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A Lindström Theorem for Modal Logic

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In the semantics of concurrent programs modal logics are used to give logical descriptions of bisimulations and other notions of process equivalence (see [3]). Hence, from a computational point of view it is important to gain a thorough understanding of the relation between modal logic and bisimulations. Independently, the connection between equivalence relations on classes of models and notions of logical equivalence is an important topic in model theory (see [1, Chapter 19]). Van Benthem [2] characterizes modal formulas as first-order formulas whose truth is preserved by bisimulations between models. And De Rijke [5] shows that two models are modally equivalent iff they have bisimilar ultrapowers. In this talkI discuss a further characterization result, namely a modal analogue of Lindström's [4] well-known characterization of first-order logic.

Lindström's result states that, given a suitable explication of a 'classical logic', first-order logic is the strongest logic to possess the Compactness and Löwenheim-Skolem properties. To prove a modal analogue we first define abstract modal logics; the key property here is that bisimilar models should be indistinguishable for abstract modal logics. Next, we say that a modal logic $(\mathcal{L}, \models_{\mathcal{L}})$ has a notion of finite rank if there is a function from $\mathcal{L}$-formulas to natural numbers that gives an upperbound on the depth of the states that have to be inspected to determine the truth of a given formula; here, the depth of a state is the smallest number of relational steps it is removed from the point of evaluation.

The main results are the following.

Theorem 1 *Standard modal logic is the strongest modal logic that has a notion of finite rank.*

Theorem 2 *Standard modal logic is the strongest modal logic whose formulas are preserved under ultraproducts over ω.*

References

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[5] de Rijke, M.: 'Model theory for modal logic'. Unpublished manuscript. CWI, Amsterdam, 1994.