1. **(2 points) Monotonicity of distance measures**: Use the Stinespring representation to deduce the following monotonicity properties. For all states $\rho_A, \sigma_A$ and channels $\Phi_{A \to B}$,

\[
T(\Phi_{A \to B}[\rho_A], \Phi_{A \to B}[\sigma_A]) \leq T(\rho_A, \sigma_A) \quad \text{and} \quad F(\Phi_{A \to B}[\rho_A], \Phi_{A \to B}[\sigma_A]) \geq F(\rho_A, \sigma_A).
\]

2. **(3 points) Depolarizing channel**: Consider the following trace-preserving superoperator on $L(\mathcal{H})$, where $\dim \mathcal{H} = d$ and $\lambda \in \mathbb{R}$ is a parameter:

\[
D_\lambda[M] = \lambda M + (1 - \lambda) \text{Tr}[M] \frac{1}{d}
\]

(a) Compute the Choi operator of $D_\lambda$ for any value of $\lambda$.

(b) For which values of $\lambda$ is $D_\lambda$ a quantum channel?

3. **(5 points) Kraus and Stinespring**: Find Kraus and Stinespring representations for the following quantum channels:

(a) **Partial trace**: $\Phi[M_{A_E}] = \text{Tr}_E[M_{A_E}]$

(b) **Add state**: $\Phi[M_A] = M_A \otimes \sigma_B$ for a state $\sigma_B$.

(c) **Measure and prepare**: $\Phi[M] = \sum_{x \in \Sigma} \langle x| M_A \langle x| \otimes \sigma_{B,x}$, where $|x\rangle$ denotes the standard basis of $\mathcal{H}_A = \mathbb{C}^\Sigma$ and $\sigma_{B,x}$ is an arbitrary state for each $x \in \Sigma$.

4. **(2 points) Quantum to classical channels**: Let $\mathcal{H}_A$ be an arbitrary Hilbert space and $\mathcal{H}_X = \mathbb{C}^\Omega$. Assume that $\Phi_{A \to X}$ is a quantum channel such that $\Phi_{A \to X}[\rho_A]$ is classical for every state $\rho_A$. Show that there exists a measurement $\mu_A : \Omega \to \text{PSD}(\mathcal{H}_A)$ such that

\[
\Phi_{A \to X}[\rho_A] = \sum_{x \in \Omega} \text{Tr}[\mu_A(x) \rho_A] |x\rangle\langle x| \quad \forall \rho_A.
\]

*Hint: Use Practice Problem 4.1.*