INTRODUCTION TO MODAL LOGIC 2017
HOMEWORK 6

- Deadline: December 12 — at the **beginning** of class.
- Grading is from 0 to 100 points.
- Results from the exercise class may be used in the proofs
- Success!

(1) (50pt) (Exercise 4.4.2 in BdRV). Let **KvB** := **K** + vB, where vB is the axiom

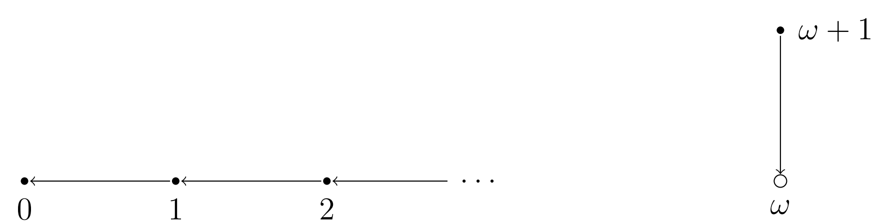
$$\Box\Diamond\top \to \Box(\Box(\Box p \to p) \to p).$$

Let $\mathfrak{J} = (J, R, A)$ be the general frame defined as follows. The domain J of $\mathfrak{J}$ is $\mathbb{N} \cup \{\omega, \omega+1\}$, i.e., the set of natural numbers together with two further points ω and $\omega + 1$, and

$$R := \{(\omega+1, \omega), (\omega, \omega), (\omega, n) : n \in \mathbb{N}\} \cup \{(n, m) : m < n\},$$

see the figure below. The collection of admissible sets A consists of the subsets of J such that X is finite and $\omega \notin X$ or X is cofinite and $\omega \in X$.

(a) Show that $\mathfrak{J}$ is indeed a general frame;
(b) Show that $\mathfrak{J} \Vdash$ vB;
(c) Show that the formula $\Box\Diamond\top \to \Box\bot$ is valid on any Kripke frame which validates the axiom vB. (A bit tricky!);
(d) Show that $\mathfrak{J} \nVdash \Box\Diamond\top \to \Box\bot$;
(e) Conclude that **KvB** is a Kripke incomplete (consistent) normal modal logic.



$\mathfrak{J}$

(2) (30pt)

(a) Let Σ be a set of formulas and A any element of $At(\Sigma)$. Show that for all $\langle\pi^*\rangle\varphi \in \neg\mathrm{FL}(\Sigma)$: $\langle\pi^*\rangle\varphi \in A$ iff ($\varphi \in A$ or $\langle\pi\rangle\langle\pi^*\rangle\varphi \in A$).

(b) Show that if $[\pi^*](p \to [\pi]p) \to (p \to [\pi^*]p)$ is valid on a frame (W, R_π, R_{π^*}), then $R_{\pi^*} \subseteq (R_\pi)^*$

(3) (20pt) Which classes of neighborhood frames do the following modal formulas define?

(a) $\neg\Box\bot$,
(b) $\Box p \to p$,
(c) $\Box p \vee \Box\neg p$,
(d) $\Box\Box p \to \Box p$.

Justify your solution with proof.