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Editorial: Efficacy of Diagrammatic Reasoning

Visual Logic, Language, and Information

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Consider the three interlocking circles of the FoLLI logo. Under one interpretation it could be a Venn Diagram, representing all the possible intersections of three sets, or perhaps it belongs to the system of Euler's Circles, in which case it illustrates three sets which have a non-empty intersection. Indeed, it could be used to represent interacting processes, or overlapping (physical) parts of a machine. Of course, the formal and computational properties of the diagram system change with respect to its interpretation. Try to add a fourth circle which overlaps each of the double intersections, but not the triple intersection. That any such attempt is impossible is a special case of a theorem of convex topology known as Helly's theorem (Eggleston, 1969). While this fact presents no problems for the Venn interpretation, it is disastrous for Euler's system – where it leads the user to an incorrect inference (that if every triple of four sets has a non-empty intersection, then they have a quadruple intersection). Thus *spatial* properties of diagrammatic objects can lead to new inferential problems over and above the ones that we know and love from traditional linear proof calculi (see Lemon and Pratt, 1997a).

So, is it worth using a form of representation that may exhibit such novel problems? Discussions about the *benefits* of using diagrammatic representations in logic go back a long way. Aristotle was certainly familiar with the idea of using a stylized tree figure to represent the relationships between (and successive sub-divisions of) such things as different species. While it is debatable whether such tree diagrams exhibit any interesting diagrammatic properties (see Stenning and Lemon, 1999), nearly all cultures made early use of maps (see Bagrow and Skelton, 1964), where there is a clear advantage in the representation of geographic space by diagrammatic space (as Russell, 1923, argued). There seems to have been little activity on the logic diagram front until 1761, when the Swiss mathematician Leonhard Euler proposed using circles to illustrate relations between sets and to generate solutions for problems in class logic (Euler, 1768). In the 1880s John Venn greatly improved upon this method by using diagrams of overlapping regions (i.e., topological models) to illustrate truth-conditions of propositions (Venn, 1881). Indeed, Frege originally presented first-order logic using a diagrammatic notation, and the next major contribution to diagrammatic logic was made by Charles S. Peirce at the start of the twentieth century (Peirce, 1933). Nowadays, of course, diagrams play an important role in problem solving for mathematics, education, physics, geography, computer science and science generally.

So, when are we today? And what do we take to be the benefits of using diagrammatic representations? Practically speaking, it is desirable to find efficient representation languages for programming, specification, and Human-Computer Interaction (HCI) generally, of which diagram systems are often thought to be the paradigm case. Theoretically too, a rigorous understanding of different representation languages, especially graphical ones, is a necessity for the evaluation of certain hypotheses in cognitive science and artificial intelligence. Thus the analysis of graphical presentations of information is an active research area, which attracts a diverse range of interests. Currently though, there are rather few established results investigating their computational and semantical properties. It is probably fair to say that apart from a few special cases, general logical aspects of diagrammatic reasoning are far from being well-understood.

The key concern facing anyone interested in diagrammatic reasoning – logicians, philosophers, computer scientists, and cognitive scientists alike – is best summarized as the *efficacy of diagrammatic representation*: what makes a diagrammatic representation particularly useful, efficient, or appropriate for solving a certain type of problem? This special issue contains four different approaches to the matter of efficacy of diagrammatic reasoning, each from a different angle, ranging from philosophy and cognitive science to theorem proving and hardware verification. In this introduction we first provide some general background information on diagrammatic reasoning; against this general background we briefly summarize the main contributions of the four papers in the special issue.

A Brief Overview of the Field

From the late 1960s onwards there has been a steady stream of research on the subject of diagrammatic approach. Nelson Goodman's 1968 book *Languages of Art* is the first to attempt to deal analytically with the issues of representation and notation for pictorial systems. Since then many philosophical distinctions have been proposed with regard to the analysis of visual information (e.g., Barwise and Shimojima, 1995; Cummins, 1996; Dretske, 1981; Howell, 1976).

Proposals for computational analyses of diagrammatic representations have been made by Levesque (1988) (see his “vivid knowledge bases”), by Larkin and Simon (1987), and in a recent paper by Grigni et al. (1995); see also Lemon and Pratt (1997b). On the practical side, Funt’s “Whisper” is an A.I. program which exploits the potential of diagrams to reduce the cost of inference in qualitative reasoning tasks (Funt, 1980). More recently, the issues have come under scrutiny from cognitive psychologists, originating with interest in the “imagery debate” (see, e.g., Kosslyn, 1994; Pylyshyn, 1981) and HCI, as well as from computer scientists with regard to visual programming languages and knowledge representation (e.g., Harel’s Higraphs, 1988; Sowa’s Conceptual Graphs, 1984). Whereas formal methods have successfully been applied to standard (linear) programming languages, there is currently a variety of important open questions regarding the formal semantics of visual programming languages.

Researchers in qualitative and spatial reasoning have also found diagrammatic representations to be of direct relevance (e.g., Gooday and Cohn, 1996), as have those interested in the role of diagrams in mathematics, logic (Hammer, 1995; Lemon and Pratt, 1999; Shin, 1995; Stenning and Oberlander, 1995), and physics – where problems are often solved using “Law Encoding Diagrams” (see Cheng, 1996). Here it seems that diagram systems are more or less effective according to their being more or less “analogous” to their problem domains. In the case of spatial reasoning, diagrams are usually effective since they exhibit the same structural properties as two-dimensional (physical) situations; but in mathematics it is found that diagrams generally match only a few of the relevant constraints on the more abstract structures involved; and in physics the efficacy of diagrammatic reasoning must be investigated with respect to how diagrams might be used to encode physical laws (e.g., conservation of momentum).

Another interesting research area is the investigation of diagrammatic theorem proving (for example, Jamnik et al., 1999; Barker-Plummer and Bailin, 1997; Gelernter, 1959), where mathematical proofs using diagrams have been automated. Efficacy is an issue here in the sense that some diagrams reveal the structure of a problem domain in such a way that it is easy (for humans at least) to “see” a solution. Related to this is the idea that diagram systems also have their place in the teaching of mathematics and logic, where it is investigated whether graphical representations can aid reasoning or improve the learnability of logical concepts (see, e.g., Barwise and Etchemendy, 1995).

In general, then, diagrammatic representation can be seen as an important species of analogical or surrogate representation – where constraints on problem domains are (to some extent at least) preserved in their representations (see Barwise and Shimojima, 1995; Cummins, 1996; Shimojima, 1996). This type of “spatial metaphor” in representation has even been extended to the analysis of meaning and mental representation (see, for example, work on “conceptual spaces” in cognitive science, Gärdenfors, 1996).

Back to Efficacy

With these generalities out of the way, let us return to the theme of this special issue, and to the contributions which we received. The theme of efficacy of diagrammatic representation has to do with the various claims that diagrams are semantically and computationally effective for problem solving in certain domains. The importance of this theme has been recognized by researchers in logic, cognitive science, and computer science. Indeed, recent years have witnessed a growing number of publications, conferences, and research projects related to the analysis of diagrammatic representation, worldwide.

In response to the increasing interest and volume of research in this area, but the relative scarcity of formal approaches, we felt that the time was right for a special issue focusing on formal investigations and explanations of efficacy of graphical representations of information. In our call for papers we suggested the following (non-exhaustive) list of appropriate themes:

1. complexity-theoretic aspects of diagrammatic reasoning;
2. analyses of visual specification and programming languages;
3. diagrammatic logics or proofs systems; formal semantics for systems of diagrammatic representation; spatial logic/qualitative reasoning and the analysis of diagrammatic representation;
4. cognitive analyses of diagrammatic representations; diagrammicity of "conceptual structures"; the imagery debate;
5. applications of diagrammatic reasoning to artificial intelligence; the role of diagrammatic representations in proof presentations in mathematics and physics; and
6. comparisons of different "forms of representation" with respect to their efficacy; the selection and construction of appropriate forms of representation for a given problem.

The submissions that we received covered all of the above topics, and more. After a careful selection procedure, we decided to accept four papers for inclusion in this special issue, both because of their individual qualities and because together they represent a broad, accessible, and representative picture of today's interest and research directions in the area of diagrammatic representation.

How, then, do the contributions to this special issue address the themes? In her contribution entitled "Reconstituting Beta Graphs into an Efficacious System," Sun-Joo Shin considers Peirce's Beta Graphs (a system for performing inferences in first-order logic). Starting from the observation that logicians have strongly preferred first-order natural deductive systems over Beta Graphs even though they are equivalent, she identifies a number of reasons for this preference. One of the main reasons is that the inference rules for Beta Graphs are hard to understand and, therefore, hard to apply for actual deductions. Shin reformulates the Beta rules to show more fine-grained symmetries built around visual features of the Beta system. It is argued that, when visual features are discovered and fully used, we obtain a more effective deductive system for Beta Graphs. Shin's contribution thus addresses the third, fourth, and sixth of the themes in the Call for Papers.

The second contribution, by Jamnik, Green and Bundy, is entitled "On Automating Diagrammatic Proofs of Arithmetic Arguments." The authors argue that there is a set of mathematical problems which humans can solve more easily by the use of diagrammatic proofs. Insight into the structure of a problem is often more readily available using diagrams rather than algebraic representations – and it seems that diagrammatic proofs capture an intuitive notion of truthfulness that humans find easy to see and understand. Jamnik and her co-authors investigate and automate such diagrammatic reasoning about mathematical theorems. Concrete, rather than general diagrams are used to prove particular instances of a universally quantified theorem. An abstracted schematic proof of the universally quantified theorem is then induced from these proof instances. An argument confirming the soundness of the abstraction of the schematic proof from the proof instances is obtained by establishing the correctness of schematic proofs in the meta-theory of diagrams. These ideas have been implemented in a system called DIAMOND, which is presented here. This paper addresses the third, fifth, and sixth of our themes.

Timing diagrams are popular in hardware design, and have been formalized for use in reasoning tasks such as computer-aided verification. These efforts have largely treated timing diagrams as interfaces to established notations for which verification is decidable, and this trend has restricted timing diagrams to expressing only regular language properties. In her contribution "Timing Diagrams: Formalization and Algorithmic Verification," Fisler presents a timing diagram logic capable of expressing certain context-free and context-sensitive properties. It is shown that verification is decidable for properties expressible in this logic. More specifically, Fisler shows that containment of ω-regular languages generated by Büchi automata in timing diagram languages is decidable. The result relies on a correlation between timing diagram and reversal bounded counter machine languages. This paper addresses the first two of our themes.

In the final contribution to this special issue, Oberlander, Monaghan, Cox, Stenning and Tobin discuss matters related to themes 3, 4, 5, and 6 in a paper called "Unnatural Language Processing: An Empirical Study of Multimodal Proof Styles." Computer-based logic proofs are taken to be a form of "unnatural" language in which the process and structure of proof generation can be observed in considerable detail. In particular, the authors have been studying how students respond to multimodal logic teaching. Performance measures have indicated that students' pre-existing cognitive styles have a significant impact on teaching outcome, and a large corpus of proofs has been gathered via automatic logging of proof development. This paper applies a series of techniques, including corpus statistical methods, to the proof logs. The results indicate that students' cognitive styles influence the structure of their logical discourse, via differing methods of handling abstract information in diagrams, and transferring information between modalities.

The papers included in this special issue exhibit the breadth of current research in visual logic, language, and information – a field whose technical and theoretical impact is yet to be fully realized. We believe that the issue of efficacy and its formal analysis forms a focal point for progress in this area. It promotes a paradigm of formal research into diagrammatic reasoning which has seldom been exemplified.

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