INTRODUCTION TO MODAL LOGIC 2015
HOMEWORK 7

• Deadline: December 1 — at the beginning of class.
• Grading is from 0 to 100 points.
• Results from the exercise class may be used in the proofs
• Success!

(1) (50pt) (From the 2014 Exam)
(a) Let $\Sigma$ be a finite subformula closed set. Let $\mathcal{M} = (W, R, V)$ be a model such that $(W, R)$ is a rooted transitive reflexive frame with no branching to the right. Show that a transitive filtration of $\mathcal{M}$ through $\Sigma$ is a rooted reflexive transitive frame with no branching to the right. See Homework 6 for the definition of no branching to the right. (Hint: start by showing that if $r$ is a root of $\mathcal{M}$, then $[r]$ is a root of the filtrated model $\mathcal{M}_\Sigma$.)

Recall that a reflexive and transitive frame $(W, R)$ is rooted if there is $x \in W$ such that for each $y \in W$ we have $Rxy$.

(b) Deduce that $\textbf{S4.3}$ has the finite model property. See Homework 6 for the definition of $\textbf{S4.3}$.

(c) Deduce that $\textbf{S4.3}$ is decidable.

(2) (25pt) Let $\Sigma$ be a set of formulas and $A$ any element of $\text{At}(\Sigma)$. Show that for all $\langle \pi^* \rangle \varphi \in \neg \text{FL}(\Sigma)$: $\langle \pi^* \rangle \varphi \in A$ iff $(\varphi \in A \text{ or } \langle \pi \rangle \langle \pi^* \rangle \varphi \in A)$.

(3) (25pt) Show that if $[\pi^*](p \rightarrow [\pi]p) \rightarrow (p \rightarrow [\pi^*]p)$ is valid on a frame $(W, R_\pi, R_{\pi^*})$, then $R_{\pi^*} \subseteq (R_\pi)^*$