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Third International Colloquium on Cognitive Science

ICCS-93

Donostia-San Sebastian 4-8 May 1993

Contributed Papers

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on
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Contributed Papers

Airenti, G. and M. Colombetti: Language as a window on subjective ontology.

Aliaga, F. and E. de Bustos: Contextual effects of shifting mood: the case of Spanish subjunctive.

Alonso Fernandez & J. M. Marroquín Cirión: Emotion and Planning.

Arias, L.; A. Riviére, E. Sarriá: Emotion and action: Understanding of false belief in verbal communication.

Asher, N. and Bras, M.: An analysis of temporal structure in French texts within the framework of a formal semantic theory of discourse structure.

Aurnague, M., L. Vieu, A. Borillo, M. Borillo: Connecting linguistic and visual space: a natural reasoning approach.

Avrahami, J.: The emergence of events.

Bairaktaris, D. and J.P. Levy: Memory Consolidation in Mean Field Theory Auto-Associators.

Blackburn, P. and Y, Venema: The logic of dynamic implication.

Blutner, R.: Nonmonotonic logic, dynamic semantics, and adverbs of generic quantification.

Broncano, F.: Truth, rationality , fitness and other explanatory properties of success.

Carston, R.: Conjunction, juxtaposition and relevance.

Divitini, M., C. Simone, R. Tinini: Representing inconsistent and dynamic knowledge for supporting communication within cooperative groups.

Edmonson, W.: General cognitive principles and the structure of behaviour.

Espinal, T.: Marked Negation: a case study in the grammar-cognition interface.

Etxeberria, A. and A. Moreno: Conectionism and computation: a complementary proposal to characterize cognition.

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García-Carpintero, M.: Teleology still works.

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Ifantidou, E. : Sentence Adverbials, parentheticals and non-truth-conditional semantics.

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Kandel, S.; L-J. Boë, J-P Orliguet: Relationships between action and perception: Visual detection of coarticulatory anticipation in handwriting.

Korta, K. and J.M. Larrazabal: Intention in Dialogue.

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Counting Objects

Wiebe van der Hoek[1]   Maarten de Rijke[2]

Key words: knowledge representation, modal logic, generalized quantifiers

1 Introduction

Terminological languages provide a means for expressing knowledge about hierarchies of sets of objects with common properties. A very important thing is to be able to count the number of objects sharing a property. This talk unifies several approaches to formalizing such counting abilities. Taking the family of terminological $\mathcal{AL}$-languages from [3] as our starting point, we establish connections with a modal system from [4, 5] in which one can reason about arbitrary finite quantities, and with quantifier formalisms from [1, 8, 9]. This cross-fertilization is quite profitable in that it yields complexity results for both modal and generalized quantifier systems; it also yields complete axiomatizations of subsumption in members of the family of $\mathcal{AL}$-languages.

2 The basic systems

We introduce our main formalisms. In recent years modal logicians have considered a number of enriched modal systems that bear on issues of knowledge representation. For example, Schild [6] relates standard modal logic to the $\mathcal{ALC}$-language of [7], and propositional dynamic logic to a terminological language called $\mathcal{TSL}$ to obtain complexity results for terminological reasoning. Below we relate a *graded modal logic* to a whole class of alternative terminological systems, viz. the family of $\mathcal{AL}$-languages, whose main distinguishing feature it is that they can be used to reason about finite quantities by means of number restrictions.

Here are the details. Terminological expressions are built up using *concepts* and *roles* by means of a number of constructs. Suppose that female and young are primitive concepts, and child and relative are primitive roles. On the domain of all humans one can use intersection $\sqcap$ and complementation $-$ to describe the set of "youngsters that are not female" as young $\sqcap$ $-$female. Most terminological languages provide *restricted quantification*, that is, quantification over roles. "Women whose children are all young" are described by female $\sqcap$ (ALL child young). An important construct found in many terminological languages is *number restrictions* of the form $(\geq n\ R)$. Thus, female $\sqcap$ ($\geq$ 3 relative) describes the set of all women having at least three relatives.

Definition 2.1 (Donini et al. [3]) The basic terminological language $\mathcal{AL}$ has *concepts* $C, D, \ldots$ that are built up from primitive concepts (denoted by A) and primitive *roles* (denoted by R) according to the rule $C ::= \top \mid \bot \mid A \mid \neg A \mid C_1 \sqcap C_2 \mid (\text{ALL } R\ C) \mid (\text{SOME } R\ \top)$; in $\mathcal{AL}$ roles are always primitive.

Models for the $\mathcal{AL}$-languages have the form $(\mathcal{D}, \cdot^{\mathcal{I}})$, consisting of a set $\mathcal{D}$, the domain, and an interpretation function $\cdot^{\mathcal{I}}$ that maps concepts to subsets of $\mathcal{D}$, and roles to binary relations on $\mathcal{D}$ such that $\top^{\mathcal{I}} = \mathcal{D}$ and $\bot^{\mathcal{I}} = \emptyset$, as usual, while

$$\begin{array}{rclrcl} (C_1 \sqcap C_2)^{\mathcal{I}} &=& C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}, & (\text{ALL } R\ C)^{\mathcal{I}} &=& \{\, d \in \mathcal{D} : \forall y\, (dR^{\mathcal{I}}y \rightarrow y \in C^{\mathcal{I}})\,\}, \\ (\neg C)^{\mathcal{I}} &=& \mathcal{D} \setminus C^{\mathcal{I}}, & (\text{SOME } R\ \top)^{\mathcal{I}} &=& \{\, d \in \mathcal{D} : \exists y\, (dR^{\mathcal{I}}y)\,\}. \end{array}$$

Definition 2.2 Languages extending $\mathcal{AL}$ are obtained by adding to $\mathcal{AL}$ one of the following constructs:

$\mathcal{U}$ *union* of concepts, written $C \sqcup D$, with $(C \sqcup D)^{\mathcal{I}} = C^{\mathcal{I}} \cup D^{\mathcal{I}}$;

$\mathcal{E}$ *full existential quantification*, written as (SOME R C), defined by $(\text{SOME } R\ C)^{\mathcal{I}} = \{\, d \in \mathcal{D}^{\mathcal{I}} : \exists y\, (dR^{\mathcal{I}}y \wedge y \in C^{\mathcal{I}})\,\}$;

$\mathcal{C}$ *complement* of non-primitive concepts, written as $\neg C$, with $(\neg C)^{\mathcal{I}} = \mathcal{D} \setminus C^{\mathcal{I}}$;

$\mathcal{N}$ *number restrictions*, written as $(\geq n\ R)$ and $(\leq n\ R)$, where n ranges over the nonnegative integers, with, for each $\bowtie \in \{\geq, \leq\}$, $(\bowtie n\ R)^{\mathcal{I}} = \{\, d \in \mathcal{D} : |\{\, y : dR^{\mathcal{I}}y \,\}| \bowtie n \}$;

$\mathcal{R}$ *intersection of roles*, written as $Q \sqcap R$, with $(Q \sqcap R)^{\mathcal{I}} = Q^{\mathcal{I}} \cap R^{\mathcal{I}}$.

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Following [3], suffixing the name of any of the above constructs to '$\mathcal{AL}$' denotes the addition of the construct to the basic $\mathcal{AL}$-language; e.g., $\mathcal{ALUN}$ extends $\mathcal{AL}$ by allowing unions and number restrictions.

Let us quickly move on to modal logic. Just like some of the $\mathcal{AL}$-languages, our modal language also has the important ability to count. To be precise, formulas of the *graded modal system* $\mathbf{Gr}(\mathbf{K_{(m)}})$ are built up from a set of proposition letters Φ, and a set of (unary) modal operators $Op = \{ \langle R_i \rangle_n \mid i = 1, \ldots, m,$ and $n \in \mathbb{N}\}$, according to the rule $\phi ::= p \mid \bot \mid \top \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid O\phi$, where p is in Φ and O is in Op. We write $[R_i]_n$ for $\neg\langle R_i \rangle_n \neg$. The semantics for $\mathbf{Gr}(\mathbf{K_{(m)}})$ is based on structures $M = (W, R_1, \ldots, R_m, V)$ where $W \neq \emptyset$ is the domain, $R_i \subseteq W^2$, and V is a valuation assigning subsets of W to elements of Φ. The truth conditions are $M, x \models p$ iff $x \in V(p)$; $M, x \models \neg\phi$ iff $M, x \not\models \phi$; $M, x \models \phi \wedge \psi$ iff $M, x \models \phi$ and $M, x \models \psi$; and

$$M, x \models \langle R_i \rangle_n \phi \iff |\{ y : xR_iy \text{ and } M, y \models \phi \}| > n,$$

where, for a set X, $|X|$ denotes its cardinality. So $[R_i]_n\phi$ is true at a state x iff the number of R_i-successors of x at which ϕ fails, does not exceed n. The ordinary diamond $\Diamond$ and box $\Box$ from standard modal logic whose semantics are based on a binary relation R, can be defined here as $\langle R \rangle_0$ and $[R]_0$, respectively.

The obvious translation δ taking $\mathcal{AL}$-expressions to modal ones maps primitive concepts A to proposition letters p, and roles R to binary relations R; δ commutes with the Boolean constructs, while $\delta(\text{ALL } R\ C) = [R]\delta(C)$, and $\delta(\geq n\ R) = \langle R \rangle_{n-1}\top$. Moreover, there is a transformation Δ taking models $(\mathcal{D}, \cdot^{\mathcal{I}})$ for terminological languages to models $M = (W, \{ R \}_{R \in \mathcal{R}}, V)$[3] for modal languages, such that given $(\mathcal{D}, \cdot^{\mathcal{I}})$, we have $x \in C^{\mathcal{I}}$ iff $\Delta(\mathcal{D}, \cdot^{\mathcal{I}}), x \models \delta(C)$. By means of such correspondences results for modal logics translate effortlessly into results for terminological languages, and conversely.

Of the extensions of $\mathcal{AL}$, the language $\mathcal{ALUECN}$ is the one that resembles $\mathbf{Gr}(\mathbf{K_{(m)}})$ most. However, in $\mathcal{ALUECN}$ one can only reason about numbers of R-related things, not about numbers of R-related things that satisfy some property C.[4] From a modal logic point of view, this restriction is not a very natural one, but in some cases it yields an improvement in complexity, as can be deduced from [3]. Moreover, some researchers in the terminological field don't consider this restriction (cf. [2]).

Here's our third main character. A unary (binary) *generalized quantifier* is a function assigning to every set M a unary (binary) predicate Q_M of subsets of M. E.g., the universal quantifier $\forall$ has $\forall_M = \{ M \}$, and the Tarskian numerical quantifiers $\exists_{\geq n}$ have $(\exists_{\geq n})_M = \{ X \subseteq M : |X| \geq n \}$. Given a set M, the analogy between the latter numerical quantifiers and the graded modal operators $\langle R \rangle_n$ is made precise by taking $R = M \times M$ (the universal relation); then, by some abuse of notation, we have $X \in (\exists_{\geq n})_M$ iff in M (considered as a modal model now) the formula $\langle R \rangle_{n-1} X$ is true at some state.

3 Exploiting the connections

To summarize, the formalisms introduced above deal with domains that may or may not have structure, and these formalisms may or may not have 'counting abilities'. The overall picture is the following:

	no counting	counting
structure	terminological languages modal logic	terminological languages modal logic
no structure	generalized quantifiers modal logic	generalized quantifiers modal logic

We will use modal logic to relate the formalisms introduced so far: every system presented here has a modal counterpart. Besides that, its main contribution here are its techniques for establishing axiomatic completeness results.

Our main system is $\mathbf{Gr}(\mathbf{K_{(m)}})$; recall that it is designed for reasoning about domains that are structured, or equipped with m binary relations. Its axioms are the usual K-axioms and rules for $\langle R_i \rangle_0$ and $[R_i]_0$; on top of that it has the following axioms (for any $k, l \in \mathbb{N}$):

A1 $\quad [R_i]_0(\phi \to \psi) \to (\langle R_i \rangle_k \phi \to \langle R_i \rangle_k \psi)$,

A2 $\quad [R_i]_0 \neg(\phi \wedge \psi) \to ((\langle R_i \rangle !_k \phi \wedge \langle R_i \rangle !_l \psi) \to \langle R_i \rangle !_{k+l}(\phi \vee \psi))$,

where $\langle R_i \rangle !_k \phi$ holds at a state iff it has precisely k R_i-successors at which ϕ holds. Semantically speaking, axiom *A1* guarantees that a formula ψ is true in at least as many points as any stronger formula ϕ. *A2* expresses a notion of *additivity*: the number of objects satisfying one of two mutually exclusive formulas is simply the sum of the number of objects satisfying each of those formulas separately.

The main tasks in terminological reasoning are satisfiability and subsumption checking. By our observations in §2 satisfiability in a terminological system is equivalent to satisfiability in the corresponding

[3] We tacitly assume that $\mathcal{R} = \{ R_1, \ldots, R_m \}$.

[4] I.e., $\mathcal{ALUECN}$ does not allow for concepts of the form $(\geq n\ R\ C)$.

modal language, and subsumption is equivalent to derivability. Thus, modulo a translation, $\mathbf{Gr}(\mathbf{K}_{(\mathbf{m})})$ completely axiomatizes subsumption in its corresponding terminological language.

At this point there are two options: one can extend or restrict the language, and one can alter the structure of the models. Many extensions and restrictions of the $\mathcal{AL}$-languages have been considered in the literature on terminological languages. Each of these corresponds to a modal language; axiomatic completeness results for them can be given that use essentially the same techniques as for $\mathbf{Gr}(\mathbf{K}_{(\mathbf{m})})$, although in some cases special provisions have to be made because of the absence of some of the Boolean connectives (cf. [4, 5]).

The second option may lead to generalized quantifier theory, namely when we remove all (relational) structure in the modal models. More precisely, consider the graded modal formulas built up with $Op = \{ \langle R \rangle_n : n \in \mathbb{N} \}$, and assume that (this single) R is interpreted as the universal relation. This modal language is strong enough to simulate any first-order definable quantifier ([5]). Validity in this language can be axiomatized completely by the system $\mathbf{Gr}(\mathbf{S5})$ which extends $\mathbf{Gr}(\mathbf{K}_{(\mathbf{m})})$ with the following axioms:

A3 $\langle R \rangle_{k+1}\phi \rightarrow \langle R \rangle_k \phi$,

A4 $[R]_0 \phi \rightarrow \phi$,

A5 $\langle R \rangle_k \phi \rightarrow [R]_0 \langle R \rangle_k \phi$.

Clearly, these axioms express essential properties of the Tarskian numerical quantifiers. As with $\mathbf{Gr}(\mathbf{K}_{(\mathbf{m})})$ and the terminological domain, the axiomatic completeness of $\mathbf{Gr}(\mathbf{S5})$ and the techniques used to establish it have direct applications to existing axiomatic calculi in generalized quantifier theory ([4, 5]).

Traditionally, matters of complexity have been at the centre of attention of research in terminological languages. Given the connections between $\mathcal{AL}$-languages and the graded modal language, one might hope for a transfer of complexity results of the former domain to the latter. Indeed, despite the fact that $\mathbf{Gr}(\mathbf{K}_{(\mathbf{m})})$ does not have an exact counterpart in the $\mathcal{AL}$-family, the techniques developped in [3] to determine the complexity of $\mathcal{AL}$-languages can be applied to $\mathbf{Gr}(\mathbf{K}_{(\mathbf{m})})$, yielding the following result.

Theorem 3.1 *Satisfiability in the system* $\mathbf{Gr}(\mathbf{K}_{(\mathbf{m})})$ *is PSPACE-complete.*

(Compare: satisfiability in the multi-modal system $\mathbf{K}_{(\mathbf{m})}$ or even in $\mathbf{K}$ is also PSPACE-complete.)

Complexity results for related modal systems can also be derived from complexity results in terminological logic, either via a 'direct' transfer, or by tinkering with the proof techniques from the latter domain (cf. [4]). Likewise, complexity results for some calculi in generalized quantifier theory can be obtained by using or adapting known complexity results and techniques for **S5**-like modal systems (again, cf. [4]).

What does generalized quantifier theory have to bring in here? One important issue in generalized quantifier theory has been the syntactic characterization of semantic properties (cf. [9]). E.g., a first-order formula $\phi(P_0, P_1)$ defines an upward monotonic quantifier (in its left-hand) argument iff it's equivalent to a formula in which P_0 occurs only positively (in the usual syntactic sense). (A quantifier Q is upward monotonic in its left-hand argument if QXY and $X \subseteq X'$ imply $QX'Y$.)

In [5] such issues of syntactic characterizations have been lifted to the modal domain, to modal operators simulating both first- and some higher-order quantifiers. We would like to suggest that a similar move towards syntactic characterizations of semantic properties should be undertaken in the terminological domain. First, because this paves the way for a better theoretical understanding of an area "whose development has mainly been implementation-driven and rather ad hoc," as Brink *et al.* [2] put it. And second, as practical applications force one to reason not only with static information, but also with changing data, and thus about changing taxonomies, it is useful to have syntactic characterizations of e.g. relations between concepts that remain unaltered under certain updates of a taxonomy; a terminological version of the above monotonicity property, for example, would describe all binary relations between concepts that are insensitive to increases in the size of their left-hand arguments.

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