared modulus ρ of the Dirac wave function specifies the density of particle paths!* This is a completely geometric result, independent of any association with probabilities. Of course, for experimental purposes the density of paths can be interpreted as a particle probability density, but no inherent randomness in nature is thereby implied.

The bottom line is that QED scattering is fundamentally about paths.

Our STA formulation reveals another aspect of QED that has been generally overlooked and may be fundamental; namely, the existence of zitter solutions and the possibility that they may describe a fundamental feature

of the electron. As we have seen, zitter wave functions with opposite chirality can be obtained from a general wave function Ψ by projection with a lightlike "zitter boost"

$$\Sigma_\pm = \gamma_\pm\gamma_0 = (\gamma_0 \pm \gamma_2)\gamma_0 = 1 \pm \boldsymbol{\sigma}_2. \tag{167}$$

Thus, for *Majorana states* introduced in (75), we obtain

$$\Psi_\pm(x) = \Psi(x)\Sigma_\pm = (\rho e^{i\beta})^{1/2} R\,\Sigma_\pm, \tag{168}$$

where, as before, $R = R(x)$ is a general spacetime rotor, though we may wish to make the phase explicit by writing $R = Ve^{\mathbf{i}\varphi}$. Then the β-factor can also be incorporated into the rotor R, to give us

$$e^{i\beta/2}Ve^{\mathbf{i}\varphi}\Sigma_+ = Ve^{i\boldsymbol{\sigma}_3\varphi}e^{i\boldsymbol{\sigma}_2\beta/2}\Sigma_+, \tag{169}$$

because the Σ_+ factor converts it to a rotation:

$$e^{i\beta/2}\Sigma_+ = e^{i\boldsymbol{\sigma}_2\beta/2}\Sigma_+ = \Sigma_+ e^{i\boldsymbol{\sigma}_2\beta/2}. \tag{170}$$

Note that the β-rotation will occur before the phase-rotation in expressions for local observables given below. Thus, *the β-factor tilts the spin vector before the phase rotation in the spin plane.* In other words, it is a "geometric tilting factor." At last we have here a clear geometric meaning for the parameter β! And we have already noted that β is inextricably connected with spin as shown by the conservation law (96) and its appearance in Pauli-Schrödinger theory in Section V.

In the next two Sections we see how these puzzling features of β suddenly make perfect sense with a simple adjustment in the Dirac equation with far-reaching physical implications.

VI. ELECTRON ZITTER

Soon after Dirac's spectacular success in accounting for electron spin and the Hydrogen spectrum, it was discovered that the electron and its antiparticle were conjoined in the Dirac equation like a pair of Siamese twins. Though this was touted as a fabulous prediction as soon as the positron was discovered, the doctors of Quantum Mechanics concluded that the twins must be surgically separated. The diagnosis and surgical procedure is most completely explained by early practitioners such as Furry and Oppenheimer [33]. The eventual outcome, of course, is known as QED today.

In this Section we reexamine the anatomy of the Dirac equation to identify structural features that can guide a cleaner separation of the twins. In particular, we identify zitter as the "beating heart" of the electron and note that it has been split in two in the standard twin separation, only to be "sewn back together" in QED. Consequently, we introduce a new surgical procedure that highlights zitter as a central property of the electron.

Dirac's strong endorsement [34] of Schrödinger's *zitterbewegung* [35] as a fundamental property of the electron has remained unchallenged to this day, though it plays little more than a metaphorical role in standard quantum mechanics and QED. However, evidence is mounting that zitterbewegung is a real physical effect, observable, for example, in Bose-Einstein condensates [36] and semiconductors [37]. Analysis with a variant of the model proposed here even suggests that zitterbewegung has been observed already as a resonance in electron channeling [10, 38]. That experiment should be repeated at higher resolution to confirm the result and identify possible fine structure in the resonance [39].

Theoretical analysis of *zitterbewegung*, or just *zitter*, requires a formulation in terms of local observables. We have already noted that the zitter frequency is inherent in the phase of the Dirac wave function. But Schrödinger claimed more, namely, that it is to be interpreted as a frequency of position oscillations at the speed of light about a mean velocity, and it has been further claimed that association of electron spin with circular zitter was implicit in his analysis [40].

Fortunately, the adjustment required to incorporate zitter into standard Dirac theory is fairly straightforward, so we can brief. Accordingly, we define the *"Zitter Particle Model"* (ZPM) to restore those fluctuations. More generally, we see that lightlike zitter velocity factors the Dirac Lagrangian into separate electron and positron parts.

Revision of the Dirac equation to describe an electron with lightlike paths can be neatly formalized with the projection operators in (168). Zitter boosts possess the reversion, idempotence and orthogonality properties

$$\widetilde{\Sigma}_\pm = \Sigma_\mp, \quad (\Sigma_\pm)^2 = 2\Sigma_\pm, \quad \Sigma_\pm\widetilde{\Sigma}_\pm = 0, \tag{171}$$

as well as

$$\Sigma_\pm - \widetilde{\Sigma}_\mp = \pm 2\gamma_2\gamma_0 = \pm 2\boldsymbol{\sigma}_2, \tag{172}$$

which specifies the timelike plane of the zitter boost. Consequently, we have lightlike local observables for electron current:

$$\tfrac{1}{2}\Psi_+\gamma_0\widetilde{\Psi}_+ = \Psi_+\gamma_0\widetilde{\Psi} = \Psi(\gamma_0 + \gamma_2)\widetilde{\Psi} = \rho u, \tag{173}$$

and for spin bivector:

$$\tfrac{1}{2}\Psi_+ i\boldsymbol{\sigma}_3\widetilde{\Psi}_+ = \Psi(\gamma_0 + \gamma_2)\gamma_1\widetilde{\Psi} = \rho e^{i\beta}ue_1, \tag{174}$$

or, for $S = isu$,

$$\tfrac{1}{2}\hbar\Psi_+ i\boldsymbol{\sigma}_3\widetilde{\Psi} = \tfrac{1}{2}\hbar\Psi\gamma_1\gamma_+\widetilde{\Psi} = \rho e^{i\beta}S. \tag{175}$$

The bottom line is a claim that observables of the wave function $\Psi_+(x)$ describe a congruence (or fibration, if you will) of lightlike helical paths with the circular period of an electron clock. Next we aim to extract individual fibrations from the wave function to create a well-defined particle model of electron motion.

We assume that the lightlike helical path of a fiber in the wave function has a well-defined center of curvature

with a timelike path with velocity $v = v(\tau)$ that we identify as a Center of Mass (CM) for the electron. Accordingly, we regard the electron as a *particle* with intrinsic spin and internal clock.

A classical model of the electron as a point charge circulating with a lightlike velocity $u(\tau)$ around a center of mass with timelike velocity $v(\tau)$ identified with the Dirac current has been proposed independently by Martin Rivas [41]. It is in general agreement with the zitter model developed here.

A. Zitter Particle Model

It should be recognized that the Dirac equation by itself does not imply any relation of the wave function to electron velocity. A fundamental question in Dirac Theory, therefore, is how to relate observables in the Dirac equation to particle position or path. The *zitter particle model* (ZPM) presented here models electron velocity as a lightlike vector and defines a complete set of local observables consistent with that.

We have seen that the hand of the electron clock rotates with the zitter frequency, so it is natural to identify the velocity of circulation with the vector e_2 while e_1 is the direction of the zitter radius vector. Since there are two senses to the circulation corresponding to *electron/positron*, we have two null vector particle velocities:

$$e_\pm = v \pm e_2 = R\gamma_\pm \widetilde{R}, \quad \text{with} \quad \gamma_\pm = \gamma_0 \pm \gamma_2. \tag{176}$$

It suffices to restrict our attention to the electron case and redefine the local observables to incorporate zitter. Our choice of sign here is a convention in agreement with [10].

Accordingly, we define the electron's "*zitter velocity*" u by

$$u = R\gamma_+\widetilde{R} = v + e_2 \tag{177}$$

and we define the " *spin bivector*" S introduced in (174) by

$$S = \frac{\hbar}{2} R\gamma_1\gamma_+\widetilde{R} = hu, \quad \text{where} \quad h = \frac{\hbar}{2}e_1 \tag{178}$$

is identified as the *hand* $h = h(\tau)$ of the electron clock.

Note that the null velocity $u^2 = 0$ implies a null spin bivector $S^2 = 0$. Using the identities

$$ue_1 = e_0e_1 + ie_3e_0 = ie_3u, \tag{179}$$

we can write S in the several equivalent forms:

$$S = ius = hu = (h + is)v = hu + \overline{S}. \tag{180}$$

To further designate the vector h, let me coin the term "*spinet*" (that which spins) as counterpart of the "spin" (vector) s. In the same spirit let me call S the electron "*spindle*" and note how it integrates spin and spinet into a single mechanism. The overbar designates a "*zitter average*," that is, an average over the zitter period $\tau_e = 2\pi/\omega_e$. So the "*linear velocity*" $v = \bar{u}$ is an average of the chiral velocity u, and, since $\bar{h} = 0$, the spin bivector $\overline{S} = ivs$ is the zitter average of the *spindle* S.

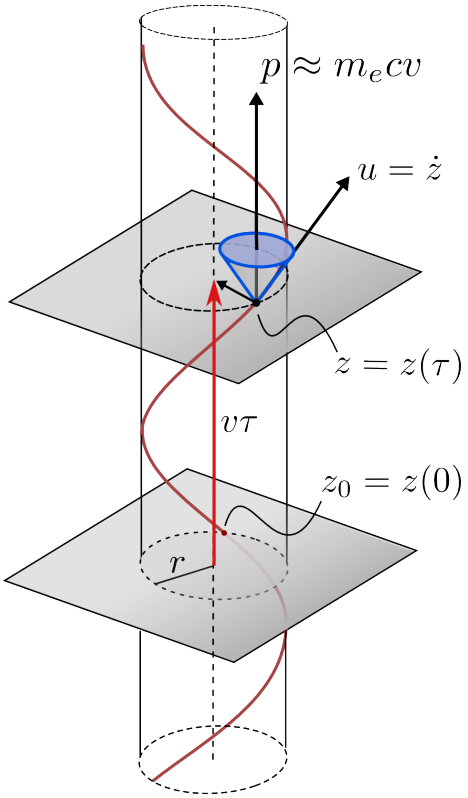

FIG. 2. In the Zitter Particle Model the electron path is an oriented lightlike helix with an opposite orientation (chirality) for electron and positron.

B. Zitter Kinematics

The electron's local observables are now restricted to a comoving frame attached to the particle path:

$$e_\mu = e_\mu(\tau) = R\gamma_\mu\widetilde{R}, \tag{181}$$

where $R = R(\tau)$ is a rotor with spin vector $s = (\hbar/2)e_3$ and spin bivector $S = isu$ as defined above. We note from (202) that its angular velocity is specified by the bivector

$$\Omega \equiv \Omega(z(\tau)) = 2\dot{R}\widetilde{R}, \tag{182}$$

so

$$\dot{e}_\mu = \Omega \cdot e_\mu \tag{183}$$

on the electron path, As illustrated in FIG. 2.

As developed to this point, our Zitter Particle model has much in common with classical models for a "particle

with spin" considered by many authors [10], so it is of interest to see what they can contribute to our analysis. It is reassuring to know that the self consistency of those models was established by derivation from a Lagrangian in [10]. Since the kinematic details align perfectly with our present model, we can restrict our attention to the key kinematical equation studied there. In particular, the relevant equation of motion for the rotor $R = R(\tau)$ has the strange but simple form:

$$\hbar\dot{R}\gamma_+\gamma_1 = \frac{\hbar}{2}\Omega R\gamma_+\gamma_1 = pR\gamma_+ + i\beta R. \tag{184}$$

It's interpretation is greatly facilitated as an equivalent equation in terms of local observables. Namely,

$$\Omega S = pu + i\beta, \tag{185}$$

where (178) gives us $S = hu = (\hbar/2)e_1u$. The bivector part of this expression gives us the spin equation of motion:

$$\dot{S} = \Omega \times S = p \wedge u. \tag{186}$$

And the scalar part gives us an expression for *particle energy*:

$$p \cdot u = \Omega \cdot S = \dot{h} \cdot u > 0. \tag{187}$$

We cannot divide (185) by the null vector u to solve for p, but we can divide by v to get a comparable expression for *momentum*

$$p = (p \cdot v)u + \dot{S} \cdot v. \tag{188}$$

Accordingly, we can identify the coefficient for the first term as a (possibly variable) *dynamical mass*,

$$p \cdot v/c \equiv m_d, \tag{189}$$

while the second term describes intrinsic angular momentum in the zitter that we call *spin momentum*:

$$q \equiv \dot{S} \cdot v = -v \cdot \dot{S}. \tag{190}$$

By the way, (184) also gives us an explicit expression for acceleration of the *zitter velocity*

$$\dot{u} = \Omega \cdot u = p \cdot S. \tag{191}$$

Finally, to complete the analysis of (185), we use (180) to solve its pseudoscalar part for the parameter $\beta = \beta(z(\tau))$ as a function of the particle path:

$$\beta = \langle \Omega s u \rangle = u \cdot \Omega \cdot s = \dot{u} \cdot s = u \cdot \dot{s}. \tag{192}$$

This is our first clear clue for a physical interpretation of β, expressing it as a relation between electron spin and velocity.

Since the spin momentum is an unfamiliar concept in conventional quantum mechanics, it will be worth our time next to examine its properties. Using $S = ius$ we get

$$q \cdot v = (\dot{s} \wedge u + s \wedge \dot{u}) \cdot v = (\dot{s} \wedge u) \cdot v = 0. \tag{193}$$

Spin and kinetic momenta are orthogonal to one another, because

$$q \cdot v = (\dot{S} \cdot v) \cdot v = \dot{S} \cdot (v \wedge v) = 0. \tag{194}$$

Hence, (188) gives us

$$p^2 = (m_e c v)^2 + (\dot{S} \cdot v)^2 = m_e^2 c^2 - (s \wedge \dot{u})^2. \tag{195}$$

This suggests that

$$-(s \wedge \dot{u})^2 = (\dot{S} \cdot v)^2 = -(p \wedge v)^2 \tag{196}$$

is a measure of energy (or mass) stored "in" an accelerated electron. Also, it should be understood that the "*spin momentum*" term $\dot{S} \cdot v = \dot{v} \cdot S$ describes linear momentum due to internal angular momentum, like a flywheel in a macroscopic moving body.

The ZPM is not complete until we specify its kinematics relating the particle velocity to its spacetime path. Accordingly, we define

$$r_e = \lambda_e e_1 \qquad \text{with} \qquad \lambda_e = \frac{\hbar}{2m_e c} = c/\omega_e \tag{197}$$

as the radius vector for circular zitter at the speed of light. The zitter center follows a timelike path $z = z(\tau)$ with velocity $v = \dot{z}$. Hence, the particle path $z_+ = z_+(\tau)$ with lightlike velocity

$$u = \dot{z}_+ = v(\tau) + \dot{r}_e(\tau) \tag{198}$$

as depicted in Fig.2. This integrates to

$$z_+(\tau) = z(\tau - \tau_c) + r_e(\tau), \tag{199}$$

where, as we shall see, the time shift τ_c depends on the energy from external interactions. Note that the time variable is the proper time of the zitter center.

An especially attractive feature of the ZPM is the physical interpretation it gives to e_1 and e_2 as the hand of an electron clock and its rate of motion. From (197) see that $e_1 = \hat{r}_e$ is the unit radius vector of the zitter, and from (177) we see that $e_2 = \dot{r}_e$ is the zitter velocity.

To understand the mechanics of the electron clock we introduce the concept of *canonical momentum* next.

C. Canonical Momentum

The real Dirac wave function has the *canonical form* given in (47) by

$$\Psi = \rho^{\frac{1}{2}} e^{i\beta/2} R = \Psi(x), \tag{200}$$

and we spent the bulk of this paper showing how its eight degrees of freedom can be interpreted in terms of local

observables. Let it review the main points to see how they relate to the problematic parameter β and the electron's *Canonical Momentum*.

We have assumed that the electron path is embedded smoothly in the spacetime manifold, so the comoving frame of local observables extends to a field

$$e_\mu = e_\mu(x) = R\gamma_\mu \widetilde{R}, \tag{201}$$

and the dynamics of the local observables is determined by the linear bivector-valued function

$$\Omega_\mu = \Omega_\mu(x) = 2(\partial_\mu R)\tilde{R}, \tag{202}$$

so

$$\partial_\nu e_\mu = \Omega_\nu \cdot e_\mu. \tag{203}$$

Components of the canonical momentum are then given by

$$P_\mu = \Omega_\mu \cdot S, \tag{204}$$

and the spin dynamics is specified by

$$\partial_\mu S = \Omega_\mu \times S, \tag{205}$$

where $S = hu$ in accordance with (180).

The rotor factor $R = R(x)$ in the general Dirac wave function is normalized to $R\widetilde{R} = \widetilde{R}R = 1$. It has a unique decomposition into the product

$$R = UVU_1U_2, \tag{206}$$

where rotor $V = (v\gamma_0)^{\frac{1}{2}}$ defines a boost to the electron's center of mass, with spatial rotors

$$U = U(x) = e^{-i\boldsymbol{\sigma}_3\varphi/2} \tag{207}$$

and

$$U_1U_2 = e^{-i\boldsymbol{\sigma}_3\varphi_1/2}e^{-i\boldsymbol{\sigma}_1\varphi_2/2} \tag{208}$$

that can be parametrized with scalar phase functions $\varphi(x), \varphi_1(x), \varphi_2(x)$, thus with three degrees of gauge freedom.

Let's refer to R as the electron's *canonical rotor*, to U as its *electric rotor* and to U_1U_2 as its *magnetic rotor*. For the record, the boost rotor is discussed in [3], which gives the general result

$$V = (v\gamma_0)^{\frac{1}{2}} = \frac{1 + v\gamma_0}{[2(1 + v \cdot \gamma_0)]^{1/2}}. \tag{209}$$

For quantized states indexed with familiar quantum numbers n, ℓ, m employed in atomic physics, we can write the most general rotor for an electron in the form

$$R_{n,\ell,m} = U_n V U_\ell U_m. \tag{210}$$

We will conclude in later Sections that these quantum numbers characterize the electron's magnetic field under any circumstance, not just when it is an atomic state.

A further simplification is worth considering, namely omitting the boost in (209) to reduce (210) to the spatial rotor

$$U_{n,\ell,m} = U_n U_\ell U_m. \tag{211}$$

with all the same quantum numbers but described in an *inertial system* specified by the constant timelike vector γ_0. These details are necessary for the most general electron state. But the essential part for electron motion is specified by the canonical momentum and the spin bivector as described next.

Along the electron particle path with velocity $v = v(\tau)$, eqns (204) and (205) reduce to

$$P \cdot v = \Omega \cdot S = \Omega \cdot (hu), \tag{212}$$

and

$$\dot{S} = \Omega \times S. \tag{213}$$

where $\Omega = \Omega(\tau) = \Omega(z(\tau)) = v^\mu \Omega_\mu(z(\tau))$. Applying the vector derivative evaluated on the particle path we have $\square \wedge v = (v\, d/d\tau) \wedge v = v\dot{v}$ to get

$$\Omega = v\dot{v} + (\hat{P} \cdot v)e_1 u, \tag{214}$$

with $|P| = \hbar/2$. Hence, for motion of the "electron clock hand" $h = e_1\hbar/2$ we have

$$\begin{aligned} \dot{h} &= (\dot{v}v) \cdot h + P \cdot v(e_1 u) \cdot e_1 \\ &= -(\dot{v} \cdot h)v + (P \cdot v)u. \end{aligned} \tag{215}$$

This shows the role of the canonical momentum P in evolution of the electron clock explicitly. In the electron rest frame this reduces to a generalization of (74)

$$e_1(\tau) = \gamma_1\, e^{\mathbf{i}\varphi(\tau)} = U\gamma_1 U^\dagger, \tag{216}$$

where rotor $U = U(z(\tau))$ is given by (207) and $\varphi(\tau)$ generalizes the free particle phase $\omega_e\tau$.

Remarkably, our model of the electron as a particle with circular zitter was proposed by Slater [42] well before the Dirac equation and Schrödinger's zitterbewegung. His argument linking it to the null Poynting vector of the photon may also prove prophetic. Of course, we get much more than Slater could by embedding the model in Dirac theory.

D. Extended Lorentz Force

At last we are prepared to consider specific implications of the ZPM with profound physical significance.

Electron momentum $p = p(x - z(\tau))$ is a gauge invariant vector field $p = P - \frac{e}{c}A$ independent of the density ρ. That means it is invariant under a gradient shift $P \to P' = P + \square\,\theta$ of the canonical momentum.

The momentum curl has the strikingly simple form

$$\square \wedge p = -\frac{e}{c}F + \square \wedge P, \tag{217}$$

where $F = \square \wedge A$ is the external electromagnetic field. Since $p = p(z(\tau))$ on the particle path,we have

$$v \cdot \square\, p = \dot{p} \quad \text{and} \quad \square\, p = v\dot{p}. \tag{218}$$

Thus, for momentum on the particle path we have

$$v \wedge \dot{p} = -\frac{e}{c}F + v \wedge \dot{P}, \tag{219}$$

Accordingly, on dotting (219) with the velocity v, we get the relativistically invariant Lorentz force law

$$\dot{p} = \frac{e}{c}F \cdot v + \dot{q}, \tag{220}$$

with an additional term $\dot{q}$ that we interpret as momentum released to the vacuum by acceleration of the electron due to the Lorentz force.

Based on the analysis in preceding subsections, we propose identifying q with the *spin momentum* defined by (190) and propose the specific form

$$q = q(z(\tau)) \equiv p - P = -s\dot{\beta}, \tag{221}$$

This agrees with identifying

$$\overline{T}(s) = -\rho s\dot{\beta} = \rho v \cdot (s \wedge \square\beta) \tag{222}$$

as the flux in the spin direction derived from the *Tetrode tensor* (123). We will confirm this conclusion with a more direct argument later on. And we shall see that the Lorentz force also has a spin dependent component responsible for the Stern-Gerlach effect.

Let's call (220) the *Extended Lorentz Force* (ELF) equation, because it extends the interaction $F \cdot v$ which is orthogonal to velocity v with a component that is collinear with the spin s. Note that this generalization of the Lorentz Force offers a simple solution to the long-standing problem of explaining why atomic states don't radiate away all their energy. It tells us that energy released by an accelerating electron is always orthogonal to the force and along the spin direction.

Later we will identify q with the momentum released by an electron when it emits a photon. Thereby we finesse the notorious difficulties of the much studied *Lorentz-Dirac* equation [43], such as pre-acceleration, and runaway solutions. Instead, radiative reaction is generated directly by photon production, with energy (mass) carried with momentum q.

E. Zitter Dynamics

Electron dynamics is governed by the Dirac equation (93), which has been completely reformulated in terms of local observables in (126) to make manifest its physical structure and interpretation. We repeat that equation here because of its seminal importance:

$$\rho(P - \frac{e}{c}A)e^{i\beta} = m_e c\rho v - \square(\rho e^{i\beta}S). \tag{223}$$

As we shall explain, this enables a rigorous and transparent interpretation of Dirac theory without any modification or approximation. Let's call it the *chiral Dirac* equation to emphasize it's equivalence to a conventional formulation in terms of the spinor wave function given by (46), albeit using STA instead of an equivalent matrix algebra.

Chiral Dirac (223) is a multivector equation, so it can be separated into vector and trivector parts as two independent equations. It appears that *the physical content of this equation resides entirely in its vector part*, where the Dirac current resides. The trivector part seems to be a consistency constraint among the local observables and we need not discuss it further here.

We interpret the vector part of *Chiral Dirac* as a constitutive equation for the energy-momentum content in the Dirac current:

$$\rho(P - \frac{e}{c}A)\cos\beta = m_e c\rho v - \square \cdot (\rho e^{i\beta}S). \tag{224}$$

We hold that this equation applies for all values of $\cos\beta$ with positive values for the electron and negative values for the positron. Thus we have a unified model of the electron as a single entity with a range of energy-momentum states given by $\cos\beta$. Here at last we can understand the physical significance of the mysterious parameter β.

The terms "*spinet*" and "*spindle*" were introduced to emphasize significance of the null bivector $S = hu$ given by (180). In particular, it follows from (170), that S is eigen(bivector) for a rotation with the duality factor $e^{i\beta}$ as its eigenvalue, as specified by the equation

$$e^{i\beta}S = R_e S\widetilde{R}_e = R_e^2 S. \tag{225}$$

The unique solution to this equation is the rotor R_e given by

$$R_e^2 = e^{X\beta}, \quad \text{where} \quad X = h/s = e_1 e_3 \tag{226}$$

is the bivector generator of rotations in the *spin-spinet* plane. Let's call them *chirality* rotations.

Note that the chirality rotation (225)

$$R_e S\widetilde{R}_e = R_e d\widetilde{R}_e u = e^{X\beta}S = h(\beta)u \equiv \underline{S} \tag{227}$$

can be interpreted as *a tilt of the zitter spin axis $s(\tau)$ with respect to the plane of the charge* circulating with velocity $u(\tau)$, as illustrated in the description of the Spindle in FIG. 3. For a polarized electron with velocity $v(\tau)$ collinear with the spin $s(\tau)$, the circulating charge generates a helical path with energy proportional to its *pitch* $= \cos\beta$, which gives us at last a clear physical interpretation for the parameter β; This makes it clear that the electron properties of spin and zitter reside entirely in the "*spindle*" $\underline{S} = S(\beta)$, where the underbar notation emphasizes a functional dependence on the *chirality angle* β.

The electron's spindle ring has two independent degrees of freedom. The twofold degeneracy was lifted by

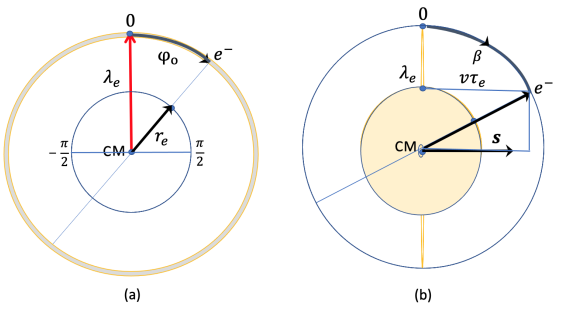

FIG. 3. **Spindle structure of an electron**: Picture the *Energy Shell* of the electron as a sphere of radius λ_e with two orthogonal cross sections. Section (a) depicts the Spindle ring with a pole fixed as a reference point, and the electron position $\mathbf{r}$ located on an *Energy Bubble* generated by accelerating the electron. Section (b) depicts the tilt angle β measuring energy $\hbar\omega \leq \hbar\omega_e$ in the bubble and its direction of propagation along the spin $\mathbf{s}$. (not to scale).

Schwinger's famous calculation for the electron's anomalous magnetic moment, raising the circular orbit to a toroidal tube with radius $\alpha_e/2\pi$. See FIG.4.

F. Conservation Laws

We see the full significance of β emerge when we identify it as a measure of stored energy in the Chiral Dirac equation (224). We identify the left side of that equation with momentum density

$$\rho \underline{p} \equiv \rho p \cos\beta = \rho(P - \frac{e}{c}A)\cos\beta. \tag{228}$$

The gauge invariant factor $(P - \frac{e}{c}A)$ includes a vector potential $A = A(x)$ for all external interactions. Specification of the scalar density $\rho = \rho(x)$ is discussed in a later Section. We will identify the factor $\cos\beta(x)$ as a measure of the energy extracted from the electron's momentum when it accelerates. This is in accord with the little known virial theorem (161) implying that $\cos\beta(x)$ is a measure of energy density in the vacuum.

With the factor β incorporated into the spindle (227), we can interpret (224) as an equation for momentum density balance:

$$\rho \underline{p} = m_e c \rho v - \Box \cdot (\rho \underline{S}). \tag{229}$$

Taking the divergence of this equation and using the conservation law (78) for the Dirac current

$$\Box \cdot (\rho v) = 0$$

along with the identity

$$\Box \cdot [\Box \cdot (\rho \underline{S})] = (\Box \wedge \Box) \cdot (\rho \underline{S}) = 0$$

then gives us the Gordon conservation law:

$$\Box \cdot (\rho \underline{p}) = 0. \tag{230}$$

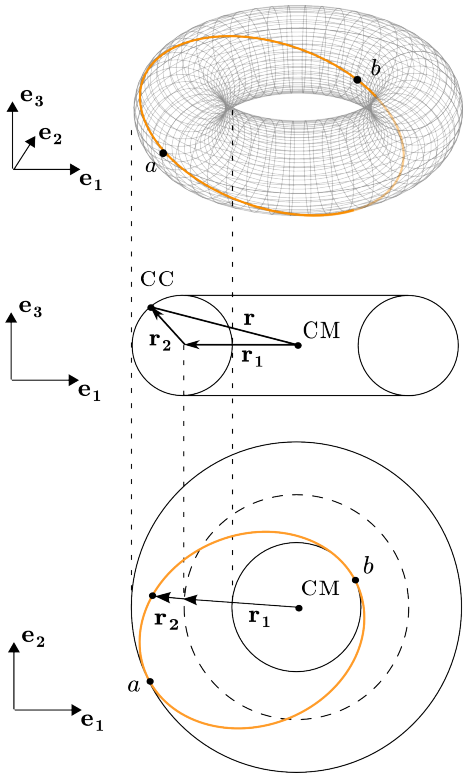

FIG. 4. In a plane rotating with angular velocity $\omega_1 \boldsymbol{\sigma}_3$ around the CM, the electron's lightlike circular orbit lies on a toroidal surface called the *electron energy shell* that projects to the orbital plane as an ellipse with antipodes a and b. Fig. from [44],

We interpret this as a conservation law for energy-momentum shared between electron and its ambient electromagnetic field.

The Chiral Dirac equation (229) is a field equation. Extracting the particle path with velocity $v = v(\tau)$ as a fiber in the field we get an explicit expression for spin dependence of electron momentum:

$$\underline{p} = m_e c v + \dot{\underline{S}} \cdot v + \underline{S} \cdot \Box \rho. \tag{231}$$

This is a completely general consequence of the Dirac equation without any approximation. Note that the last term measures deviations from the particle path as expected from a Stern-Gerlach force on electrons.

Ambiguities in the physical interpretation of the Dirac current ρv have bedeviled quantum mechanics since its inception. In (229) we identify the left side with Center of Mass (CM) flow. In contrast, we identify $\Box \cdot (\rho \underline{S})$ with a flow of charge, which is expressed in terms of a divergence because the electron charge is displaced from the CM by the zitter radius λ_e and circulates around it at the speed of light.

The chiral Dirac equation (224) is a field equation defined in terms of smooth functions $f(x)$ of spacetime points x, while electron particle paths are embedded as fibers in the field.

We have seen that zitter can be incorporated into the Dirac equation simply by replacing the timelike spin bivector $S = isv$ with the null spin bivector $S = isu$ given by (180). With this replacement, zitter is immediately incorporated into the formulation and analysis of conservation laws for energy-momentum, angular momentum and spin in Section IV, so they are ready for any application to the electron. Of course the ELF equation is just the energy-momentum law discussed above. We leave detailed treatment of orbital angular momentum and spin conservation for another time, though the fundamentals are already formulated in Section IV.

VII. FROM ZITTER TO ZILCH

As first shown in [1], the Faraday bivector $F = \mathbf{E} + i\mathbf{B}$ can be put in the unique invariant form:

$$F = \mathbf{f}e^{i\beta} = \mathbf{f}\cos\beta + i\mathbf{f}\sin\beta, \tag{232}$$

where the exponential specifies a duality transformation through an angle given by

$$\tan 2\beta = \frac{2\mathbf{E}\cdot\mathbf{B}}{\mathbf{E}^2 - \mathbf{B}^2} = \frac{iF\wedge F}{F\cdot F}. \tag{233}$$

Note that (232) determines *a rest frame in which the electric and magnetic fields are parallel without using a Lorentz transformation.* In addition, the squared magnitude of $\mathbf{f}$ is

$$\mathbf{f}^2 = [(\mathbf{E}^2 + \mathbf{B}^2)^2 - 4(\mathbf{E}\times\mathbf{B})^2]^{\frac{1}{2}}, \tag{234}$$

which is a tensor invariant of the Poynting vector for F:

$$T^\mu = \tfrac{1}{2}\tilde{F}\gamma^\mu F = \tfrac{1}{2}\tilde{\mathbf{f}}\gamma^\mu\mathbf{f}. \tag{235}$$

This belongs to a family of invariants of the electromagnetic field called *Zilch tensors* [45, 46]. All these tensors are independent of the value for the angle $\beta = \beta(x)$ in (232). Ironically, that angle is the most important feature of Zilch, so we give it a special name: **Zilch angle** or **Zilch function** or just **Zilch** and propose to designate it with a special symbol, such as

$$\phi = \phi(x) \quad \text{or} \quad \Phi = \Phi(x), \tag{236}$$

where the "null slash" symbol is suggestive of the vacuum. One reason the significance of this quantity has been overlooked heretofore is that its value depends on coupling the Dirac equation with its ambient EM field, and that coupling is not often considered in classical EM theory.

Indeed, we shall show that the **Zilch** angle can be identified with the function $\beta = \beta(x)$ in the canonical form (47) for the Dirac wave function, so we can write

$$\phi = \phi(x) \quad \Leftrightarrow \quad \beta = \beta(x) \tag{237}$$

interchangeably. For reasons explained in preceding Sections, this implies immediately that **Zilch** *can be interpreted as a measure of energy density in the electromagnetic vacuum.* That calls for analysis of the β role in energy exchange between electron and field which we consider below.

Like Maxwell's equation, the Dirac equation is a field equation defined on all of spacetime. The ***Zilch*** *function $\beta = \beta(x) = \beta(\mathbf{x}, ct)$ defines a mapping between these two equations as well as an embedding of particle paths in the Dirac field.* This Section presents details of that embedding for a comprehensive theory of the electron.

A. Dynamics with Zilch: Quantum Force & Current

Recall that $v = v(\tau) = e_0$ is the CM velocity, while $u = u(\tau)$ is the particle velocity of the electron charge circulating in a spacelike plane specified by the unit bivector $e_2(\tau)e_1(\tau) = e_2e_1 = ie_3e_0$ designating electron spin direction. This determines a unique comoving frame attached to the electron path. Let's call it the *electron inertial frame.* Then, without loss of generality, we can use (209) to map the electron path to the lab frame specified by γ_0, putting the Dirac equation in the form given by (151):

$$(\partial_t + c\boldsymbol{\nabla})\Psi\mathbf{i}\hbar = m_ec^2\Psi^* + e(A_0 - \mathbf{A})\Psi, \tag{238}$$

where the Zilch parameter β makes an explicit appearance as a function $\beta = \beta(x)$ in the mass term with

$$\Psi^* = \gamma_0\Psi\gamma_0 = e^{-i\beta}\Psi. \tag{239}$$

This confirms Boudet's demonstration that Zilch is an essential feature of the Darwin solutions for hydrogen. And it explains how the *virial theorem* for electron energy (161) applies to an *inertial system.*

$$\langle E\rangle = m_ec^2\langle\cos\beta\rangle = m_ec^2\int d^3x\cos\beta, \tag{240}$$

further supporting the view that $\cos\beta(x)$ is a measure of energy density in the field of an electron.

By the way, the association of γ_0 with electron mass given by (239) clarifies another puzzling feature of quantum mechanics, namely the physical significance of the hermitian conjugate Ψ^*. Moreover, we shall see that γ_0 can be identified with the velocity of the electron in a spacetime *Aether.*

For an inertial frame with $v = \gamma_0$ the spin bivector S in (180) splits into spatial vector and bivector parts:

$$S = isu = isvvu = isv(1 + v\wedge u) = i\mathbf{s} - i\mathbf{s}\wedge\mathbf{u}, \tag{241}$$

which reduces to the formally *complex vector*

$$\boldsymbol{S} = i\mathbf{s} + \mathbf{s}\times\mathbf{u}, \tag{242}$$

which we refer to as *inertial spin.*

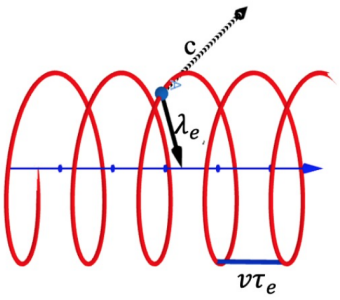

FIG. 5. The lightlike helical path of an electron with zitter radius λ_e, constant speed v and zitter period τ_e.

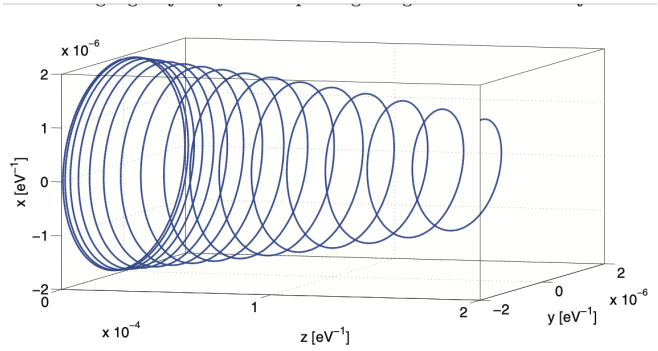

FIG. 6. Lightlike helical path of an electron with constant acceleration from rest. (Figure from [48])

Having decided on the primacy of inertial systems specified by the timelike vector $v = \gamma_0$, we are free to change notation and represent the electron *CM velocity* by $\mathbf{v} = \mathbf{v}(t)$ and speed by $v = |\mathbf{v}|$. Then, for constant speed the electron traverses a helical path, as shown in FIG. 5.

$$\dot{\boldsymbol{S}} = -i(\mathbf{s} \wedge \dot{\mathbf{u}}) = \mathbf{s} \times \dot{\mathbf{u}}. \tag{243}$$

The proper velocity of electron charge is a lightlike null vector $u = u(\tau)$. However, its projection into an inertial frame is a scalar $u^2 = u_0^2 - \mathbf{u}^2 = 0$. Accordingly, the zitter velocity $\mathbf{u} = \mathbf{u}(\tau) = \mathbf{u}(ct) \equiv \mathbf{u}(t)$ executes a helical screw motion along its spin vector $\mathbf{s}$. For constant acceleration, the path is given by FIG 5. The general *spindle structure* of electron states is described in FIG. 3 and specified by equations (225) and (226).

With these preliminaries setting the stage, we are prepared to address the fundamental problem of evaluating the Dirac equation on a particle path. Accordingly, in an inertial frame the vector derivative of the wave function on a particle path reduces to

$$(\partial_t + c\boldsymbol{\nabla})\Psi = \dot{\Psi}, \tag{244}$$

whence the Dirac equation (238) reduces to

$$\dot{\Psi}\, \mathbf{i}\hbar = m_e c^2 \Psi^* + e(A_0 - \mathbf{A})\Psi, \tag{245}$$

with $\Psi = \Psi(z(\tau)) = \Psi(\mathbf{z}(t), t)$. Here at last we have a completely general version of the Dirac equation with the zitter incorporated in the structure of the wave function to specify electron paths. To emphasize its fundamental importance let's call it the **Zitter-Dirac equation**. Our next task is to make its structure explicit with specific solutions so we can study their implications.

But first it should be noted that there are no probabilities involved in this equation, because it describes a specific path rather than the ensemble of paths considered by the Copenhagen interpretation. To be sure, there is a measurement problem because the electron is zipping around at the speed of light, but now we see Heisenberg's uncertainty principle as a consequence of the electron's zitter radius, a purely classical concept.

Another consequence of the classical interpretation for **zitter-Dirac** is that there is no question of gauge invariance. For eA_0 can be identified as an electric potential, while $e\mathbf{A}$ is a momentum potential, just as Maxwell surmised in his original formulation of electrodynamics [47]. To emphasize the interpretation of the vector potential $e\mathbf{A}$ as *momentum imparted to the electron by the vacuum*, Of course, electron and positron have opposite chirality expressed by opposite signs of the electric charge.

A general method for solving the zitter-Dirac equation that elucidates its geometric structure is given by the classical *Frenet equations* for 3D curves. As first described in [49], that is most efficiently done by introducing a comoving frame of local observables $\{\mathbf{e}_k = U\boldsymbol{\sigma}_k\widetilde{U} \mid k = 1, 2, 3\}$ with the equation of motion

$$\dot{\mathbf{e}}_k = \boldsymbol{\omega} \times \mathbf{e}_k \tag{246}$$

and angular velocity

$$\boldsymbol{\omega} = \kappa_1 \mathbf{e}_3 + \kappa_2 \mathbf{e}_1 = U(\kappa_1 \boldsymbol{\sigma}_3 + \kappa_2 \boldsymbol{\sigma}_1)\widetilde{U}, \tag{247}$$

where κ_1 is the (first) *curvature* and κ_2 is the (second) *curvature or torsion* of the curve. Finally, the equation of motion can be reduced to a simpler spinor equation:

$$\dot{U} = \tfrac{1}{2} U i(\kappa_1 \boldsymbol{\sigma}_3 + \kappa_2 \boldsymbol{\sigma}_1). \tag{248}$$

Call it the *Spinor Frenet equation.*

In the general case, the rotor $U = U(t)$ factors into a product $U_3 U_1 U_2$ of three simple rotors, but for constant spin $\mathbf{s}$ it can be reduced to two rotors with scalar phase angles $\theta_m = \theta_m(t)$ given in

$$U(t) = U_1 U_2 = e^{\frac{1}{2}\theta_1 i\boldsymbol{\sigma}_3} e^{\frac{1}{2}\theta_2 i\boldsymbol{\sigma}_1}. \tag{249}$$

Accordingly, we can now write the rotor for an electron with constant spin in the explicit form

$$U(t) = e^{\frac{1}{2} i\boldsymbol{\omega}_e t} e^{\frac{1}{2} i\mathbf{e}_1 \beta(t)}, \tag{250}$$

and its path for constant speed is illustrated in FIG. 5, without showing a possible phase shift when the signal is

emitted. The rotor for constant acceleration is illustrated in FIG. 6

The electron *spindle structure* specified by these equations is described by FIG. 3 in Section VI. We emphasize, however, that the *Zilch function* $\beta = \beta(x) = \beta(\mathbf{x}, t)$ is a scalar-valued field that permeates all of spacetime with values $\beta = \beta(\tau)$ on the path of a zilch signal. Thereby it may serve as an electromagneic Aether, much as Dirac [2] and others have proposed.

B. Birth of a photon

In the preceding subsection we described motion of an electron in an inertial frame with *zilch functions* given by (249) and (250). In subsequent Sections we will use the same functions to describe **Zilch signals** emitted into the Aether by an electron at rest. Moreover, we will freely switch between these alternative interpretations when convenience and context permits.

Here we apply the same equations to describe production of a photon from a freely moving electron. To set the stage, we interpret kinetic energy in free electron zitter as a dynamical mass

$$m_e c^2/2 = \mathbf{\Omega_e} \cdot \boldsymbol{S} = (-i\boldsymbol{\omega_e}) \cdot (i\mathbf{s}) = \boldsymbol{\omega_e} \cdot \mathbf{s} = \hbar\omega_e/2. \quad (251)$$

Then the classical energy equipartition theorem tells us that the total energy is composed of equal parts kinetic energy K and potential energy V given by

$$E = m_e c^2 = K_e + eV_e = \hbar\omega_e, \quad (252)$$

where $V = eV_e = e^2/2r$ is potential energy with respect to the electron CM. This is consistent with the energy equipartition theorem derived from the Dirac equation (240).

When a electron is accelerated, the force law (220) requires that a packet of energy (a photon) be released to the vacuum to maintain the balance of *bound energy* (189). De Broglie realized that this energy release is equivalent to creating a positron with negative energy $(-e)V_e = -e^2/2r$. That will be recognized as a version of Dirac's "hole theory" argument to justify the existence of the positron, which was surely the inspiration for de Broglie's theory that the *photon must be composed of an electron-positron pair* [50].

Though de Broglie persisted in his claim that the Dirac equation must explain the photon, he was never able to bring his argument to a successful conclusion. Here we show how to realize de Broglie's proposal with the zitter particle model depicted in Fig.1 for both electron and positron. We simply assume that both electron and positron have helical paths with a common center, but separated with a spacelike interval less than a Compton wavelength.

As described in Fig. 7, we assume that photon production originates at the CM with the creation of an $e^- e^+$ pair with dynamical mass

$$m_\gamma \equiv m_e \cos\beta, \quad (253)$$

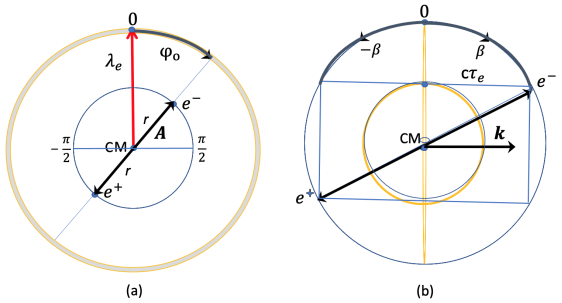

FIG. 7. **Spindle structure of a photon**: Picture the photon *Energy Shell* as a sphere of radius λ_e with two orthogonal cross sections. Section (a) depicts the Spindle ring for a photon composed of an electron-positron pair e^+e^- with fixed separation $2r$ circulating at the speed of light with polarization angle φ_0, where the range $0 \le \varphi_0 \le \pm\pi$ designates left and right circular polarizations. Section (b) shows the photon vector $\boldsymbol{k}$ and tilt angle β in the photon bubble filled with energy $\hbar\omega_e$ (not to scale). When the bubble is filled with energy $2m_e c^2$ the photon splits into an electron-positron pair.

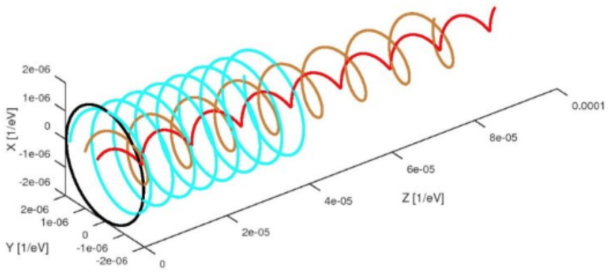

FIG. 8. Picture the photon as a moving Spindle ring with angular momentum $\hbar$ that generates an electromagnetic wave with amplitude normalized to its energy. (Figure from [48] with a different but related interpretation)

where β is the chirality angle given by (228) and shown in Fig. 3. In other words, the electron spindle generates a photon with energy

$$E_\gamma = m_e c^2 \cos\beta = \frac{2e^2}{r} = \hbar\omega_\gamma, \quad (254)$$

and the energy shift in the process of radiation is given by

$$E/\hbar = (-i\boldsymbol{\omega}) \cdot (i\hat{\mathbf{s}}) = \boldsymbol{\omega} \cdot \hat{\mathbf{s}} = \boldsymbol{\omega}\, \hat{\mathbf{s}} = \omega_e \pm \omega_\gamma, \quad (255)$$

where the constraint $\boldsymbol{\omega} \wedge \mathbf{s} = 0$ ensures that propagation of the photon is aligned with the spin axis, while $\pm$ designates opposite circulations. Thus, the electron spindle functions as a *circular antenna* generating photons while its charge circulates around the *spindle ring*.

As depicted on Fig. 5 for the rest frame of this electron-positron system, the particle motion projects to a circle with zitter radius λ_e where electron and positron are lo-

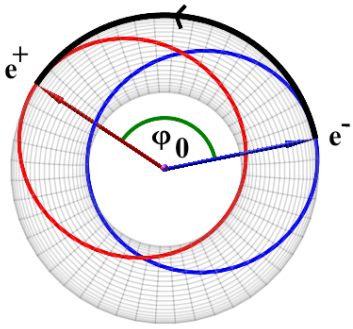

FIG. 9. The photon can be modeled as an electron-positron pair located on a toroidal ring (or energy shell) with a fixed angular separation φ_0 designating its polarization and complementary Villarceau circles for electron and positron. The fusion of these circles with a suitable phase lag generates a toroidal photon path.

cated with fixed angular separation φ_0 and spin angular momentum $2\mathbf{s} = \hbar\boldsymbol{\sigma}_3$.

Now we assume that the e^-e^+ segment embedded in the photon ring, and the spin and charge stored within it, propagates as a *photon* along a lightlike path with tangent vector k and helical path like the one depicted in Fig. 4. It is amusing to think of the e^- and the e^+ as terminals of a battery that drives a current of constant energy $\hbar\omega_\gamma$ around the ring.

As the photon moves, it generates an electromagnetic field that can be represented algebraically by a vector potential

$$\mathbf{A} = \omega_\gamma \, \mathbf{e} \, e^{i\hat{\boldsymbol{k}}\varphi}, \tag{256}$$

with amplitude proportional to the frequency ω_γ and phase $\varphi(t) = k \cdot z + \varphi_0$ with $k \cdot z = \omega t - \boldsymbol{k} \cdot \mathbf{z}(t)$, while the initial phase is set by a fixed vector $\mathbf{e}$ with $\mathbf{e} \cdot \mathbf{s} = 0$. Note that the variables for time t and frequency ω_γ are set as initial conditions in the instantaneous rest frame of the electron emitting the photon. We note also that the photon carries energy $\hbar\omega_\gamma \leq 2m_e c^2$ with the limiting value given for pair production

The photon propagates at the speed of light with momentum $p = \hbar k$, so $p^2 = 0$. The photon energy $\hbar\omega_\gamma = p \cdot v$ is determined by the proper velocity $v = v(\tau)$ of the source when it is emitted. Therefore, without loss of generality, we can describe photon emission in the instantaneous rest frame of the electron given by $v = \gamma_0$, so we have the momentum spacetime split $p\gamma_0 = \hbar(\omega_\gamma + \boldsymbol{k})$. Then from the rest frame independence of our photon model we conclude that *the momentum of the emitted photon* $\mathbf{p} = \hbar\boldsymbol{k}$ *must be collinear with with electron spin* $\mathbf{s}$. Among other things, this general result accounts for the *headlight effect* in cyclotron radiation.

We complete our picture of the photon by projecting its lightlike path in spacetime into the spacelike path shown in Fig. 8. That depicts the photon ring generating a circularly polarized electromagnetic wave with various frequencies as it propagates with velocity $c\hat{\boldsymbol{k}}$.

Finally, incorporating the toroidal structure of electron's anomalous magnetic moment described in Fig. 8 thickens the photon ring. Thus, the field generated in each cycle is akin to a smoke ring, so the whole wave train consists of a chain of discrete "circulating smoke rings" much like the "vortex atoms" proposed by Lord Kelvin in the nineteenth century. That is consistent with the experimentally observed countability of photons. The structure of the photon "smoke ring" is pictured in Fig. 9. Its toroidal path [51] is depicted in Fig. 10 .

The problem remains to square our model of the photon with what is known about electromagnetic radiation discussed in [44]. As explained there, the motion of a photon is governed by Maxwell's equation

$$\partial_0 F = -\boldsymbol{\nabla} F, \tag{257}$$

with the constraint $F^2 = 0$. For a photon we can write $F = \mathbf{E} + i\mathbf{B} = \mathbf{E}(1 + \hat{\boldsymbol{k}})$, where the electric field $\mathbf{E} = \partial_0 \mathbf{A}$ is determined by the vector potential given by (256).

The photon field $F = F(\varphi)$ has the same functional form as a plane wave, but its phase function $\varphi = k \cdot z(\tau)$ is centered on a lightlike curve $z = z(\tau)$. Hence, the phase is given by $\varphi = \omega t - \boldsymbol{k} \cdot \mathbf{z}$. and Maxwell's equation (258) reduces to the eigenvalue equation

$$\boldsymbol{k} F = \pm F\omega, \tag{258}$$

where the signs correspond to states of left/right circular polarization. Moreover, we can decompose F into the canonical form

$$F = \mathbf{E} + i\mathbf{B} = \boldsymbol{f} Z, \tag{259}$$

where

$$Z = Z(\varphi) = \rho e^{i\varphi} \tag{260}$$

can be regarded as a complex impedance of the photon singularity, and $\boldsymbol{f}$ is a *polarization bivector* with various forms given in [44]. In particular, we can write

$$\boldsymbol{f} = \hat{\mathbf{e}} \, e^{i\hat{\mathbf{k}}\varphi_0} = \hat{\mathbf{e}}(cos\,\varphi_0 + i\hat{\mathbf{k}}\, sin\,\varphi_0) = \mathbf{e} + i\mathbf{b}, \tag{261}$$

where φ_0 is the polarization angle in Fig. 5 and $\boldsymbol{f}\boldsymbol{f}^\dagger = 1$. The parameter ρ in (260) requires some explanation which we present in the next Section.

Finally, we complete our model of the photon by mentioning a recent demonstration that **the photon has a well defined center of mass (CM) just like the electron [52].** Among other things, this means that optical properties such as amplitudes, phases and correlations (even quantum entanglement) can be deduced directly from measurements of light intensity.

VIII. MASS AND ENERGY DENSITY

Newton supposed that some kind of medium is needed to transmit gravitational interactions, but he wisely declined to speculate about its properties. In the eighteenth

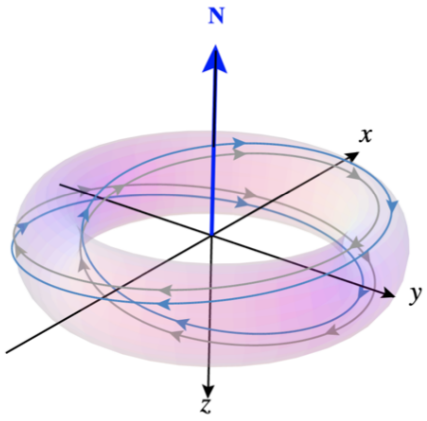

FIG. 10. The grey line represents the photon's axis of travel around the toroidal flow. This is a left-circularly-polarised photon. The spin axis in aligned with magnetic north $\boldsymbol{N}$. Figure from [53]

century Maxwell led the way in developing a theory of the vacuum that culminated in an electromagnetic explanation for the properties of light. Regarding the vacuum as a dielectric medium with variable permittivity $\varepsilon = \varepsilon(x)$ and permeability $\mu = \mu(x)$ at each spacetime point x, Maxwell's condition for the propagation of light in a vacuum is given by

$$\varepsilon\mu = 1/c^2 = \varepsilon_0\mu_0. \tag{262}$$

From that starting point Seymour Blinder [54, 55] has shown that polarization of the vacuum in the neighborhood of a classical electron is uniquely determined by the very simple assumptions that

(1) the energy density of the electron field is proportional to the charge density, and

(2) the total energy in the field determines the electron mass.

$$\varepsilon(r) = \varepsilon_0 \exp\left(\frac{\lambda_c}{r}\right), \tag{263}$$

where

$$\lambda_c = \frac{1}{2}\frac{e^2}{m_e c^2} = \alpha_e \lambda_e \tag{264}$$

is recognized as half the *classical electron radius* expressed as a product of the fine structure α_e and the zitter electron radius λ_e. I call lt the *Blinder ansatz.*

This puts that λ_e into new perspective as the *radius of vacuum polarization.* Absorbing the constant ε_0 into the units defining electric charge, we interpret its inverse $\varepsilon(r)$ as a *material density* of the electromagnetic vacuum:

$$\rho_e(r) = \rho_e(x - z_e(\tau)) = \varepsilon^{-1}(r) = e^{-\lambda_c/r}, \tag{265}$$

where $r = (x - z(\tau)) \cdot v$ is the *classical retarded distance* from the position $z(\tau)$ of the electron's *Center of Mass* CM.

Note that the position $z_e(\tau)$ of *Center of Charge* CC is separated from the CM by the zitter radius

$$|z_e(\tau) - z(\tau)| = \lambda_e, \tag{266}$$

and at that separation

$$\rho_e(z_e(\tau) - z(\tau)) = e^{-\lambda_c/\lambda_e} = e^{-\alpha_e} \tag{267}$$

can be interpreted as a dimensionless measure of the strength of a vacuum singularity,

That singles out the Dirac current $e\rho_e v$, or more precisely, the fluctuating null current $e\rho u$ specified by (177), as the source of the electron's electric field.

Accordingly, we follow Blinder in identifying each electron as a singularity in the universal spacetime vacuum. The ansatz explains how energy (aka mass) is stored in the vacuum. And we note that the Extended Lorentz Force (220) serves as a mechanism for installing or removing energy in the vacuum, in particular, for creating or absorbing photons.

To put the Blinder ansatz in context, we recall from (47) and (206) that the Dirac wave function $\Psi = \Psi(x)$ has the general form

$$\Psi = \rho^{1/2} e^{i\beta/2} R \qquad \text{where} \qquad R = UVU_1U_2 \tag{268}$$

is a unique factored form of the rotor $R = R(x)$. We focus our attention here on the factored form $\Psi = \psi R'$, where the factor

$$\psi = \rho^{1/2} U(\varphi) = e^{-(\alpha + i\boldsymbol{\sigma}_3\varphi)}. \tag{269}$$

has the familiar form for the Schrödinger wave function given by (131), but the rotor $U = U(x)$ can be identified now as the generator for the *spinet*, the hand of the electron clock given by (207). On a particle path the variations of this phase in this wave function are given by $\varphi = \varphi(x - \dot{z}(\tau))$, and governed by the Dirac equation. That is well established in non-relativistic as well as relativistic QM.

On the other hand, the physical significance of the scalar factor ρ in (269) has long been a subject of intense debate, in particular, as to whether it implies that QM is inherently probabilistic. *To resolve the issue, de Broglie claimed we must be able to define an alternative wave function that does not involve probabilities.* But he was not able to show how to do it. Let is see how the *Blinder ansatz* solves that problem.

A. Real de Broglie waves and the superposition principle

For a single electron with density $\rho_e(\tau) = \rho_e(z(\tau))$ along its path $z = z(\tau)$ given by (265), we define a wave function with amplitude

$$\psi_e = \rho_e^{1/2} e^{-\mathbf{i}\varphi/2} = e^{-(\lambda_c/r + \mathbf{i}\omega_e\tau)/2} = \psi_e(r, \tau), \tag{270}$$

Let's call this a *real de Broglie wave*. The usual Schrödinger wave function ψ_S has the same kind of complex function form. Indeed, linearity of the Dirac equation allows a superposition of solutions with a complex "Schrödinger" factor

$$\psi_S = C\psi_e \tag{271}$$

that can be identified as a probability amplitude in accordance with standard QM. It seems to have been overlooked, however, that the probability amplitude ψ_S may have an irreducible complex factor ψ_e with a physical interpretation that does not involve probabilities.

That observation may be sufficient to resolve the contentious debate between realist and subjectivist interpretations of QM. Of course, experiment must be the arbiter for the debate. To be specific, we need experiments on single electrons to measure their real de Broglie wave properties directly. This is not the place for details, but a few remarks will point to possibilities.

In particular, we look to measure the electron's *Blinder density* $\rho_e = \psi_e\psi_e^\dagger$ in the vacuum near a fixed surface. One well developed experimental domain where this is relevant is diffraction of single electrons and photons. The key issue there is the mechanism for momentum transfer between the particle and diffracting slits [56]. A related issue is the role of Blinder density and evanescent waves in the Goos—Hänchen and Imbert—Fedorov beam shifts [57]. A third issue is the role of real de Broglie waves in explaining tunneling and tunneling times [29].

As a technical point about de Broglie waves, we note a crucial feature of the ansatz is that $\rho(z_e(\tau)) = 0$ everywhere along the electron path $z_e(\tau)$, and thus at a single point on any 3-D spacelike hypersurface. At that point the phase in the wave function is undetermined, so it can be multivalued. This mechanism serves also to pick out a particle path in the Pilot Wave theory of Section V. Indeed, it has the surprising implication that the *Quantum potential* in Pilot wave theory vanishes on the electron path [44], with important consequences that have not been previously considered.

It is no accident that the Blinder constant λ_c is simply proportional to the London penetration depth in superconductivity [58], since both describe fundamental properties of the electron. Some of the many implications have been discussed by Hirsch [59]. Here we identify the London penetration depth with the irreducible amplitude ψ_e in the electron wave function (270), so it is an essential element in electron structure.

We conclude that the electron is nature's most basic superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is energy in the electron's electromagnetic field.

B. Singularity structure

The Blinder function defined for the electron by (265) has an alternative formulation suggesting a general property of vacuum singularities not limited to the electron or photon. Thus, we write the electron's Blinder exponent in the form

$$\lambda_c/r = \frac{e^2/\hbar c}{m_e c/\hbar v \cdot (x - z_e)} = \frac{\alpha_e}{k_e \cdot (x - z_e(\tau))}. \tag{272}$$

This suggests that any particle with kinetic momentum $k = p/\hbar$ and position $z(\tau)$ will have a Blinder function of the form

$$\rho = e^{-\alpha_e/k\cdot(x - z(\tau))}, \tag{273}$$

so the particle is located at $\rho = 0$, and we drop the subscript on k_e to allow k to be a null as well as timelike. There is no longer a suggestion here that the exponent is the Coulomb potential of a charged particle. Here the fine structure constant α_e acts as a kind of general scaling constant for vacuum singularities, so it may play that role even in strong interactions, as argued by MacGregor [60].

Like the electron, the photon is a singularity in the electromagnetic vacuum field with density $\rho = \rho(z)$ plausibly described by (273). That gives the photon a size and shape. One might worry that the photon density (273) could propagate like the electron's Coulomb potential to influence the photon's surroundings in a way that has not been observed. However, it is a general theorem [29] that influence from a null surface, like the boundary of a photon path in the present model, will propagate only along that boundary. In free space the photon moves in a straight line. However, in a wave guide or optical fiber, the path is shaped by the material walls that modify the parameter ρ.

Note that incorporating the Blinder function into our model of the photon increases the degrees of freedom for the vector potential $\mathbf{A}$ specified in (256) from 1 to 2. Thus, with $\mathbf{e} \cdot \boldsymbol{k} = 0$ the polarization vector $\mathbf{e}$ can be generated by rotor $U = U_1U_2$ and written

$$\mathbf{e} = U\boldsymbol{\sigma}_1\tilde{U}. \tag{274}$$

as specified before by (208) and (210). This provides strong theoretical grounds for predicting the existence of quantized toroidal states for individual photons. Experimentalists will be proud to announce that they got there first! It seems they have already detected toroidal states in the diffraction of individual photons! [61] However, that may be a beginning rather than the end of the story, a presage of a richer landscape of toroidal states in elementary particle theory to be considered at another time.

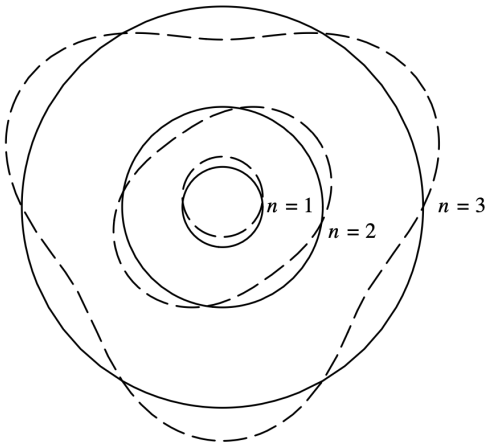

FIG. 11. Zitterbewegung oscillations in atomic states. (Figure from [62]). Now we can understand them quantitatively as paths on a torus generated by the electron's anomalous magnetic moment.

IX. ZITTER-DIRAC EQUATION

Now we are prepared to bring our long investigation into the physical interpretation of Dirac's equation for the electron to a definitive conclusion.

We reframe the analysis in Section VIIA to show how directly the results flow from the Dirac equation itself.

We suppose now that when the Zilch signal is received by a near or adjacent electron with an aligned spin, a *resonant coupling* of the zitter phases can be set up between the two electrons that can evidently be identified with the **Quantum Force** proposed and studied at length by Jorge Hirsch [59].

Free from external perturbation, the strength of the resonant coupling by two electrons is not limited by distance. For example, the electrons may be located on opposite plates of a capacitor. Moreover, in an electric circuit the signals from similar pairs in a capacitor can be bundled collectively as filaments of an electric current that carries energy but no charge. We identify it as the celebrated **Displacement current** that Maxwell postulated to complete his theory of electrodynamics. Though this current carries no charge, it does generate a magnetic field so it must play a role in electromagnetic induction. Accordingly, **we identify the zilch signal as the central line of magnetic force connecting two electrons**, in other words, the **Quantum Force**. We explore the many implications of that idea in the following Sections.

A. Displacement current: Zilch signal

As explained with (249), for constant spin the rotor equation for a *Zilch signal* can be written in the explicit forms

$$U(t) = e^{\frac{1}{2}\theta_1 i\boldsymbol{\sigma}_3} e^{\frac{1}{2}\theta_2 i\boldsymbol{\sigma}_1} = e^{\frac{1}{2}i\boldsymbol{\omega}_e t} e^{\frac{1}{2}i\mathbf{e}_1\beta(t)} \tag{275}$$

and its path for constant β is illustrated in Fig. 5, without showing an initial phase shift when the signal is emitted.

For the de Broglie wave function (270), direct chiral projection onto the spindle gives us

$$\Psi(\mathbf{x},t) = \rho_e(\mathbf{x},t)^{\frac{1}{2}}\, e^{\frac{1}{2}i\boldsymbol{\omega}_e t} e^{\frac{1}{2}i\mathbf{e}_1\beta(t)}. \tag{276}$$

This equation can be interpreted as an accelerating electron or a magnetic zilch signal, depending on initial conditions (Fig. 6). The electron's Spinet action can be pictured as a propeller with variable pitch that drives the the electron through the vacuum zilch. The zilch signal can be identified with the *Displacement current* $\partial_t \mathbf{D}$ in standard electromagnetic theory. This has simplifications first recognized by Dirac.

In 1951, Dirac published a short article entitled *Is there an Aether?* [2, 63] Therein he proposed that *all inertial forces are due to local motion of the vacuum*, to which he ascribed the velocity (in his notation):

$$U = \frac{-q}{mc}A \quad \text{with} \quad U^2 = c^2, \tag{277}$$

where A is the relativistic version of Maxwell's vector potential. This is completely consistent with the zitter-Dirac equation (245), so it is free from the problems of gauge invariance that bedevils conventional quantum mechanics. We consider its implications for Maxwell's equation below.

The year 1951 is a good date to mark completion of Dirac's contribution to quantum mechanics with a salutation to Maxwell.

B. Atomic Structure

For integer values of the *zilch phase* the wave function (276) reduces immediately to a general solution for the Bohr atom where $n = 1, 2, ...$ is the *principal quantum number*:

$$\Psi_n(\mathbf{x},t) = \rho_e(\mathbf{x},t)^{\frac{1}{2}}\, e^{\frac{1}{2}i\boldsymbol{\omega}_e t} e^{\frac{1}{2}i\mathbf{e}_1\beta_n(t)}. \tag{278}$$

This is the solution depicted in Fig. 11. Generalization to include angular momentum and magnetic quantum numbers as specified by (210) is given by the wave function

$$\Psi_{n,\ell,m} = \Psi_n(\mathbf{x},t) U_\ell U_m. \tag{279}$$

This, in fact is the form of the general solution of the Dirac equation found by [30] and described in Section VB. However, this presentation shows that the quantum numbers $< n, \ell, m >$ belong to the magnetic zilch field and not to the electric charge of the electron. Indeed, there is clear experimental evidence that zilch can be separated from charge in diffraction, as discussed below.

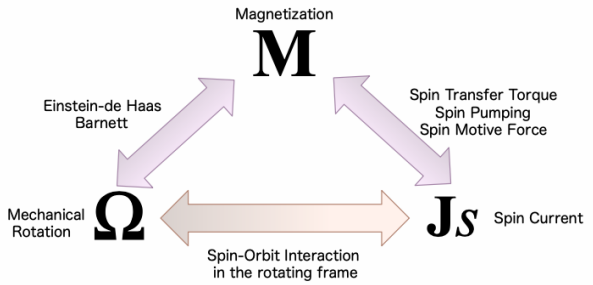

FIG. 12. Spin transfer torque [64]

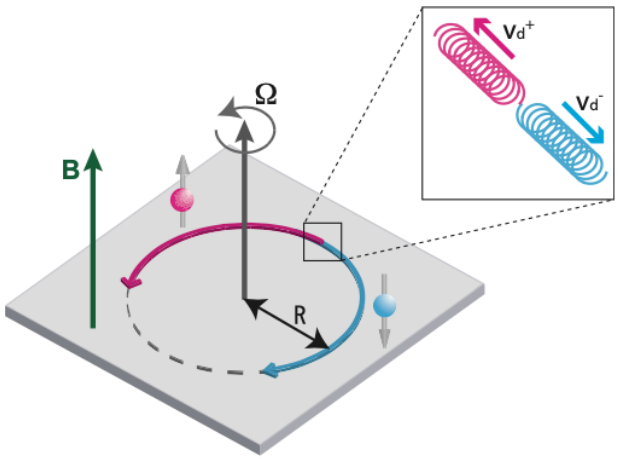

FIG. 13. Effects of mechanical rotation on spin currents. Solution for the equation of motion is a superposition of two cyclotron motions with different frequencies. The drift velocity of the up-(down-) electron is $\mathbf{v}_d^+$ ($\mathbf{v}_d^-$) parallel to the azimuthal direction denoted by $\hat{\phi}$.[64]

On the other hand, the Dirac equation also determines discrete paths for the electron in hydrogen, as specified at length in [44]). The standard objection to these solutions is that they are not energetically stable, because an accelerating charge radiates away energy. Now, however, we see that atomic electrons can be stabilized by the zilch field in which they are embedded, as explained next.

C. Radiation with Zilch

Equation (276) is an exact solution of the Zitter-Dirac equation, but it has been overlooked until the analysis of local observables in this paper identified the Zilch function $\beta(\mathbf{x}, t)$ as the universal substrate of the electromagnetic vacuum. This confirms the identification (221) of $q = -s\dot{\beta}$ as momentum released to the vacuum by acceleration of the electron due to the extended Lorentz force (ELF):

$$\dot{p} = \frac{e}{c} F \cdot v + \dot{q}. \tag{280}$$

Although this is relativistically invariant, we have seen that projection into the electron's rest system has special physical significance. In Section V we saw that the Dirac equation implies a force on an electron given by (140), which is worth repeating here.

$$\mathbf{f} = e[\mathbf{E} + \mathbf{v} \times \mathbf{B}/c] + \frac{e}{mc} \grave{\nabla} \grave{\mathbf{B}} \cdot \mathbf{s}, \tag{281}$$

and we confirm that this must be augmented by release of momentum given by $\boldsymbol{q} = -\mathbf{s}\dot{\beta}$ as explained in Section VID.

D. Spintronics

Spintronics (aka spin-based electronics) is concerned with manipulation of spin degrees of freedom in solid state systems along with electronic charge used in traditional semiconductor electronics. Its applications are vast, from information storage and transfer to quantum computing, but it has lacked a coherent theoretical foundation in quantum mechanics. In a stunning *tour de force* that foundation has been supplied in [64] and described brilliantly with two excellent figures: FIG. 12 and FIG.13. They found exact solutions of the Dirac equation for a rotating frame using techniques that were available more than eighty years ago but obscured by a deficient understanding of electron spin. Happily, the solutions are much easier to derive from the zitter-Dirac equation (245), and they take the form given by (276).

In particular, as shown in FIG. 13, an external Lorentz force with cyclotron frequency ω_c drives individual electrons with opposite spins to circulate in opposite directions specified by the factors

$$e^{\pm \frac{1}{2} \kappa i \mathbf{e}_1 \omega_c}, \tag{282}$$

where κ is a scalar constant specifying density of states for the material in which the electrons are embedded.

E. Stern-Gerlach and AB effects

Note that the last term in (281) is just what is needed to produce a Stern-Gerlach effect for electrons. It is a consequence of the spindle structure of the electron. Problems involved in detecting it experimentally have been deeply discussed by Batelaan [65]. The solution promises to provide a fundamental device for separating spin up and down states of the electron with rich implications for spintronics and quantum computing.

Batelaan [66] has also conducted ground breaking research on Aharonov-Bohm effect in Fig. 14. However,

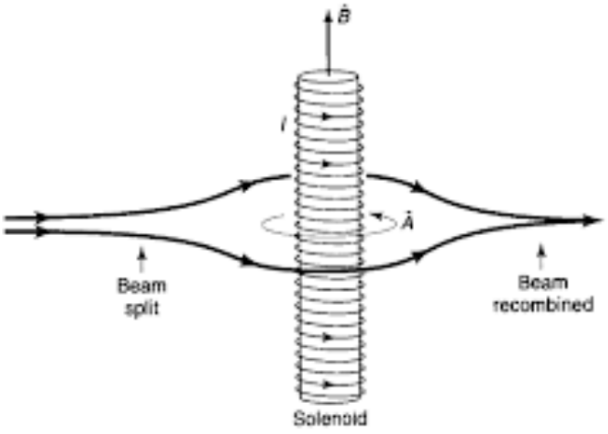

FIG. 14. The AB effect is explained by a nonconservative force of classical origin. [67]

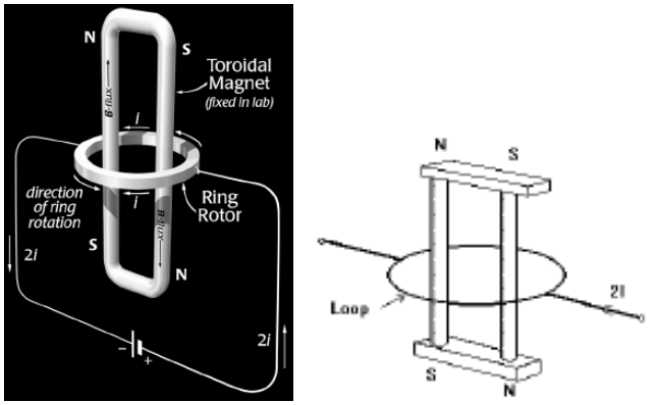

FIG. 15. The Marinov Motor does useful work on mechanical systems by extracting magnetic energy from the vacuum [69, 70]

Wesley [67] explains that the vector potential in the figure has a perfectly satisfactory classical explanation as motional induction [68] with a nonconservative force proportional to $\dot{\mathbf{A}}$. According to the zitter Dirac equation (245) there is nothing problematic about this force. The mistake in previous analysis was failure to recognize that the Dirac equation implies that the Lorentz force (which assumes the electron is a point charge) must be generalized to an Extended Lorentz Force (ELF) to account for electron zitter.

F. Marinov Motor

The design and performance of the Marinov motor has been thoroughly discussed by Phipps [69, 70]. But something crucial is missing from the account, namely the spin of the electron in the electric current. We can explain the significance of that fact here and leave the rest to Phipps. The spin of an electron free to move in a magnet naturally aligns itself with the spins in the magnetic substrate. When the magnet moves it generates an electric current $\mathbf{J} = \hat{\mathbf{s}}D = \mathbf{D}$ which we can identify with Maxwell's *Displacement current*. As we have seen before, this current generates a magnetic field which propagates in the vacuum. When the closed circuit of magnets in 15 is given a push, it starts to rotate around a vertical axis, and generates a displacement current $\partial_t D$ that drives a current $2D = \dot{\beta}$ around a closed circuit and continuously releases energy into the vacuum.

G. Particle Diffraction by Zilch

Maxwell–Dirac theory has unique implications for the problem of electron and photon diffraction, indeed, for particle diffraction in general. The main problem with particle theories of diffraction is identifying a plausible mechanism for momentum exchange between each diffracted particle and the slits, a causal link which is missing from all accounts of diffraction by standard wave mechanics and Pilot Wave theory. For each scattered particle momentum transfer is directly observable, whereas the diffraction pattern conserves momentum only as a statistical average. Evidently the only way to account for this fact is by reducing diffraction to quantized momentum exchange between each particle and slit. To that end, a detailed analysis of optical diffraction patterns explained by photon momentum exchange is given by Mobley [56].

The double-slit diffraction pattern for light has been long been regarded as *prima facie* evidence for wave-particle duality. However, refinement of experimental technique in recent decades has produced a growing body of evidence for an alternative explanation for diffraction, namely: *quantized momentum exchange* between particles and the diffracting slits.

The statistical build-up of a diffraction pattern one particle at a time is essentially the same for photons and electrons [65, 71]. A comparable result has been found for diffraction of neutrons and atoms [72, 73] as well as molecules as large as C_{60} [74, 75]. It even extends to diffraction of optical vortex knots [61]. And our analysis in Section IV of excitations in the free field of an electron suggests similar results in electron diffraction.

The fact that the build-up of a diffraction pattern is essentially the same for such a wide variety of particles calls for a theory that explains the essential mechanism producing the result. Such a theory has been constructed by maverick physicist J. P. Wesley [76], though it has found little recognition in the physics community despite its high relevance to the wave-particle puzzle.

Since Wesley's own account is so clear and straightforward, we can settle for a brief summary of significant points here. Wesley shows that the 2-slit diffraction pattern can be constructed from particle trajectories generated by the scalar wave equation, given a suitable energy

$E = \hbar\omega$ and wave number $\mathbf{k} = \mathbf{p}/\omega$.

Now we identify the universal Zilch function $\Phi = \Phi(x)$ as the causal mechanism for diffraction. Then we suppose that vanishing electric and magnetic fields outside the diffraction slits are generated by a vector potential $A = A(x)$ with $\Box \wedge A = 0$, so locally, at least, A is a gradient. Assuming the same for the canonical momentum P in (204) , we have a gauge invariant *phase gradient*

$$\Box\Phi = P - \frac{e}{c}A. \tag{283}$$

This provides a promising mechanism for quantized momentum transfer in diffraction. For we know that quantized states in QM are determined by boundary conditions on the phase. Successful calculation of diffraction patterns along these lines would provide strong evidence for the following claim: the vacuum surrounding electromagnetically inert matter is permeated by a vector potential with vanishing curl. Evidently the same mechanism can explain the extended Aharonov-Bohm (AB) effect [66]. One concludes, then, that the causal agents for diffraction and the AB effect are one and the same: a universal vector potential permeating the vacuum (or, *Aether*, if you will) of all spacetime, much as proposed by Dirac [2].

Considering the similarity of electron and photon diffraction patterns, we should expect the same mechanism to explain both, especially if photons are composed of electron-positron pairs as proposed in the preceding Section. Indeed, the evolution of path density for the electron is determined by the Dirac equation, which gives

$$\Box^2\Phi = -m_e c\dot{z} \cdot \Box \ln \rho. \tag{284}$$

For a photon with propagation vector k, the analog is

$$k \cdot \Box \ln \rho = \Box^2\Phi/\hbar, \tag{285}$$

where, of course, ρ is the path density for photons, just as it is for electrons. Accordingly, we conclude that diffraction is "caused" by the vacuum surrounding material objects. In other words, *diffraction is refraction by the vacuum*!

Strictly speaking, the density (impedance) of the vacuum should be incorporated into any vector potential by writing $\mathcal{A} = \rho A$, with a new notation to distinguish it from the usual vector potential, whether or not it is the gradient of a scalar field. The Aether can then be regarded as a conserved fluid (with $\Box \cdot \mathcal{A} = 0$) flowing through spacetime with particle singularities (electron, photon or whatever) in the density swept along. This picture has a beautiful macroscopic analog describing diffraction of a macro particle in a classical fluid [78].

H. Gravito-electromagnetism (GEM)

The relation of gravity to electrodynamics has been a fundamental question since the validation of Maxwell's

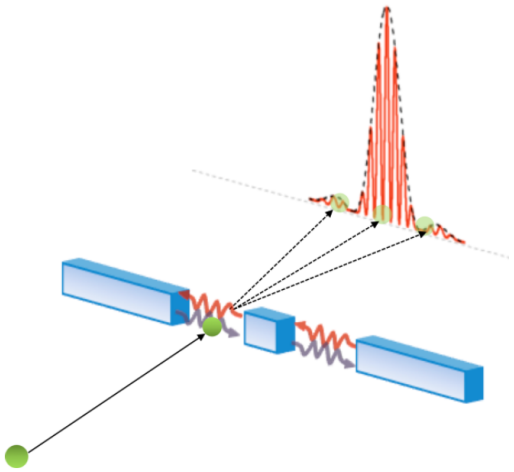

FIG. 16. Diffraction mechanism. Vacuum field modes are selected by the double-slit structure, which in turn guide the electron particle motion. (Figure from H. Batelaan [77])

equation. Famously, Einstein was inspired by Mach in creating General Relativity (GR) as a universal theory of gravity but was greatly disappointed to conclude that GR is incompatible with *Mach's Principle*. The possibility that gravity is an inherent property of the vacuum is given new currency by Dirac's proposal (277) which enables us to identify the Aether with Zilch. The role of the constant $c^2 = 1/\mu\epsilon$ is especially noteworthy as the Blinder ansatz identifies electron mass $m_e = E/c^2$ with energy stored in the vacuum, while others ([79]) have related it to Mach's Principle. How this enables unification of gravity with electromagnetism (GEM) is thoroughly discussed by [80]. That can be compared with the more conventional approach to gravity from GR reviewed by [81]. A formulation *"Einstein's Gravity"* compatible with GEM is given by [82, 83]. This has practical implications for lunar laser ranging measurements and the behavior of clocks in a global positioning system as discussed by [84]. A definitive study of Mach's Principle from a classical point of view has been given by Andre Assis [85].

I. Quantum Force and Superconductivity

As depicted in FIG 3, the *electron energy shell* is a sphere with a diameter of a Compton radius $2\lambda_e$. Consequently, Coulomb repulsion keeps conducting electrons in a metal separated by that distance under normal conditions. However, as described in FIG. 17, when neighboring electrons have parallel spins they can form a resonant state where their centers are separated by less than $2\lambda_e$. We emphasize that this model of a bound electron pair was derived by Martin Rivas from deep analysis of the Dirac equation with a point particle interpretation. We propose it as a model for the *Cooper pair* in super con-

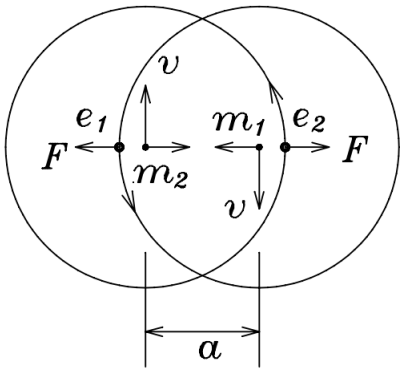

FIG. 17. Initial position and velocity of the center of mass and charges for a bound motion of a two-electron system with parallel spins. The circles would correspond to the trajectories of the charges if considered free. The interacting Coulomb force F is computed in terms of the separation distance between the charges. [41]

ductivity theory, and we call the mean separation of the of the circulating electrons the *Rivas distance*.

States where spin currents exist in the absence of external fields have been thoroughly studied by Jorge Hirsch to describe the superconducting state of metals [86] and aromatic molecules [86]. We to refer to work as the *Hirsch theory of superconductivity* to distinguish it from *BCS theory*. In a subsequent paper we expand the present model for a Cooper pair into a unified theory of the Pauli Principle and nuclear structure.

X. ONTOLOGY CUM EPISTEMOLOGY

When long-standing scientific debates are finally resolved, it invariably turns out that both sides are correct in positive assertions about their own position but incorrect in negative assertions about the opposing position.

The Great Debate over the interpretation of quantum mechanics can be cast as a dialectic between Einstein's emphasis on ontology and Bohr's emphasis on epistemology [87]. This paper offers a new perspective on the debate by focusing on the local observables determined by the Dirac wave function.

Since the Dirac equation is a linear differential equation, the superposition principle can be used to introduce probabilities in initial conditions and construct the wave packets of standard quantum mechanics with the Born interpretation of ρ as probability density. This establishes full compatibility between the Pilot Wave and Born interpretations of Dirac wave functions.

A definitive analysis of zitter in the Dirac wave function was given in Section IVC and proposed as a defining property of the electron. That shows that zitter can be regarded as electron phase incorporated in charge oscillations around its Center of Mass. That completes the description of the electron in Born-Dirac theory as a particle with intrinsic spin and zitter in its motion.

Concerning the Born rule for interpreting the wave function in Quantum Mechanics: There is no doubt that probability is essential for interpreting experiments. Indeed, overcoming the failure of Old Quantum Theory to account for intensities of spectral lines was one of the first great victories for Quantum Mechanics and Born's rule for statistical interpretation.

Moreover, it comes not to destroy QM but to fulfill! For the particle model offers an ontic interpretation to QM, while QM offers an empirically significant way to assign probabilities to particle states. The particle model provides electron states with definite position, momentum, spin and (zitter) phase.

The Born rule offers a way to assign probabilities to these states. However, such probabilities do not imply uncertainties inherent in Nature as often claimed. Rather, they express limitations in our knowledge and control of specific states, best described by Bayesian probability theory so ably expounded by E. T. Jaynes [25, 87].

Of course, the present approach calls for reconsideration of many arguments and applications of standard quantum mechanics, especially those involving zitter and the Heisenberg Uncertainty Relations, which are already burdened by many conflicting interpretations [88].

A new perspective on the Great Debate on the interpretation of QM is introduced by incorporating Zilch as well as Zitter in our electron model, as is evident in preceding examples.

XI. ACKNOWLEDGEMENT

I dedicate this paper to my good friend Roget Boudet, who contributed significantly to its development. Roget died on August 31, 2016 at the age of 88. These remarks serve as an obituary.

Roget was a fiercely independent and scrupulous fellow, with a strong social conscience. I know nothing of his early life, but I believe he worked as a sailor in his youth, and I know he enjoyed sailing throughout his life. He received his doctorate in mathematics and held the rank of professor at *Aix-en-Provence* in France until his retirement.

Roget told me that his doctoral advisor was the last of the true experts on "Classical Geometry," and spoke of him with great reverence. Roget continued that work in his thesis, but I don't know any of the details. On the other hand, Roget could not avoid a heavy dose of Bourbaki, which was dominant in French mathematics education, even to the point of naming the real number line in French schools as "*Rue du Bourbaki!*" He studied Bourbaki with due respect but maintained his natural skepticism born of his schooling in classical geometry. That helped him develop proficiency with the complex

FIG. 18. Roget Boudet (1928-2016) French mathematician and incorruptible scholar in the classical tradition

calculations in quantum electrodynamics that he studied diligently over many years. Taking nothing for granted, he worked out every detail for himself. That makes his published books [30, 89] uniquely valuable, especially as a context and background for the present article.

Roget was the first person to understand and appreciate my original paper on Real Dirac Theory in 1967 [8]. He was so impressed that he immediately flew from France to my home in Arizona to visit me. Then he hosted me and my family in France during my sabbatical in 1973.

Thereafter, Roget continued to serve for the rest of his life as a devoted apostle for *Space Time Algebra* in the French mathematics community, though conversion proved to be as difficult in France as the rest of the world. The slow diffusion of STA and Geometric Algebra in the scientific community is described in [90]. It is most gratifying that Roget independently reworked my papers on the electron in his own way over many decades, thereby giving independent support for the main ideas.

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