Name:

UvA Student ID:

General comments.

1. The time for this exam is 2 hours (120 minutes).
2. There are 100 points in the exam.
3. Make sure that you have your name and student ID on each of the sheets you are handing in.
4. If you have any questions, please indicate this silently and someone will come to you. Answers to questions that are relevant for everyone will be announced publicly.
5. No talking during the exam.
6. Cell phones must be switched off and stowed.

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(1) (40pt) Consider the Kripke frame $\mathcal{F} = (W, R)$ shown below. The arrows denote the accessibility relation $R$ — for example, $R(3, 1)$ holds, but $R(1, 3)$ does not.

(a) At which points is the formulas $\Box p$ true? Justify your solution.

(b) At which points is the formulas $\Box \Box p$ true? Justify your solution.

(c) Let $V$ be the valuation into $\mathcal{F}$ given by $V(p) = \{2, 5, 6\}$, as shown in the diagram. Is the formula $\varphi = \Box (p \rightarrow \Box p)$ true at world 3 in the Kripke model $(\mathcal{F}, V)$. Justify your solution.

(d) Find a valuation $V$ into $\mathcal{F}$ with $V(p) = \{2, 5, 6\}$ and such that the formula $\psi = \Box \Box (\Diamond \Diamond p \rightarrow \Diamond \Diamond q) \land \Box (q \rightarrow \neg p)$ is true at world 3 in the Kripke model $(\mathcal{F}, V)$. Justify your solution.

(2) (30pt)

(a) Let $\mathcal{M} = (W, R, V)$ and $\mathcal{M}' = (W', R', V')$ be Kripke models. Define when $B \subseteq W \times W'$ is a bisimulation.

(b) Show that the following property on frames is not modally definable: every state has at least one predecessor.

(c) Let $\mathcal{M} = (W, R, V)$ be the model shown above, with $V(p) = \{2, 5, 6\}$. Let $\Sigma = \{\Box p, p\}$. Draw a diagram of the smallest filtration $\mathcal{M}^\sigma$ of $\mathcal{M}$ through $\Sigma$. Indicate which subformulas of $\Box p$ are true at which worlds of $\mathcal{M}^\sigma$. 
(3) (30pt) Use the Sahlqvist algorithm to compute the first-order correspondents of the formulas:

(a) $\Box p \land p \to \Box \Box p$,

(b) $\Diamond p \to \Box \Diamond p$. 