EXERCISE CLASS 2-12-2016:
EXERCISES ON PDL

(1) Correspondence for PDL-formulas.
(a) \( \langle \pi_1; \pi_2 \rangle p \leftrightarrow \langle \pi_1 \rangle \langle \pi_2 \rangle p \) is valid on a frame \((W, R_{\pi_1}, R_{\pi_2})\) iff \(R_{\pi_1; \pi_2} = R_{\pi_1} \circ R_{\pi_2} \).
(b) \( \langle \pi_1 \cup \pi_2 \rangle p \leftrightarrow \langle \pi_1 \rangle p \lor \langle \pi_2 \rangle p \) is valid on a frame \((W, R_{\pi_1}, R_{\pi_2})\) iff \(R_{\pi_1 \cup \pi_2} = R_{\pi_1} \cup R_{\pi_2} \).
(c) \( p \lor \langle \pi \rangle \langle \pi^* \rangle p \to \langle \pi^* \rangle p \) is valid on a frame \((W, R_{\pi}, R_{\pi^*})\) iff \((R_{\pi})^* \subseteq R_{\pi^*} \).

(2) A logic \( L \) is compact for the class \( C \) of Kripke frames if the following condition is met: for every set of formulas \( \Sigma \), if every finite subset of \( \Sigma \) is satisfiable in a model based on a frame in \( C \), then \( \Sigma \) itself can be satisfied in a model based on a frame in \( C \). Show that PDL is not compact for the class of regular frames.

(3) Show that the induction axiom \([\pi^*](p \to [\pi]p) \to (p \to [\pi^*]p)\) is not canonical.

(4) Let \( \Sigma \) be a non-empty finite set of formulas in the language of PDL. Prove that \( \text{At}(\Sigma) = \{ \Gamma \cap \neg FL(\Sigma) : \Gamma \text{ a PDL-MCS} \} \).

(5) Prove the properties (i)–(iv) of atoms listed in Lemma 4.81.

(6) Show that the finite models defined in the PDL completeness proof can be obtained (up to isomorphism) via certain filtrations.

1. ADDITIONAL EXERCISES

(1) Explain why PDL is a decidable logic.

(2) Let \( \Sigma \) be a non-empty finite set of PDL-formulas. Show that \( \vdash_{\text{PDL}} \bigvee_{A \in \text{At}(\Sigma)} \hat{A} \).

(3) Let \( \Sigma = \{ \varphi \} \) be a singleton set (of formulas in the language of PDL). Show that \( \neg FL(\Sigma) \) is finite. \( \text{Hint: This is not so easy.} \)

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\( ^1 \)In BdRV \( R_{\pi_1} \circ R_{\pi_2} \) is also denoted \( R_{\pi_1; \pi_2} \).