

A VIRTUAL REALITY SYSTEM FOR VISUALIZING GEOMETRIC ALGEBRAS

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Summary of the Abstract

A virtual reality system for visualizing the several geometric algebras (including the conformal geometric algebra and projective geometric algebra) is described. Users can create geometric objects in 3D, using their hands to position points and vectors in space, the stereo headset giving a 3D view of the world.

Introduction

Numerous systems have been developed for visualizing the spaces of geometric algebras. All of these systems have been either for 2D algebras, or for projects of three space onto a flat screen. In this work, we describe a virtual reality system for visualizing geometric algebras in 3D, using a Meta Quest III headset. While the system works for several algebras, the focus of the work is on the Conformal Geometric Algebra.

This work follows from GABLE and GAViewer, so we will focus our discussion of previous work on visualization to those two packages, although other packages exist for viewing and interacting with geometric algebras.

GABLE was a prototype Matlab package supporting a few geometric algebras. The input to GABLE was primarily textual, and the output was Matlab graphics, with the drawn objects being vectors, bivectors, and trivectors (Figure 1). Interaction of objects was fairly limited, although some support (via Matlab) for changing the view and creating animations was available. The default view for bivectors was as circles, with the area of the circle being the magnitude of the bivector, and with line segments around the boundary of the circle to indicate the orientation of the bivector. Trivectors were drawn as line segments of a sphere (to allow one to see inside the sphere), again with line segments indicating the orientation of the trivector.

GAViewer was built on geometry libraries created with Gaigen. In addition to having circular and spherical representations of bivectors and trivectors (Figure 2(a)-(c)), it also had a factored representation of these objects, depicting them as parallelograms and parallelepipeds (Figure 2(c),(d)). More importantly, there was better support for the conformal model, and for drawing objects in the conformal models (Figure 3). While

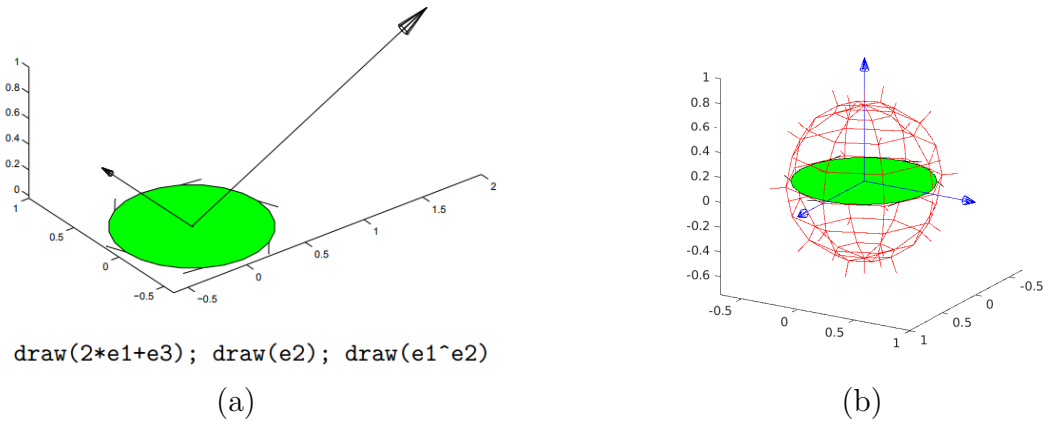


Figure 1: GABLE. (a) Example text input and graphics output; (b) Drawing of vectors, bivectors, and trivectors.

GAVIEWER has significantly better means for interacting with objects (the scene could easily be translated and rotated with the mouse), the primary method for entering data is still through a text window. Further, the 3D depth cues were from a mix of shading, perspective, and rotation.

With virtual reality headsets (Figure 4), improved visualization and interaction with objects in a geometric algebra is possible. The obvious graphics improvement is the stereo image, giving better 3D visualization of objects with binocular vision. Additionally, the user can create and manipulate objects either directly with their hands or with controllers.

With Meta Quest, a virtual world is created, with a camera placed at the viewer's eye location (Figure 5). The rendered scene is then superimposed on the real world (Figure 6). The user can interact with the objects in the scene, such as grabbing the tip of a vector with a controller or with their hand, and changing the vector by moving the tip with their hand in 3D.

The goal of this work is to create a tutorial along the lines of the GABLE tutorial, but allowing for interaction with the constructed objects. Further, this virtual reality system will have support for CGA [2], allowing the user to create points, and then create circles and spheres as the outer product of points.

References

- [1] Leo Dorst, Stephen Mann, and Tim Bouma. GABLE: A Matlab tutorial for geometric algebra. Available at <https://cs.uwaterloo.ca/~smann/GABLE/>, 1999.
- [2] Leo Dorst, Daniel Fontijne, and Stephen Mann. *Geometric Algebra for Computer Science*. Morgan-Kaufmann, 2007.

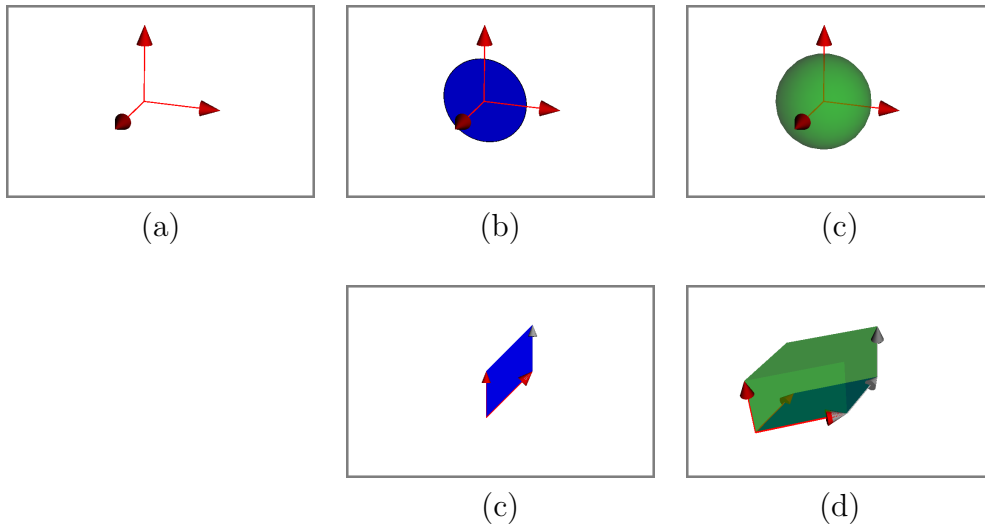


Figure 2: GAViewer: (a) vectors, (b) bivectors, (c) trivectors, (c) factored bivectors, (d) factored trivectors.

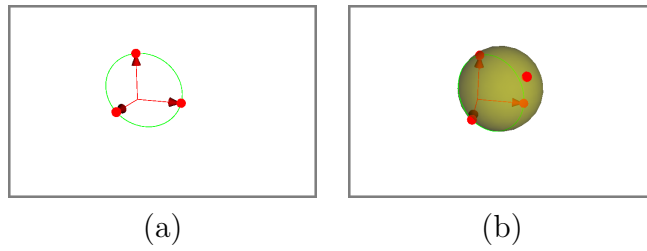


Figure 3: GAViewer: (a) Circle as outer product of three conformal points, (b) Sphere as outer product of four conformal points.



Figure 4: Left: Meta Quest III

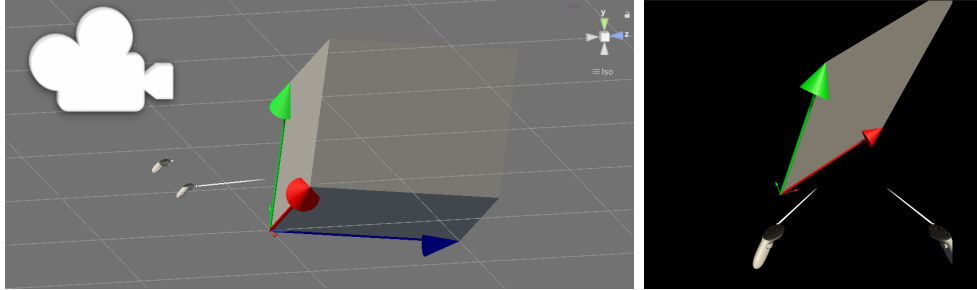


Figure 5: Left: a virtual world, specifying where the viewer is relative to the objects in the scene. Right: the scene rendered from the camera's view.

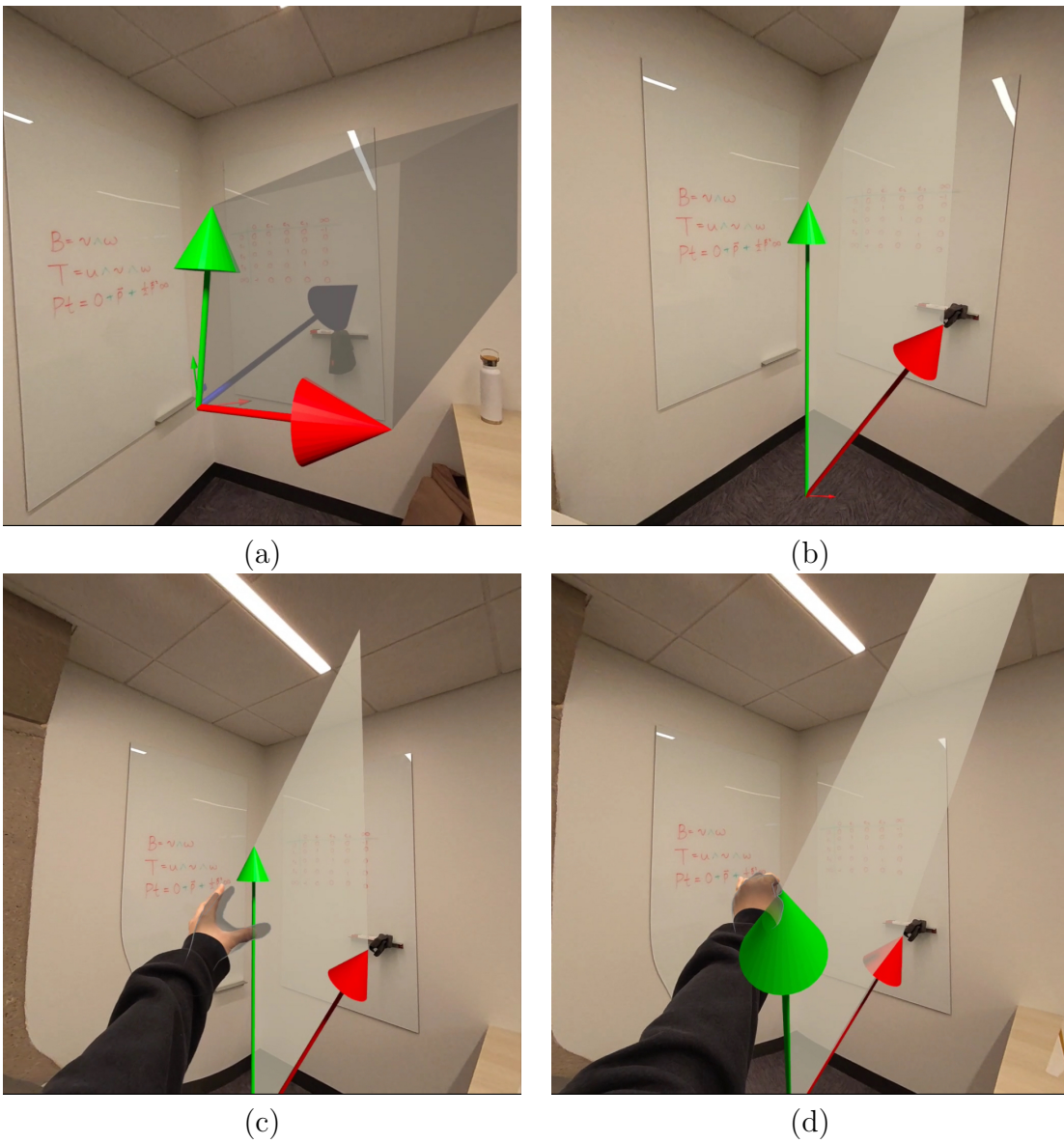


Figure 6: (a) A bivector; (b) a trivector; (c) and (d) grabbing and adjusting one of the vectors of the trivector.

- [3] Daniel Fontijne. Gaigen 2: a geometric algebra implementation generator. In *Proceedings of GPCE'06*, 2006.