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Collective modes of the quantum one-component plasma in a magnetic field

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We derive the collective modes of a quantum one-component plasma in a magnetic field by using a projection operator technique. With the help of these modes the long-time behaviour of the time correlation functions for the charge density, the current density and the energy density is determined in the limit of small wavenumbers.

1. Introduction

In a previous paper [1] we have established a set of equilibrium fluctuation formulas for the quantum one-component plasma (OCP) in a magnetic field. The method we used was the so-called equation-of-motion method. In this way we have been able to derive a complete set of fluctuation formulas for the charge density, the current density and the energy density.

Once these fluctuation formulas are available a discussion of time-dependent correlations comes within reach. In general, the asymptotic decay of these time correlations is determined by the collective modes that are sustained by the system. In deriving these modes the equilibrium fluctuation properties of the system are used in an essential way.

We will calculate the mode spectrum by starting from the microscopic balance equations for the charge density, the current density and the energy density. Subsequently, projection operator methods will be employed to derive an eigenvalue equation from which the modes can be determined. For a classical plasma in a magnetic field this method has been used before [2]. One expects that the basic features of the modes and frequencies as found in the classical theory, such as the dependence on the angle between the wavevector and the magnetic field, will survive in the quantum case. In particular, the gyroplasmon frequencies will certainly play a role in the quantum plasma as well. However, the dynamical behaviour of the quantum system will be more complicated than that of the classical system, since in the quantum case the anisotropy due to the magnetic field is a more profound feature of the system. In fact, even the equilibrium properties are influenced by this anisotropy.

The mode frequencies of a quantum plasma in a magnetic field have been studied before in the context of the random phase approximation (RPA) [3]. In that approximation some features of the mode spectrum are missed, however. In particular, dissipative damping effects in the mode frequencies are not included. In the following we will avoid the use of RPA.

The paper is organised as follows. In section 2 we will introduce some notations and write down the balance equations. In section 3 we will derive the collective modes and mode frequencies in successive orders of the wavenumber, for general orientations of the wavevector with respect to the magnetic field. In section 4 we will treat the special case of a wavevector that is perpendicular to the magnetic field. As it turns out, the mode spectrum for that case cannot be found by starting from the treatment of section 3. In section 5 we will discuss the long-time asymptotic behaviour of the time correlation functions for the charge density, the current density and the energy density. Lastly, in the appendix we will determine a few fluctuation formulas that we need in the derivation of the modes and that do not appear in [1].

2. Balance equations

The system we are considering is a magnetized OCP in the fluid phase. It consists of N particles of charge e and mass m in a volume V. The particles move in a neutralizing background of charge density $-q_V = -eN/V$. The external magnetic field $\boldsymbol{B}$ is uniform and time-independent.

In order to derive the collective modes we shall need the balance equations of the charge density, the current density and the energy density. In second quantization the Fourier transforms of these quantities are defined as

$$Q(\boldsymbol{k}) = \frac{e}{V} \sum_{\boldsymbol{k}'} \psi^{\dagger}(\boldsymbol{k}' - \boldsymbol{k})\, \psi(\boldsymbol{k}')\,, \tag{1}$$

$$\boldsymbol{J}(\boldsymbol{k}) = \frac{e\hbar}{2mV} \sum_{\boldsymbol{k}'} \psi^{\dagger}(\boldsymbol{k}' - \boldsymbol{k}) \left(2\boldsymbol{k}' - \boldsymbol{k} - \frac{ie}{\hbar c} \boldsymbol{B} \wedge \nabla_{\boldsymbol{k}'}\right) \psi(\boldsymbol{k}')\,, \tag{2}$$

$$E(\boldsymbol{k}) = \frac{\hbar^2}{8mV} \sum_{\boldsymbol{k}'} \psi^\dagger(\boldsymbol{k}'-\boldsymbol{k}) \left(2\boldsymbol{k}' - \boldsymbol{k} - \frac{\mathrm{i}e}{\hbar c} \boldsymbol{B} \wedge \nabla_{\boldsymbol{k}'}\right)^2 \psi(\boldsymbol{k}')$$
$$- \frac{1}{2V^3} \sum_{\boldsymbol{q}(\neq \boldsymbol{0}, \neq \boldsymbol{k})} \frac{e^2 \boldsymbol{q}\cdot(\boldsymbol{k}-\boldsymbol{q})}{q^2(\boldsymbol{k}-\boldsymbol{q})^2} \sum_{\boldsymbol{k}'\boldsymbol{k}''} \psi^\dagger(\boldsymbol{k}'-\boldsymbol{k}+\boldsymbol{q})\, \psi^\dagger(\boldsymbol{k}''-\boldsymbol{q})\, \psi(\boldsymbol{k}'')\, \psi(\boldsymbol{k}') \,. \tag{3}$$

The set of all Fourier-transformed local operators span a Hilbert space with a scalar product given by the Kubo–Mori expression

$$\langle A(\boldsymbol{k}) \mid B(\boldsymbol{k})\rangle = \beta^{-1} \int_0^\beta \mathrm{d}\tau \frac{1}{V} \langle [A^\dagger(\boldsymbol{k})]_\tau B(\boldsymbol{k})\rangle_{\mathrm{T}} \,, \tag{4}$$

where we introduced the Dirac bra–ket notation. The integrand is an imaginary-time-dependent Green function, with the subscript T indicating truncation. In the canonical ensemble it is defined as

$$\frac{1}{V} \langle [A^\dagger(\boldsymbol{k})]_\tau B(\boldsymbol{k})\rangle = Z^{-1}\, \mathrm{tr}\{\exp[-(\beta-\tau)H]\, A^\dagger(\boldsymbol{k}) \exp(-\tau H)\, B(\boldsymbol{k})\} \,, \tag{5}$$

in which H is the Hamiltonian, Z is the canonical partition function and β is the inverse temperature.

Instead of the operators given in (1)–(3) we will make use of the kets

$$|A_0(\boldsymbol{k})\rangle := \frac{\sqrt{\beta}}{k} |Q(\boldsymbol{k})\rangle \,, \tag{6}$$

$$|A_i(\boldsymbol{k})\rangle := \frac{\sqrt{\beta}}{\omega_{\mathrm{p}}} |J_i(\boldsymbol{k})\rangle \,, \qquad i = 1, 2, 3 \,, \tag{7}$$

$$|A_4(\boldsymbol{k})\rangle := \sqrt{\frac{k_{\mathrm{B}}\beta^2}{n c_V}}\, |E(\boldsymbol{k})\rangle \,, \tag{8}$$

and the corresponding adjoints. Here $\omega_{\mathrm{p}} = (ne^2/m)^{1/2}$ is the plasmon frequency, c_V is the specific heat per particle and n is the particle density. The ket vectors are chosen such that for small wavevectors their scalar products get the form

$$\langle A_a(\boldsymbol{k}) | A_b(\boldsymbol{k})\rangle = \delta_{ab} + \mathcal{O}(k) \,. \tag{9}$$

This statement can be checked by employing the fluctuation formulas derived in [1]. In the following we shall be making frequent use of results from that paper, in fact so often that we will not state it explicitly anymore. Occasionally, we shall need fluctuation formulas that have not been given in [1]. These have been collected in the appendix.

To formulate the balance equations for the quantities defined in (6)–(8) it is convenient to introduce the Liouville superoperator L, which acts in the Hilbert space of operators. It is defined as

$$LA = \frac{1}{\hbar}[H, A] . \tag{10}$$

Having introduced these notations we can finally write the balance equations as

$$L|A_0(\boldsymbol{k})\rangle = -\omega_{\mathrm{p}}\hat{\boldsymbol{k}}\cdot|\boldsymbol{A}(\boldsymbol{k})\rangle , \tag{11}$$

$$L|\boldsymbol{A}(\boldsymbol{k})\rangle = -\omega_{\mathrm{p}}\hat{\boldsymbol{k}}|A_0(\boldsymbol{k})\rangle - \mathrm{i}\omega_{\mathrm{c}}|\boldsymbol{A}(\boldsymbol{k})\rangle \wedge \hat{\boldsymbol{B}} - \frac{e\sqrt{\beta}}{m\omega_{\mathrm{p}}}\boldsymbol{k}\cdot|\boldsymbol{T}(\boldsymbol{k})\rangle , \tag{12}$$

$$L|A_4(\boldsymbol{k})\rangle = -\sqrt{\frac{k_{\mathrm{B}}\beta^2}{nc_V}}\,\boldsymbol{k}\cdot|\boldsymbol{J}_{\mathrm{E}}(\boldsymbol{k})\rangle . \tag{13}$$

Here $\omega_{\mathrm{c}} = (e/mc)|\boldsymbol{B}|$ is the cyclotron frequency, $\boldsymbol{T}(\boldsymbol{k})$ is the pressure tensor defined in [1] and $\boldsymbol{J}_{\mathrm{E}}(\boldsymbol{k})$ is the energy flow density. In general, we shall write $\hat{\boldsymbol{p}}$ for the unit vector in the direction of $\boldsymbol{p}$.

3. Collective modes

The time-dependent correlation function $C_{ab}(\boldsymbol{k}, t)$ associated to two quantities $A_a(\boldsymbol{k})$ and $A_b(\boldsymbol{k})$ from the set (6)–(8) is defined as

$$C_{ab}(\boldsymbol{k}, t) = \langle A_a(\boldsymbol{k})|\, \mathrm{e}^{\mathrm{i}Lt}|A_b(\boldsymbol{k})\rangle . \tag{14}$$

The (one-sided) Fourier-transformed counterpart of this function

$$C_{ab}(\boldsymbol{k}, z) = -\mathrm{i}\int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}zt} C_{ab}(\boldsymbol{k}, t) = \langle A_a(\boldsymbol{k})|\, \frac{1}{z+L}\, |A_b(\boldsymbol{k})\rangle , \tag{15}$$

with $\mathrm{Im}\, z > 0$, satisfies an equation that follows by employing the projection-operator formalism:

$$\sum_{cd} [zC_{ac}(\boldsymbol{k}) - \Omega_{ac}(\boldsymbol{k}) - M_{ac}(\boldsymbol{k}, z)]C_{cd}(\boldsymbol{k})^{-1}C_{db}(\boldsymbol{k}, z) = C_{ab}(\boldsymbol{k}) . \tag{16}$$

Here the summation indices take the values 0 to 4. Furthermore,

$$C_{ab}(\boldsymbol{k}) = C_{ab}(\boldsymbol{k}, t = 0) = \langle A_a(\boldsymbol{k}) | A_b(\boldsymbol{k}) \rangle \tag{17}$$

is the static correlation function, while the matrices

$$\Omega_{ab}(\boldsymbol{k}) = -\langle A_a(\boldsymbol{k}) | L | A_b(\boldsymbol{k}) \rangle , \tag{18}$$

$$M_{ab}(\boldsymbol{k}, z) = \langle A_a(\boldsymbol{k}) | LQ \frac{1}{z + QLQ} QL | A_b(\boldsymbol{k}) \rangle \tag{19}$$

are proportional to the direct and the indirect part of the frequency matrix, respectively. The projection operator Q projects on the complement of the part of Hilbert space spanned by the ket vectors (6)–(8):

$$Q = 1 - P = 1 - \sum_{ab} |A_a(\boldsymbol{k})\rangle C_{ab}(\boldsymbol{k})^{-1} \langle A_b(\boldsymbol{k})| . \tag{20}$$

The collective mode frequencies are the eigenvalues of the frequency matrix for small wavenumbers k. The collective modes are the corresponding eigenvectors. We will calculate these eigenvalues and eigenvectors from the generalized eigenvalue problem defined by putting the determinant of the matrix in the first factor of (16) equal to 0. In doing so we shall follow a perturbative procedure. First, we calculate the modes in zeroth order of k. Subsequently, these zeroth-order modes will be used as a basis to calculate the first-order modes, and then we move on to the second order. In the following we shall indicate the order in k by a superscript. So $A^{(n)}$ is the nth coefficient in the expansion of A in powers of k, while $A^{(0+1+2)}$, for example, stands for the expansion of A up to second order in k.

3.1. Zeroth order

From the balance equations it follows that one has

$$QL|A_a(\boldsymbol{k})\rangle = \mathcal{O}(k) , \tag{21}$$

for all $a = 0, \ldots, 4$. Hence, the indirect part of the frequency matrix is at least of order k^2, which means we can forget about this part while determining the eigenvalues and eigenvectors in zeroth and first order of k. Upon evaluating (17) and (18) in zeroth order one finds

$$\begin{vmatrix} z & -\omega_{\mathrm{p}}\hat{\boldsymbol{k}} & 0 \\ -\omega_{\mathrm{p}}\hat{\boldsymbol{k}} & z+\mathrm{i}\omega_{\mathrm{c}}\boldsymbol{\varepsilon}\cdot\hat{\boldsymbol{B}} & 0 \\ 0 & 0 & z \end{vmatrix} = 0\,, \tag{22}$$

where ε is the Levi-Civita tensor. The eigenvalues are

$$z_{\lambda\rho}^{(0)} = \rho\omega_\lambda\,, \qquad \lambda, \rho = \pm 1\,, \tag{23}$$

$$z_{\mathrm{T}}^{(0)} = 0\,, \tag{24}$$

where $\omega_\lambda = \frac{1}{2}\sqrt{\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2 + 2\omega_{\mathrm{p}}\omega_{\mathrm{c}}\cos\vartheta} + \frac{1}{2}\lambda\sqrt{\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2 - 2\omega_{\mathrm{p}}\omega_{\mathrm{c}}\cos\vartheta}$. We defined ϑ as the angle between $\boldsymbol{k}$ and $\boldsymbol{B}$. The corresponding eigenvectors have the form

$$|A_{\lambda\rho}^{(0)}(\boldsymbol{k})\rangle = C_\lambda[|A_0(\boldsymbol{k})\rangle + \boldsymbol{v}_{\lambda\rho}\cdot|\boldsymbol{A}(\boldsymbol{k})\rangle]\,, \tag{25}$$

$$|A_{\mathrm{T}}^{(0)}(\boldsymbol{k})\rangle = |A_4(\boldsymbol{k})\rangle\,. \tag{26}$$

Here $\boldsymbol{v}_{\lambda\rho}$ is the vector

$$\boldsymbol{v}_{\lambda\rho}(\hat{\boldsymbol{k}}) = \frac{\rho\omega_{\mathrm{p}}\omega_\lambda}{\omega_\lambda^2 - \omega_{\mathrm{c}}^2}\hat{\boldsymbol{k}}_\perp + \frac{\rho\omega_{\mathrm{p}}}{\omega_\lambda}\hat{\boldsymbol{k}}_\parallel - \frac{\mathrm{i}\omega_{\mathrm{p}}\omega_{\mathrm{c}}}{\omega_\lambda^2 - \omega_{\mathrm{c}}^2}\hat{\boldsymbol{k}}\wedge\hat{\boldsymbol{B}}\,, \tag{27}$$

with $\hat{\boldsymbol{k}}_\parallel = \cos\vartheta\,\hat{\boldsymbol{B}}$ and $\hat{\boldsymbol{k}}_\perp = \hat{\boldsymbol{k}} - \hat{\boldsymbol{k}}_\parallel$. The eigenvectors have been normalized. The normalization constant in eq. (25) is given by $C_\lambda = [\frac{1}{2}(\omega_\lambda^2 - \omega_{\mathrm{c}}^2)/(\omega_\lambda^2 - \omega_{-\lambda}^2)]^{1/2}$. The eigenvalues found here are closely analogous to those obtained in classical theory [2]. In particular, the eigenvalues (23) are the gyroplasmon frequencies.

3.2. First order

As a basis we will use the zeroth-order eigenvectors. The eigenvalue equation becomes

$$\begin{vmatrix} z-\omega_+ & 0 & 0 & 0 & (zR_+ + S_+)k \\ 0 & z+\omega_+ & 0 & 0 & (zR_+ - S_+)k \\ 0 & 0 & z-\omega_- & 0 & (zR_- + S_-)k \\ 0 & 0 & 0 & z+\omega_- & (zR_- - S_-)k \\ (zR_+ + S_+)k & (zR_+ - S_+)k & (zR_- + S_-) & (zR_- - S_-)k & z(1+Uk) \end{vmatrix} = 0\,, \tag{28}$$

with the abbreviations

$$R_\lambda = C_\lambda \sqrt{\frac{k_B\beta}{nc_V}} \frac{e}{m\omega_p^2}\left(\frac{\partial(\beta T_B)}{\partial\beta} + e_V - \frac{3}{2}\frac{\omega_\lambda^2 - \omega_p^2}{\omega_c^2}\frac{\partial}{\partial\beta}(\beta\, \delta T_B)\right), \tag{29}$$

$$S_\lambda = -C_\lambda \sqrt{\frac{k_B\beta}{nc_V}} \frac{e\omega_\lambda}{m\omega_p^2}\left(T_B + e_V - \frac{3}{2}\frac{\omega_\lambda^2 - \omega_p^2}{\omega_c^2}\delta T_B\right), \tag{30}$$

$$U = \langle A_T^{(0)}(\boldsymbol{k})|A_T^{(0)}(\boldsymbol{k})\rangle^{(1)}. \tag{31}$$

We introduced the pressure in the direction of the field $T_B = \hat{\boldsymbol{B}}\cdot\langle \boldsymbol{T}\rangle\cdot\hat{\boldsymbol{B}}$ and the anisotropic part of the pressure tensor $\delta T_B = T_B - \frac{1}{3}\mathrm{tr}\langle \boldsymbol{T}\rangle$. Furthermore, the energy density is $e_V = \langle E\rangle$. In determining the matrix in (28) we used a few fluctuation formulas which do not appear in [1] and which have been listed in the appendix. An explicit expression for U is not available. However, it will not be needed in the following.

The eigenvalues are unchanged as compared to the zeroth order. However, the eigenvectors do gain extra terms in first order:

$$|A_{\lambda\rho}^{(0+1)}(\boldsymbol{k})\rangle = C_\lambda\Big[|A_0(\boldsymbol{k})\rangle + \boldsymbol{v}_{\lambda\rho}\cdot|\boldsymbol{A}(\boldsymbol{k})\rangle - \frac{ek}{m\omega_p^2}\sqrt{\frac{k_B\beta}{nc_V}}\,\beta\frac{\partial}{\partial\beta}\left(T_B - \frac{3}{2}\frac{\omega_\lambda^2-\omega_p^2}{\omega_c^2}\delta T_B\right)|A_4(\boldsymbol{k})\rangle\Big], \tag{32}$$

$$|A_T^{(0+1)}(\boldsymbol{k})\rangle = C_T^{(0+1)}\Big[|A_4(\boldsymbol{k})\rangle - \frac{ek}{m\omega_p^2}\sqrt{\frac{k_B\beta}{nc_V}}\Big((T_B + e_V)|A_0(\boldsymbol{k})\rangle + \frac{3\mathrm{i}\omega_p}{2\omega_c}\delta T_B\,(\hat{\boldsymbol{k}}\wedge\hat{\boldsymbol{B}})\cdot|\boldsymbol{A}(\boldsymbol{k})\rangle\Big)\Big], \tag{33}$$

where $C_T^{(0+1)} = 1 - \frac{1}{2}Uk$.

3.3. Second order

The second-order corrections to the first-order mode frequencies $z_{\lambda\rho}^{(0+1)} = \rho\omega_\lambda$ and $z_T^{(0+1)} = 0$ follow from perturbation theory. The expression for the mode frequencies up to second order is

$$z_a = \Big(-\langle A_a(\boldsymbol{k})|L|A_a(\boldsymbol{k})\rangle + \langle A_a(\boldsymbol{k})|LQ\frac{1}{z_a + QLQ}QL|A_a(\boldsymbol{k})\rangle\Big)\langle A_a(\boldsymbol{k})|A_a(\boldsymbol{k})\rangle^{-1}, \tag{34}$$

in which we have to insert, on the right-hand-side, the first-order eigenvectors and the zeroth-order frequencies. The results are

$$z_{\lambda\rho}^{(0+1+2)} = \rho\omega_\lambda - \mathrm{i}k^2 D_{\lambda\rho}(\hat{\boldsymbol{k}}) , \tag{35}$$

$$z_{\mathrm{T}}^{(0+1+2)} = -\mathrm{i}k^2 D_{\mathrm{T}}(\hat{\boldsymbol{k}}) . \tag{36}$$

Here the second-order contributions, which determine the dispersion and the damping of the modes in leading order of k, are

$$\begin{aligned} D_{\lambda\rho}(\hat{\boldsymbol{k}}) = \mathrm{i}C_\lambda^2 \Big\{ & \frac{k_{\mathrm{B}}\beta^3}{n^2 m c_V \omega_{\mathrm{p}}^2} \rho\omega_\lambda \left[\frac{\partial}{\partial\beta} \left(T_B - \frac{3}{2} \frac{\omega_\lambda^2 - \omega_{\mathrm{p}}^2}{\omega_{\mathrm{c}}^2} \delta T_B \right) \right]^2 \\ & + \frac{\beta}{\sqrt{nm}} \left(\langle Q(\boldsymbol{k}) | \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{v}_{\lambda\rho} \rangle^{(2)} + \frac{1}{\omega_{\mathrm{p}}} \langle \boldsymbol{v}_{\lambda\rho} \cdot \boldsymbol{J}(\boldsymbol{k}) | \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{v}_{\lambda\rho} \rangle^{(1)} \right) \\ & + \frac{\beta}{nm} \langle \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{v}_{\lambda\rho} | Q \frac{1}{\rho\omega_\lambda + QLQ} Q | \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{v}_{\lambda\rho} \rangle^{(0)} \Big\} , \end{aligned} \tag{37}$$

$$D_T(\hat{\boldsymbol{k}}) = \mathrm{i} \frac{k_{\mathrm{B}}\beta^2}{n c_V} \langle \hat{\boldsymbol{k}} \cdot \boldsymbol{J}_{\mathrm{E}}(\boldsymbol{k}) | Q \frac{1}{QLQ} Q | \hat{\boldsymbol{k}} \cdot \boldsymbol{J}_{\mathrm{E}}(\boldsymbol{k}) \rangle^{(0)} . \tag{38}$$

One expects that the damping of the modes is governed by the indirect part of the frequency matrix and not by the direct part, since the latter is determined by the Hermitian operator L. Indeed, one may prove that the first two lines of (37) are purely imaginary, so that they contribute to the dispersion only.

At this point a comparison with the classical result is useful. The classical limit of $z_{\mathrm{T}}^{(0+1+2)}$ is trivially obtained. It coincides with the expression found in [2]. To derive the classical limit of $z_{\lambda\rho}^{(0+1+2)}$ we use the fact that the pressure tensor becomes isotropic then, so that δT_B drops out and T_B equals p. Furthermore, we need the classical counterparts of the fluctuation formulas involving Q, $\boldsymbol{J}$ and $\boldsymbol{T}$. From [4] we find in the present notation:

$$\langle Q(\boldsymbol{k}) | \boldsymbol{T}(\boldsymbol{k}) \rangle_{\mathrm{cl}}^{(2)} = \frac{k^2}{\beta n e \kappa_{\mathrm{T}}} \boldsymbol{U} , \tag{39}$$

$$\langle \boldsymbol{J}(\boldsymbol{k}) | \boldsymbol{T}(\boldsymbol{k}) \rangle_{\mathrm{cl}}^{(1)} = 0 . \tag{40}$$

Notice the appearance of the classical isothermal compressibility κ_{T}. In the quantum case an explicit thermodynamic result for the linear combination of fluctuation expressions occurring in (37) is not readily available. In particular, it is not clear whether this combination is related to the compressibility for the quantum OCP as well.

In this way we arrive at a classical expression for $z_{\lambda\rho,\mathrm{cl}}^{(0+1+2)}$ which is consistent with that given in [2]:

$$z_{\lambda\rho,\mathrm{cl}}^{(0+1+2)} = \rho\omega_\lambda(1 + k^2 c_\mathrm{s}^2 \omega_\mathrm{p}^{-2} C_\lambda^2) + k^2 C_\lambda^2 \frac{\beta}{nm} \langle \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{v}_{\lambda\rho} | Q \frac{1}{\rho\omega_\lambda + QLQ} Q | \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{v}_{\lambda\rho} \rangle_\mathrm{cl}^{(0)} . \tag{41}$$

Here c_s is the sound velocity of the classical OCP, which is given by

$$c_\mathrm{s}^2 = \frac{1}{nm\kappa_\mathrm{T}} + \frac{k_\mathrm{B}\beta^3}{n^2 m c_V} \left(\frac{\partial p}{\partial \beta}\right)^2 . \tag{42}$$

In closing this section we remark that it is possible to write the indirect parts of the frequency matrix, appearing in $z_\mathrm{T}^{(0+1+2)}$ and $z_{\lambda\rho}^{(0+1+2)}$, in terms of thermal conductivity coefficients and dynamical viscosity coefficients. The procedure is the same as in the classical case (for the details, see [2]).

4. Collective modes, the degenerate case

In the previous section we calculated the collective modes for the quantum OCP. There, we silently assumed that the wavevector $\boldsymbol{k}$ and the magnetic field $\boldsymbol{B}$ are not orthogonal, i.e., that one has $\cos\vartheta \neq 0$. In the opposite case of mutually perpendicular field and wavevector, the eigenvalues are degenerate, and perturbation theory is no longer valid in the form we used it. As a consequence, we have to go through the calculations again in order to find the modes for $\cos\vartheta = 0$.

4.1. Zeroth order

In zeroth order, the eigenvalue equation is the same as in the non-degenerate case, i.e., it is given by (22). The eigenvalues and eigenvectors do change, however, because $\cos\vartheta = 0$. They read

$$z_\rho^{(0)} = \rho\omega , \qquad \rho = \pm 1 , \tag{43}$$

$$z = 0 , \quad (3\times) , \tag{44}$$

with $\omega = (\omega_\mathrm{p}^2 + \omega_\mathrm{c}^2)^{1/2}$. The corresponding eigenvectors are

$$|A_\rho^{(0)}(\boldsymbol{k})\rangle = C[|A_0(\boldsymbol{k})\rangle + \boldsymbol{w}_\rho \cdot |\boldsymbol{A}(\boldsymbol{k})\rangle] , \tag{45}$$

$$|A_{\mathrm{V}}^{(0)}(\boldsymbol{k})\rangle = \hat{\boldsymbol{B}} \cdot |\boldsymbol{A}(\boldsymbol{k})\rangle , \tag{46}$$

$$|A_{\mathrm{C}}^{(0)}(\boldsymbol{k})\rangle = C_{\mathrm{C}}\Big(|A_0(\boldsymbol{k})\rangle + \frac{\mathrm{i}\omega_{\mathrm{p}}}{\omega_{\mathrm{c}}}(\hat{\boldsymbol{k}} \wedge \hat{\boldsymbol{B}}) \cdot |\boldsymbol{A}(\boldsymbol{k})\rangle\Big) , \tag{47}$$

$$|A_{\mathrm{T}}^{(0)}(\boldsymbol{k}) = |A_4(\boldsymbol{k})\rangle , \tag{48}$$

with the vector $\boldsymbol{w}_\rho$ defined as

$$\boldsymbol{w}_\rho(\hat{\boldsymbol{k}}) = \frac{\rho(\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2)^{1/2}}{\omega_{\mathrm{p}}}\hat{\boldsymbol{k}} - \frac{\mathrm{i}\omega_{\mathrm{c}}}{\omega_{\mathrm{p}}}\hat{\boldsymbol{k}} \wedge \hat{\boldsymbol{B}} . \tag{49}$$

The normalization constants are $C_{\mathrm{C}} = \omega_{\mathrm{c}}/\sqrt{\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2}$ and $C = \omega_{\mathrm{p}}/\sqrt{2(\omega_{\mathrm{p}}^2 + \omega_{\mathrm{c}}^2)}$.

Notice that the eigenvectors (45) for the gyroplasmon modes are found by taking the limit $\cos\vartheta \to 0$ in the $\lambda = 1$ case of (25). In contrast, the form of the eigenvectors, as given in (46)–(48), is a choice: because of the degeneracy in the $z = 0$ eigenvalues we could have chosen a different set. For the thermal mode (48) we took the same form as in (26). On the other hand, we did not choose the eigenvectors $|A_{\mathrm{V}}^{(0)}\rangle$ and $|A_{\mathrm{C}}^{(0)}\rangle$ to be the limiting cases of the $\lambda = -1$ eigenvectors in (25). The reason for this choice will become clear later on. In their present form the eigenvectors (46)–(47) are closely analogous to those found in classical theory, the first describing a viscous mode and the second the convective-cell mode.

4.2. *First order*

Upon adopting as a basis the eigenvectors found in zeroth order in k we get the eigenvalue equation in first order as

$$\begin{vmatrix} z-\omega & 0 & 0 & 0 & (z\mathcal{R}+\mathcal{S})k \\ 0 & z+\omega & 0 & 0 & (z\mathcal{R}-\mathcal{S})k \\ 0 & 0 & z & 0 & 0 \\ 0 & 0 & 0 & z & z\mathcal{T}k \\ (z\mathcal{R}+\mathcal{S})k & (z\mathcal{R}-\mathcal{S})k & 0 & z\mathcal{T}k & z(1+Uk) \end{vmatrix} = 0 , \tag{50}$$

with

$$\mathcal{R} = C\sqrt{\frac{k_{\mathrm{B}}\beta}{nc_V}}\frac{e}{m\omega_{\mathrm{p}}^2}\Big(\frac{\partial(\beta T_B)}{\partial\beta} + e_V - \frac{3}{2}\frac{\partial}{\partial\beta}(\beta\,\delta T_B)\Big) , \tag{51}$$

$$\mathscr{S} = -C\sqrt{\frac{k_{\mathrm{B}}\beta}{nc_V}}\,\frac{e\omega}{m\omega_{\mathrm{p}}^2}\left(T_B + e_V - \frac{3}{2}\,\delta T_B\right), \tag{52}$$

$$\mathscr{T} = C_{\mathrm{C}}\sqrt{\frac{k_{\mathrm{B}}\beta}{nc_V}}\,\frac{e}{m\omega_{\mathrm{p}}^2}\left(\frac{\partial(\beta T_B)}{\partial\beta} + e_V + \frac{3}{2}\,\frac{\omega_{\mathrm{p}}^2}{\omega_{\mathrm{c}}^2}\,\frac{\partial}{\partial\beta}\,(\beta\,\delta T_B)\right). \tag{53}$$

The eigenvalues that follow from (50) retain the same form as in zeroth order, so that three of them are still degenerate. The eigenvectors for the gyroplasmon modes get an additional contribution in first order:

$$\begin{aligned}|A_\rho^{(0+1)}(\boldsymbol{k})\rangle = C\Big(&|A_0(\boldsymbol{k})\rangle + \boldsymbol{w}_\rho\cdot|\boldsymbol{A}(\boldsymbol{k})\rangle\\ &-\sqrt{\frac{k_{\mathrm{B}}\beta}{nc_V}}\,\frac{ek}{m\omega_{\mathrm{p}}^2}\,\beta\,\frac{\partial}{\partial\beta}\left(T_B - \frac{3}{2}\,\delta T_B\right)|A_4(\boldsymbol{k})\rangle\Big).\end{aligned} \tag{54}$$

For the three degenerate modes we again have a freedom of choice in selecting a convenient set of eigenvectors. For the viscous mode we adopt the same form as in zeroth order. As the thermal-mode eigenvector in first order we take the same expression as in the non-degenerate case, i.e. (33). Finally, the first-order convective-cell eigenvector is determined by orthogonalization. As a consequence we get

$$\begin{aligned}|A_{\mathrm{C}}^{(0+1)}(\boldsymbol{k}) = C_{\mathrm{C}}\Big[&|A_0(\boldsymbol{k})\rangle + \frac{\mathrm{i}\omega_{\mathrm{p}}}{\omega_{\mathrm{c}}}\,(\hat{\boldsymbol{k}}\wedge\hat{\boldsymbol{B}})\cdot|\boldsymbol{A}(\boldsymbol{k})\rangle\\ &-\frac{ek}{m\omega_{\mathrm{p}}^2}\sqrt{\frac{k_{\mathrm{B}}\beta}{nc_V}}\,\beta\,\frac{\partial}{\partial\beta}\left(T_B + \frac{3}{2}\,\frac{\omega_{\mathrm{p}}^2}{\omega_{\mathrm{c}}^2}\,\delta T_B\right)|A_4(\boldsymbol{k})\rangle\Big],\end{aligned} \tag{55}$$

$$\begin{aligned}|A_{\mathrm{T}}^{(0+1)}(\boldsymbol{k})\rangle = C_{\mathrm{T}}^{(0+1)}\Big[&|A_4(\boldsymbol{k})\rangle - \frac{ek}{m\omega_{\mathrm{p}}^2}\sqrt{\frac{k_{\mathrm{B}}\beta}{nc_V}}\Big((T_B + e_V)|A_0(\boldsymbol{k})\rangle\\ &+\frac{3\mathrm{i}\omega_{\mathrm{p}}}{2\omega_{\mathrm{c}}}\,\delta T_B\,(\hat{\boldsymbol{k}}\wedge\hat{\boldsymbol{B}})\cdot|\boldsymbol{A}(\boldsymbol{k})\rangle\Big)\Big].\end{aligned} \tag{56}$$

4.3. Second order

One can find the second-order correction to the non-degenerate eigenvalues $z_\rho^{(0+1)}$ via ordinary perturbation theory, as in (34). One finds

$$z_\rho^{(0+1+2)} = \rho\omega - \mathrm{i}k^2 D_\rho(\hat{\boldsymbol{k}}), \tag{57}$$

with the coefficient

$$D_\rho(\hat{k}) = \mathrm{i}C^2 \left\{ \frac{k_{\mathrm{B}}\beta^3}{n^2 m c_V \omega_{\mathrm{p}}^2}\, \rho\omega \left[\frac{\partial}{\partial\beta} \left(T_B - \frac{3}{2}\,\delta T_B \right) \right]^2 \right.$$
$$+ \frac{\beta}{\sqrt{nm}} \left(\langle Q(\boldsymbol{k}) | \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{w}_\rho \rangle^{(2)} + \frac{1}{\omega_{\mathrm{p}}} \langle \boldsymbol{w}_\rho \cdot \boldsymbol{J}(\boldsymbol{k}) | \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{w}_\rho \rangle^{(1)} \right)$$
$$\left. + \frac{\beta}{nm} \langle \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{w}_\rho | Q \frac{1}{\rho\omega + QLQ} Q | \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \boldsymbol{w}_\rho \rangle^{(0)} \right\} . \tag{58}$$

To find the mode frequencies belonging to the other modes, one has to use a modified perturbation theory, adjusted to degenerate eigenvalues. The non-gyroplasmon part of the direct frequency matrix vanishes up to second order in k, as follows from parity and time-reversal invariance (see the appendix). One is left with the indirect part. The latter, i.e., $M_{ij}(\boldsymbol{k}, z=0)$ for $i, j = \mathrm{V, C, T}$, has to be diagonalized in order to get the eigenvalues in second order. Using once more parity considerations and time-reversal invariance one can show that all the matrix-elements with one subindex V vanish, so that the viscous mode decouples from the other two. Here it becomes clear why we have made the particular choice of eigenvectors (46)–(48). For the viscous-mode frequency we thus find in second order:

$$z_V^{(0+1+2)} = \frac{\beta k^2}{nm} \langle \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \hat{\boldsymbol{B}} | Q \frac{1}{QLQ} Q | \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) \cdot \hat{\boldsymbol{B}} \rangle^{(0)} . \tag{59}$$

The mode frequencies of the convective-cell and the thermal mode follow by diagonalizing the two-dimensional matrix $M_{ij}(\boldsymbol{k}, z=0)$ for $i, j = \mathrm{C, T}$.

5. Asymptotic behaviour of time correlation functions

In this section we will use the properties of the modes obtained above to extract information on the time behaviour of the time correlation functions for the charge density, the current density and the energy density. In particular, we shall determine the asymptotic time dependence of these functions for small wave numbers and for large times.

The asymptotic behaviour of the time correlation functions $C_{ab}(\boldsymbol{k}, t)$ for large t is determined by the singularities of the Fourier transform $C_{ab}(\boldsymbol{k}, z)$ that are closest to the real z-axis. For small k these are determined by the mode frequencies evaluated in the previous sections. Let us start from (16) for small z and k. As a basis set we take the modes (32) and (33) up to first order in k. We multiply (16) from the left by the nth left eigenvector $\xi_a^{(n)}$ of the matrix

$\Sigma_c(\Omega_{ac} + M_{ac})C_{cb}^{-1}$. On the basis we have chosen, namely $|A_{\lambda\rho}^{(0+1)}\rangle$ and $|A_T^{(0+1)}\rangle$, these eigenvectors are given by $\xi_i^{(n)} = \delta_{in} + \mathcal{O}(k^2)$. The corresponding eigenvalues are z_n, which are given by (35) and (36) up to second order in k. Hence, we arrive at the equation

$$\sum_a \xi_a^{(n)} C_{ab}(\boldsymbol{k}, z) = (z - z_n)^{-1} \sum_a \xi_a^{(n)} C_{ab}(\boldsymbol{k}) . \tag{60}$$

Correspondingly, the time correlation function $\Sigma_a \xi_a^{(n)} C_{ab}(\boldsymbol{k}, t)$ for small wavenumber has the following asymptotic behaviour for large t:

$$\sum_a \xi_a^{(n)} C_{ab}(\boldsymbol{k}, t) = \sum_a \xi_a^{(n)} C_{ab}(\boldsymbol{k}) \exp(-\mathrm{i} z_n t) . \tag{61}$$

For small k it is sufficient to consider the modes up to first order in k only. Up to this order the mode frequencies do not yet contain damping terms. Since t is large, however, the leading damping terms in the frequencies, which are of second order in k, play an important role. Including these we arrive at the following set of asymptotic formulas:

$$\langle A_{\lambda\rho}^{(0+1)}(\boldsymbol{k}) | \mathrm{e}^{\mathrm{i}Lt} | B(\boldsymbol{k}) \rangle = \langle A_{\lambda\rho}^{(0+1)}(\boldsymbol{k}) | B(\boldsymbol{k}) \rangle \exp(-\mathrm{i} z_{\lambda\rho}^{(0+1+2)} t) , \tag{62}$$

$$\langle A_{\mathrm{T}}^{(0+1)}(\boldsymbol{k}) | \mathrm{e}^{\mathrm{i}Lt} | B(\boldsymbol{k}) \rangle = \langle A_{\mathrm{T}}^{(0+1)}(\boldsymbol{k}) | B(\boldsymbol{k}) \rangle \exp(-\mathrm{i} z_{\mathrm{T}}^{(0+1+2)} t) , \tag{63}$$

where $|B(\boldsymbol{k})\rangle$ can be any linear combination of the modes up to first order in k.

It is straightforward to obtain from (62) and (63) the asymptotic expressions for the time correlation functions of the charge density, the current density and the energy density, in leading order of the wavenumber. For the charge–current time correlation functions one finds

$$C_{QQ}^{(2)}(t) = \frac{1}{2\beta} \sum_{\lambda\rho} \frac{\omega_\lambda^2 - \omega_{\mathrm{c}}^2}{\omega_\lambda^2 - \omega_{-\lambda}^2} \exp(-\mathrm{i}\rho\omega_\lambda t - D_{\lambda\rho} k^2 t) , \tag{64}$$

$$C_{JQ}^{(1)}(t) = \frac{\omega_{\mathrm{p}}}{2\beta} \sum_{\lambda\rho} \frac{\omega_\lambda^2 - \omega_{\mathrm{c}}^2}{\omega_\lambda^2 - \omega_{-\lambda}^2} \boldsymbol{v}_{\lambda\rho} \exp(-\mathrm{i}\rho\omega_\lambda t - D_{\lambda\rho} k^2 t) , \tag{65}$$

$$C_{JJ}^{(0)}(t) = \frac{\omega_{\mathrm{p}}^2}{2\beta} \sum_{\lambda\rho} \frac{\omega_\lambda^2 - \omega_{\mathrm{c}}^2}{\omega_\lambda^2 - \omega_{-\lambda}^2} \boldsymbol{v}_{\lambda\rho} \boldsymbol{v}_{\lambda\rho}^* \exp(-\mathrm{i}\rho\omega_\lambda t - D_{\lambda\rho} k^2 t) , \tag{66}$$

with the second-order dispersion and damping contributions $D_{\lambda\rho}$ as given in (37).

Likewise, we find for the energy–density auto-correlation function

$$C^{(0)}_{\mathrm{EE}}(t)=\frac{nc_V}{k_{\mathrm{B}}\beta^2}\exp(-D_Tk^2t)\,, \tag{67}$$

with D_T defined in (38).

The evaluation of the asymptotic time behaviour of the cross-correlation functions with the charge or current density on the one hand and the energy density on the other hand is somewhat more involved. It turns out that in these cross-correlation functions exponentially damped functions with both the gyroplasmon and the thermal damping coefficients appear. In fact, one finds

$$\begin{aligned}C^{(2)}_{EQ}(t)=&\frac{1}{2}\frac{e}{\beta m\omega_{\mathrm{p}}}\sum_{\lambda\rho}\frac{\omega_\lambda^2-\omega_{\mathrm{c}}^2}{\omega_\lambda^2-\omega_{-\lambda}^2}\rho\omega_\lambda^{-1}(\hat{\boldsymbol{k}}\cdot\langle\boldsymbol{T}\rangle+\hat{\boldsymbol{k}}\langle E\rangle)\cdot\boldsymbol{v}_{\lambda\rho}\\&\times\exp(-\mathrm{i}\rho\omega_\lambda t-D_{\lambda\rho}k^2t)\\&+\frac{e}{m\omega_{\mathrm{p}}^2}\frac{\partial T_B}{\partial\beta}\exp(-D_Tk^2t)\,,\end{aligned} \tag{68}$$

$$\begin{aligned}C^{(1)}_{EJ}(t)=&\frac{1}{2}\frac{e}{\beta m}\sum_{\lambda\rho}\frac{\omega_\lambda^2-\omega_{\mathrm{c}}^2}{\omega_\lambda^2-\omega_{-\lambda}^2}\rho\omega_\lambda^{-1}\boldsymbol{v}^*_{\lambda\rho}(\hat{\boldsymbol{k}}\cdot\langle\boldsymbol{T}\rangle+\hat{\boldsymbol{k}}\langle E\rangle)\cdot\boldsymbol{v}_{\lambda\rho}\\&\times\exp(-\mathrm{i}\rho\omega_\lambda t-D_{\lambda\rho}k^2t)\\&-\frac{3\mathrm{i}e}{2m\omega_{\mathrm{c}}}(\hat{\boldsymbol{k}}\wedge\hat{\boldsymbol{B}})\frac{\partial\,\delta T_B}{\partial\beta}\exp(-D_Tk^2t)\,.\end{aligned} \tag{69}$$

It is interesting to compare the results found here with the imaginary-time-dependent fluctuation formulas calculated in [1], in particular with the cross-fluctuation formulas containing the energy density. In the latter a few static terms (independent of the imaginary time) appeared. In the present formalism the counterparts of these are the contributions in (68)–(69) of which the damping is determined by D_T. In contrast, the contributions in (68)–(69) with damping governed by $D_{\lambda\rho}$ correspond to the terms in [1] that depend on the imaginary time.

Appendix

In this appendix we will list some fluctuation formulas we have used in the main text, namely those that we have not come across in [1]. Not surprisingly, the fluctuation expressions in this appendix all vanish. The first ones we consider are those needed in the derivation of the modes in first order in k. They are

$$\langle \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) | Q(\boldsymbol{k}) \rangle^{(1)} = 0 \,, \tag{A.1}$$

$$\langle \hat{\boldsymbol{k}} \cdot \boldsymbol{T}(\boldsymbol{k}) | \boldsymbol{J}(\boldsymbol{k}) \rangle^{(0)} = 0 \,, \tag{A.2}$$

$$\langle Q(\boldsymbol{k}) | Q(\boldsymbol{k}) \rangle^{(3)} = 0 \,, \tag{A.3}$$

$$\langle Q(\boldsymbol{k}) | \boldsymbol{J}(\boldsymbol{k}) \rangle^{(2)} = 0 \,, \tag{A.4}$$

$$\langle \boldsymbol{J}(\boldsymbol{k}) | \boldsymbol{J}(\boldsymbol{k}) \rangle^{(1)} = 0 \,. \tag{A.5}$$

The first two of these formulas can be proven by using (22) and (23) of [1]. In this way one can reduce the left-hand sides to linear combinations of the commutator expressions

$$\frac{1}{V} \langle [\hat{\boldsymbol{k}} \cdot \boldsymbol{T}(-\boldsymbol{k}), Q(\boldsymbol{k})] \rangle^{(1)} \tag{A.6}$$

and

$$\frac{1}{V} \langle [\hat{\boldsymbol{k}} \cdot \boldsymbol{T}(-\boldsymbol{k}), \boldsymbol{J}(\boldsymbol{k})] \rangle^{(0)} \,. \tag{A.7}$$

By explicit calculation one shows that both these expressions are linear combinations of $V^{-1} \langle \boldsymbol{J}(\boldsymbol{k}) \rangle^{(0)}$, so that they must vanish on account of spatial reflection invariance. The last three fluctuation expressions listed above can be transformed to linear combinations of the first two by using once more the same equations of [1]. Hence, they vanish as well.

A few other fluctuation formulas that are needed in deriving the modes can be proven by using the symmetry of the system with respect to spatial reflection and time reversal. These symmetries lead to the following general properties of the Kubo–Mori scalar product $\langle A(\boldsymbol{k}) | B(\boldsymbol{k}) \rangle = C_{AB}(\boldsymbol{k}, \boldsymbol{B})$ as a function of $\boldsymbol{k}$ and $\boldsymbol{B}$:

$$C_{AB}(-\boldsymbol{k}, \boldsymbol{B}) = \eta_A^P \eta_B^P C_{AB}(\boldsymbol{k}, \boldsymbol{B}) \,, \tag{A.8}$$

$$C_{AB}(\boldsymbol{k}, -\boldsymbol{B}) = \eta_A^T \eta_B^T C_{AB}(\boldsymbol{k}, \boldsymbol{B}) \,, \tag{A.9}$$

where $\eta_{A,B}^P$ and $\eta_{A,B}^T$ are ± 1 depending on the even or odd character of A and B with respect to spatial reflection and time reversal. Using these properties one easily establishes the following fluctuation formulas valid for arbitrary values of the wavenumber:

$$\langle Q(\boldsymbol{k}) | \hat{\boldsymbol{k}} \cdot \boldsymbol{J}_{\mathrm{E}}(\boldsymbol{k}) \rangle = 0 \,, \tag{A.10}$$

$$\langle E(\boldsymbol{k})|\hat{\boldsymbol{k}}\cdot\boldsymbol{J}_{\mathrm{E}}(\boldsymbol{k})\rangle = 0\,, \tag{A.11}$$

$$\langle(\hat{\boldsymbol{k}}\wedge\hat{\boldsymbol{B}})\cdot\boldsymbol{J}(\boldsymbol{k})|\hat{\boldsymbol{k}}\cdot\boldsymbol{J}_{\mathrm{E}}(\boldsymbol{k})\rangle = 0\,. \tag{A.12}$$

In fact, the first fluctuation expression should be even in $\hat{\boldsymbol{k}}$ and odd in $\hat{\boldsymbol{B}}$. Since no scalar function with these properties can be constructed the expression must vanish. The proof of the other two formulas is equally trivial.

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