

Lecture Notes

MODAL LOGIC
AND
PROCESS ALGEBRA
A Bisimulation Perspective

edited by

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Preface

This volume is an offspring of the workshop ‘Three Days of Bisimulation,’ held at the Centre for Mathematics and Computer Science (CWI), Amsterdam, in April of 1994. The editors of this volume constituted the organizing committee of the workshop.

Our motivation for organizing this workshop stemmed from our belief that modal logicians and process algebraists share a research area of common interest. A key phrase here is *formal aspects of programming languages*, comprising issues such as correctness, analysis and concurrency. The aim of the workshop was to bring together researchers from both fields; and in order to stimulate interaction between the participants we asked the speakers to concentrate on a few particular topics in the area: labeled transition systems, and especially, bisimulations.

We felt that the quality of the presentations at the workshop justified a follow-up in the form of this volume: a collection of 14 papers, all of which are on modal logic or process theory, or take bisimulations as a central perspective. The contents of this volume do not coincide exactly with the presentations at the meeting. In particular, some presentations at the workshop are not recorded here for one reason or another; furthermore, speakers delivering a tutorial at the workshop submitted research papers, and we added one paper that was written in response to the workshop.

We would like to thank those who helped us in organizing the workshop and editing this volume. To start with, the workshop was sponsored by the Netherlands Organization for Scientific Research (NWO) under project NF 102/62-356 ‘Structural and Semantic Parallels in Natural Languages and Programming Languages’. Thanks are due to the referees for their solid work. Finally, we would like to thank Jan van Eijck and Mieke Bruné for their valuable help.

Amsterdam, February 1995,
Alban Ponse, Maarten de Rijke,
and Yde Venema



Introduction

In this introductory text we briefly describe labeled transition systems and bisimulations, and their connection with modal logic and process algebra. Next, we make some remarks on the common interests of these disciplines. Finally, we give short descriptions of the contributions.

Labeled Transition Systems. Since the name ‘labeled transition system’ is used in the literature for a number of slightly variant notions, we do not give a precise definition here. Roughly speaking, labeled transition systems are graph-like structures which are used to represent the dynamic behavior of some system. A bisimulation between two such structures (which may be identical) is a relation connecting states from both structures in the following way. Two states are in the relation only if each step starting in one state, can be mimicked by a step from the other state, in such a way that the resulting states are again in the relation. Then, two states are called bisimilar when there is a bisimulation relating them.

Both in modal logic and in process algebra, transition systems and bisimilarity are regarded fundamental. But these notions also raise interest in a broader context, as some contributions to this volume show.

Modal Logic. Ever since the early 1970’s modal languages have been used as description languages for structures such as labeled transition systems. Modal formulas are evaluated locally, at a single state inside a labeled transition system; the mechanism for evaluating formulas does not take one outside the system, but instead it forces certain moves along the restricted patterns described by the modal operators. Another special feature of modal languages is that they are relatively constrained fragments of classical languages such as first-order logic; in general, modal languages avoid the quantificational power of classical languages.

Bisimulations enter the picture here as truth-preserving relations between models: bisimilar transition systems satisfy the same modal formulas. But the connection between modal logic and bisimulations is much

stronger than this statement may suggest: in a number of ways modal languages are uniquely identified by the fact that their formulas are preserved by bisimulations; several papers in the volume are related to this issue.

Process Algebra. This generic term comprises the study of concurrent (communicating) processes in an algebraic fashion. In this approach, atomic actions, algebraic operations and equational axioms are used to describe and analyze processes. Typically, a distributed system or concurrent protocol can be described as the concurrent execution of a number of elementary subprocesses, possibly employing synchronous or asynchronous communication. The external behavior of such a system can be obtained by abstracting from internal activity, and can then be matched against a ‘specification’ of the process. This allows the verification of global correctness of the system. Process algebra provides an algebraic framework for such correctness issues.

Labeled transition systems provide a convenient modeling of processes. Standard process algebra axiom systems then induce various notions of equivalence between processes, including bisimulations or versions thereof. In this way, a wealth of process equivalences has been axiomatized. Alternatively, one can start from labeled transition systems, or rather from restricted formats of ‘transition rule calculi’, define an appropriate process language and set up an algebraic process theory that guarantees certain properties of the processes thus definable. For example, bisimulation is a congruence under certain restrictions on the transition rules.

As sketched above, labeled transition systems and bisimulations are central notions in both modal logic and process algebra, though from a different perspective. The most obvious difference is that modal formulas are evaluated inside the transition system, whereas process-algebraic terms are interpreted by a structure as a whole. One might call modal logic an *internal* language of labeled transition systems and process algebra an *external* one. A second important difference concerns the role that the notion of bisimulation plays in modal logic and process theory: in modal logic it is the central structural relation, as is witnessed by the fact that almost all preservation results are based on it. In process algebra, there are many serious alternative notions employed in the identification of processes, in particular in the presence of abstraction.

Nevertheless, a number of trends combine the two perspectives on labeled transition systems and bisimulations. Here are two examples. First, the complexity of a system can be so large, and the desired properties so diverse, that correctness is not a priori obvious. Then, the expressivity of modal, ‘internal’, languages can be used in the setting of process algebra to help establishing *property correctness*. Second, there is a research line in

modal logic which seeks to *fine-tune the signature of modal languages*, in order to capture such variants of the notion of bisimulation as are studied in process theory.

Though the spectrum sketched here is wide, we think that the present volume covers a representative part. We now briefly discuss the contents of each paper, and its place in the spectrum.

Andréka, van Benthem and Németi. In this contribution simulations, that is, asymmetric bisimulations, are used to prove results in the model theory of first-order logic with finitely many variables. The main results establish that the Los-Tarski preservation theorem holds for finite-variable fragments of first-order logic if the notion ‘preservation under submodels’ is replaced by ‘preservation under simulations’.

Baeten, Bergstra and Ştefănescu. In this contribution process graphs, i.e., rooted transition systems with interior states and so-called pins, are studied. A pin is an external connection of a chip. This provides a semantical setting which refines the common one in process algebra, and in which the feedback operator from flowchart theory can be modeled. Adding structure to pins gives an algebra of which the original ACP of Bergstra and Klop is a subalgebra of a reduced model.

Bergstra and Ponse. In this paper attention is given to frames, i.e., labeled, unrooted transition systems. Frames can be seen as a semantical basis for process algebra, Floyd-Hoare logic and modal logic. A frame algebra is proposed, and adding additional features allows for an interpretation of the abovementioned formalisms.

Bonsangue and Kwiatkowska. These authors combine algebraic, logical, and topological viewpoints on labeled transition systems. They prove several results on image-compact modal frames, these being labeled transition systems endowed with a certain topology. Furthermore, they discuss a Stone-type duality theorem for the modal μ -calculus, which is an extension of ordinary modal logic with a fixed-point operator.

Caucal. In this paper, bisimulations on transition graphs of push-down automata are studied. First, the particular graph generation is discussed. With certain restrictions, the class of regular graphs of finite degree is obtained. A subclass is identified that contains graphs for which the maximal bisimulation is decidable. Finally, various classes of transition systems are interrelated.

Goldblatt. The author studies the Hennessy-Milner property. A transition system is said to have this property if the relation of logical equivalence between its points is a bisimulation. Several descriptions of transition systems with the Hennessy-Milner property are provided, and connections are made with standard constructions from modal model theory.

Groote and van Vlijmen. In this contribution a modal logic for processes

with data is introduced. In order to express and reason about particular properties of a process, such as for instance deadlock freedom, the authors introduce an extended modal logic. In this logic, reasoning about data is also possible, for instance by first-order quantification. Behavioral aspects of processes can be expressed using modalities. Furthermore, a variant of bisimulation is identified that preserves modal truth.

Hirshfeld and Moller. This contribution focuses on the decidability of bisimulations between context-free processes and commutative context-free processes. In a restricted case, even an algorithm in polynomial time appears to exist.

Hodkinson. This contribution presents a new, game-theoretic proof of Kamp's well-known theorem stating that over the class of linear, Dedekind complete flow of time, the temporal language with modal operators *Since* and *Until* is as expressive as monadic first-order logic. In a strict sense, this contribution is neither concerned with transition systems nor with bisimulations; nevertheless, we feel that this volume is a proper niche for it, since the games Hodkinson employs to determine the expressiveness of modal and first-order languages, induce structural notions of equivalence between models which are variants of bisimulations.

Hollenberg. This paper combines perspectives on bisimulations from modal logic and process theory: in the first part of the paper the author studies Hennessy-Milner classes, that is, classes of transition systems where logical equivalence is a bisimulation. Then Hollenberg applies his findings to process theory, providing a process algebraic signature with a Hennessy-Milner semantics.

De Rijke. In this contribution, the author addresses the following issue: give a unique characterization of modal logic in terms of the preservation of modal formulas under bisimulations, and an additional feature. Several answers are provided, both in terms of so-called rank functions, and, more abstractly, in terms of ultrapowers.

Rutten. This contribution contains a category-theoretic study of the notion of a bisimulation. Representing transition systems as co-algebras, the author works out the analogy with universal algebra, and thus provides simple and insightful proofs for many, known and new, results on bisimulations.

Smolka, Sokolsky and Zhang. The authors survey some results on the parallel complexity of checking whether two finite-state transition systems are bisimilar, and of model checking for the modal μ -calculus. It is shown that worthwhile speedups can in practice be obtained by parallel implementations. Also, a number of parallel complexity results are established for restricted versions of the model checking problem.

Visser, Van Benthem, De Jongh, and Renardel de Lavalette. In the final paper of this volume a special fragment of intuitionistic logic is studied.

This fragment consists of all propositional formulas without nestings of implications to the left. First, a proof theoretic analysis is offered, yielding results on interpolation, and then a model theoretic one in terms of subsimulations; among other things, this second approach leads to an analogue of Łoś's Theorem.

We believe that this collection gives a representative overview of research in labeled transition systems and bisimulations. We hope that it will add to the interaction and cooperation between researchers working in this field.