# MATHEMATICAL STRUCTURES IN LOGIC 2017
# HOMEWORK 2

- Deadline: March 6 — at the **beginning** of class.
- In exceptional cases homework can be submitted electronically (in a single pdf-file!) to Mees de Vries (`mees.devries@student.uva.nl`).
- Grading is from 0 to 100 points.
- Discussion of problems is allowed, but each student should submit a homework they themselves have written.
- Success!

(1) (40pt) Do the following equations hold in any Heyting algebra? If yes, give a proof, if not, provide a counter-example.

   (a) $(a \vee b) \to c = (a \to c) \wedge (b \to c)$,

   (b) $\neg\neg a \vee \neg a = 1$,

   (c) $\neg\neg\neg a = \neg a$,

   (d) $(a \to b) \vee (b \to a) = 1$.

   Here $\neg a = a \to 0$.

(2) (40pt)

   (a) Show that the lattice $(\mathrm{FinCofin}(\mathbb{N}), \subseteq)$ of finite and cofinite subsets of $\mathbb{N}$ forms a Boolean algebra. Show that this Boolean algebra is not complete.

   (b) Show that the lattice $(\mathrm{Fin}(\mathbb{N}) \cup \{\mathbb{N}\}, \subseteq)$ of finite subsets of $\mathbb{N}$ (together with $\mathbb{N}$) forms a complete bounded distributive lattice. Is this lattice a Heyting algebra? Justify your solution.

(3) (20pt) Let $L$ be a lattice. We say that a non-zero element $a \in L$ is *join prime* if $a \leq b \vee c$ implies $a \leq b$ or $a \leq c$. (Check exercise sheet 1 for the definition of join irreducible elements.)

   (a) Show that in a distributive lattice the join irreducible elements coincide with the join prime elements.

   (b) Give an example of a lattice having a join irreducible element which is not a join prime element.