INTRODUCTION TO MODAL LOGIC 2016
HOMEWORK 5

- Deadline: November 28 — at the **beginning** of class.
- Grading is from 0 to 100 points.
- Results from the exercise class may be used in the proofs
- Success!

(1) (30pt) (From the 2014 Exam) In the following exercise you can use that the canonical model for **S4.3** is reflexive and transitive.

(a) Show that the canonical model for the modal logic

$$\mathbf{S4.3} = \mathbf{S4} + \Box(\Box p \to q) \vee \Box(\Box q \to p)$$

has no branching to the right. Recall that a reflexive Kripke frame has no branching to the right if

$$\forall x \forall y \forall z((Rxy \wedge Rxz) \to (Ryz \vee Rzy)).$$

You are not allowed to use Sahlqvist completeness theorem.

(b) Deduce that **S4.3** is sound and complete with respect to reflexive transitive frames with no branching to the right.

(2) (30pt)

(a) Let Σ be a finite subformula closed set. Let $\mathfrak{M} = (W, R, V)$ be a model such that (W, R) is a rooted transitive reflexive frame with no branching to the right. Show that a transitive filtration of $\mathfrak{M}$ through Σ is a rooted reflexive transitive frame with no branching to the right. (Hint: start by showing that if r is a root of $\mathfrak{M}$, then $[r]$ is a root of the filtrated model $\mathfrak{M}_\Sigma$.)

Recall that a reflexive and transitive frame (W, R) is *rooted* if there is $x \in W$ such that for each $y \in W$ we have Rxy.

(b) Deduce that **S4.3** has the finite model property.

(c) Deduce that **S4.3** is decidable.

(3) (40pt) (Item (a) is from the 2014 Exam)

(a) Show that for any modal formulas φ and ψ we have

$$\vdash_{\mathbf{K}} \Box\varphi \vee \Box\psi \text{ implies } \vdash_{\mathbf{K}} \varphi \text{ or } \vdash_{\mathbf{K}} \psi.$$

(b) Show that for any modal formulas φ and ψ we have:

$$\vdash_{\mathbf{S5}} \varphi \to \Box\psi \text{ iff } \vdash_{\mathbf{S5}} \Diamond\varphi \to \psi.$$

(c) Show that the above properties do not hold for all normal modal logics. That is, give an example of normal modal logics L_1 and L_2 which do not satisfy (a) and (b), respectively.

(Hint: use soundness and completeness of **K** and **S5** with respect to Kripke frames.)