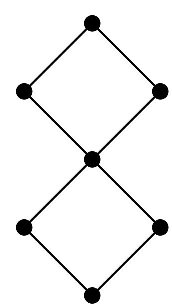
MATHEMATICAL STRUCTURES IN LOGIC 2016
HOMEWORK 5

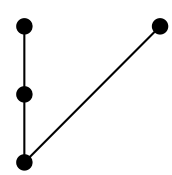
- Deadline: March 8 — at the **beginning** of class.
- In exceptional cases homework can be submitted electronically (in a single pdf-file!) to Frederik Lauridsen `f.m.lauridsen@uva.nl`
- Grading is from 0 to 10 points.
- Success!

(1) (2pt)

(a) Draw the poset dual to the Heyting algebra drawn below.



(b) Draw the Heyting algebra dual to the poset drawn below.



(2) (2pt)

(a) Let $h : A \to B$ be a Boolean homomorphism between Boolean algebras A and B. Show that if h is surjective, then its dual $h_* = h^{-1} : X_B \to X_A$ is injective.

(b) Let $f : X \to Y$ be a continuous map between Stone spaces X and Y. Show that if f is surjective, then its dual $f^* = f^{-1} : \mathsf{Clop}(Y) \to \mathsf{Clop}(X)$ is injective.

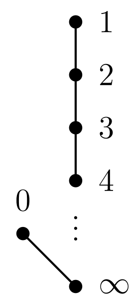
(3) (3pt) Let X be a poset. X is called *rooted* if there is $r \in X$ such that $X = \uparrow r$. Then r is called the *root* of X.

(a) Let X be an Esakia space. Show that X is rooted if and only if in its dual Heyting algebra A for each $a, b \in A$ we have $a \vee b = 1$ implies $a = 1$ or $b = 1$.

(b) Let X be a rooted Esakia space with a root r. X is called *strongly rooted* if $\{r\}$ is open. Show that X is strongly rooted if and only if the Heyting algebra dual to X has the second largest element. (Recall that in Hausdorff spaces every point is closed.)

(c) Give an example of a rooted Esakia space, which is not strongly rooted.

(4) (3pt) Consider the ordered topological space $(\mathfrak{X}, \leq)$ drawn below where the space $\mathfrak{X}$ is the Alexandroff compactification $\alpha\mathbb{N}$ of $\mathbb{N}$. In other words, the clopen sets are finite subsets of $\mathbb{N}$ and cofinite subsets of $\mathbb{N}$ together with the point ∞.



(a) Show that this space is a Priestley space.

(b) Show that it is not an Esakia space.

(c) Draw the distributive lattice dual to this space.