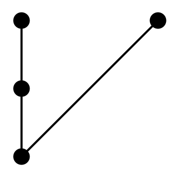
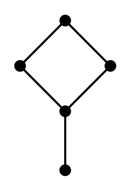
MATHEMATICAL STRUCTURES IN LOGIC 2017
HOMEWORK 3

- Deadline: March 20 — at the **beginning** of class.
- In exceptional cases homework can be submitted electronically (in a single pdf-file!) to Mees de Vries (`mees.devries@student.uva.nl`).
- Grading is from 0 to 100 points.
- Success!

(1) (20pt) Draw the Heyting algebra of all up-sets of the poset drawn below.



(2) (20pt) Let A be the Heyting algebra drawn below.



Find an embedding $\iota\colon A \hookrightarrow \prod_{i\in I} A_i$ of HAs such that for each $i \in I$ the algebra A_i is a linear HA and $\pi_i \circ \iota\colon A \to A_i$ is surjective.

(3) (20pt) Recall that a topological space is called T_0 if for each $x \neq y$ there exists an open set U such that ($x \in U$ and $y \notin U$) or ($y \in U$ and $x \notin U$).

Let $(X, \leq)$ be a pre-ordered set (i.e., $\leq$ is reflexive and transitive). Let (X, τ) be the corresponding Alexandroff topology. What is a necessary and sufficient condition on $\leq$ so that (X, τ) is a T_0-space. Justify your solution.

(4) (40pt) Let A be a Heyting algebra and $\mathrm{Rg}(A) = \{\neg\neg a : a \in A\}$. Then $\mathrm{Rg}(A)$ is a Boolean algebra, where $a \dot{\vee} b = \neg\neg(a \vee b)$. Show that

(a) $\mathrm{Rg}(A) = \{a = \neg\neg a : a \in A\}$.

(b) Show that $\neg\neg : A \to \mathrm{Rg}(A)$ is a $\vee$-homomorphism (i.e., that it commutes with the operation $\vee$. (In fact, it also commutes with $\to$ and $\wedge$, but you do not have to show that.)