Exercise 1. Compute both the largest and the smallest filtration of the model below through each of the following sets

\[ \Sigma_1 := \{p, q\} \quad \Sigma_2 := \{p, q, \lozenge p\} \quad \Sigma_3 := \{q\} \]
directedness: $\forall x_0,x_1,x_2 ((Rx_0x_1 \land Rx_1x_2) \rightarrow \exists y(Rx_1y \land Rx_2y));$

density: $\forall x_1,x_2 (x_1Rx_2 \rightarrow \exists y(Rx_1y \land Ryx_2));$

every world sees a reflexive world: $\forall x \exists y(Rxy \land Ryy).$

**Exercise 6.** We say that a class of models $K$ has the property $(\ast)$ if any formula in the language of basic modal logic which is satisfiable in some $K$-model is also satisfiable in a finite $K$-model. Show that the following classes of models has the $(\ast)$-property.

1. The class of reflexive models;
2. The class of transitive models;
3. The class of directed models;
4. The class of models with no branching to the right.

**Exercise 7.** Compute the standard translations of the following formulas

1. $\Diamond p \rightarrow \Diamond \Diamond p$
2. $\Diamond \Box p \lor \Box p$
3. $\Box (\Box p \rightarrow q) \lor \Box (\Box q \rightarrow p)$
4. $(p \land \Box (\Diamond p \rightarrow \Box q)) \rightarrow \Diamond \Box q$
5. $\Box (\Box p \rightarrow p) \rightarrow \Box p$
6. $[\pi^*](p \rightarrow [\pi]p) \rightarrow (p \rightarrow [\pi^*]p)$

**Exercise 8.** $(\ast)$ Is it possible to find a pair of pointed Kripke models $(M, s)$ and $(M', s')$ one of which is finitely branching such that $M, s \equiv M', s'$ but $M, s \not\equiv M', s'$?