1. Majorization examples:
   (a) Let \( p = (0.1, 0.7, 0.2) \) and \( q = (0.3, 0.2, 0.5) \). Determine whether \( p \prec q \) or \( q \prec p \).
   (b) Find a sequence of Robin Hood transfers that converts one distribution into the other.
   (c) Express this sequence as a single stochastic matrix and verify that this matrix is in fact doubly stochastic.
   (d) Express this matrix as a convex combination of permutations.
   (e) Find a pair of probability distributions \( p \) and \( q \) such that neither \( p \prec q \) nor \( q \prec p \).

2. Alternative definitions of majorization:
   Let \( u = (u_1, \ldots, u_n) \) be a vector and let \( r \) denote reverse sorting and \( s \) denote sorting:
   \[
   r_1(u) \⩾ r_2(u) \⩾ \cdots \⩾ r_n(u),
   \]
   \[
   s_1(u) \leq s_2(u) \leq \cdots \leq s_n(u),
   \]
   such that \( \{r_i(u) : i = 1, \ldots, n\} = \{s_i(u) : i = 1, \ldots, n\} = \{u_i : i = 1, \ldots, n\} \) as multisets.
   Let \( u \) and \( v \) be two probability distributions over \( \Sigma = \{1, \ldots, n\} \), i.e., \( u_i \geq 0, v_i \geq 0 \), and \( \sum_{i=1}^n u_i = \sum_{i=1}^n v_i = 1 \). Show that the following ways of expressing \( v \prec u \) are equivalent:
   (a) \( \sum_{i=1}^m r_i(v) \leq \sum_{i=1}^m r_i(u) \), for all \( m \in \{1, \ldots, n-1\} \).
   (b) \( \sum_{i=1}^m s_i(v) \geq \sum_{i=1}^m s_i(u) \), for all \( m \in \{1, \ldots, n-1\} \).
   (c) \( \forall t \in \mathbb{R} : \sum_{i=1}^n \max(v_i - t, 0) \leq \sum_{i=1}^n \max(u_i - t, 0) \).

3. Vectorization and partial trace:
   (a) Show that, for all \( L, R \in L(\mathcal{H}_A, \mathcal{H}_B) \),
       \[
       \text{Tr}_A[|L\rangle\langle R|] = LR^\dagger.
       \]
   (b) Let \( \Xi \in \text{SepC}(A : B) \) be given by
       \[
       \Xi(M) = \sum_{a \in \Sigma} (A_a \otimes B_a)M(A_a \otimes B_a)^\dagger,
       \]
       for all \( M \in L(\mathcal{H}_A \otimes \mathcal{H}_B) \). Show that, for all \( X \in L(\mathcal{H}_A, \mathcal{H}_B) \),
       \[
       \text{Tr}_A\left[\Xi(|X\rangle\langle X|)\right] = \sum_{a \in \Sigma} B_aX_A^\dagger A_aX^\dagger B_a^\dagger.
       \]