(1) Our primitive connectives are $\lor$, $\neg$, $\top$, $\bot$ and $\lozenge$. Let $p$ be a propositional letter that occurs in $\varphi$. Define by induction on $\varphi$: The occurrence of $p$ is positive (negative).

(2) A formula $\varphi$ is called positive (negative) in $p$ if all occurrences of $p$ are positive (negative).

- Show that if $\varphi$ is positive in $p$ then it is upward monotone in $p$, and if it is negative in $p$ then it is downward monotone in $p$.
- What about the converse? If $\varphi$ upward (downward) monotone in $p$ does it follow that $\varphi$ is positive (negative) in $p$?

(3) Can you give first-order correspondents for the following formulas?

- $\Box p \land p \to \lozenge \lozenge p$
- $\lozenge \Box p \to \Box \lozenge \lozenge p$