INTRODUCTION TO MODAL LOGIC 2015
HOMEWORK 5

• Deadline: November 10 — at the beginning of class.
• Grading is from 0 to 100 points.
• Results from the exercise class may be used in the proofs
• Success!

(1) (30pt) Prove
(a) $\vdash_K \Box \varphi \rightarrow \Box(\psi \rightarrow \varphi)$
(b) $\vdash_K (\Diamond \varphi \land \Box(\varphi \rightarrow \psi)) \rightarrow \Diamond \psi$

You may find it helpful to note that the following are propositional tautologies:
• $\varphi \rightarrow (\psi \rightarrow \varphi)$
• $\varphi \rightarrow (\varphi \lor \psi)$
• $(\varphi \rightarrow \psi) \rightarrow (\neg \psi \rightarrow \neg \varphi)$
• $(\varphi \rightarrow (\psi \rightarrow \chi)) \leftrightarrow ((\varphi \land \psi) \rightarrow \chi)$
• $\varphi \rightarrow (\psi \rightarrow (\varphi \land \psi))$

(2) (40pt) Recall that $S_5 = K + (\Box p \rightarrow p) + (\Box p \rightarrow \Box p) + (p \rightarrow \Box \Diamond p)$. Show:
(a) $\vdash_{S_5} \Diamond p \rightarrow \Box \Diamond p$
(b) $\not\vdash_{S_5} \Diamond p \rightarrow \Box p$ (You may use that $S_5$ is sound with respect to the class of frames $(W, R)$, where $R$ is an equivalence relation)

(3) (30pt)
(a) Show that if a frame $\mathfrak{F}$ is a bounded morphic image of a frame $\mathfrak{G}$, then $Log(\mathfrak{G}) \subseteq Log(\mathfrak{F})$.

(b) Let $\mathcal{C}$ be a non-empty class of frames. Prove that $Log(\mathcal{C})$ is contained in the logic of a single reflexive point or $Log(\mathcal{C})$ is contained in the logic of a single irreflexive point.

(4) (10pt) (BONUS!) The exercise is for those of you who like syntactical manipulations. $GL = K + (\Box(\Box p \rightarrow p) \rightarrow \Box p)$. Try to find a $GL$-proof of $\Box p \rightarrow \Box \Box p$, i.e. show that $\vdash_{GL} \Box p \rightarrow \Box \Box p$ (we already discussed in class how to do this semantically, here we ask for a syntactical derivation.)