INTRODUCTION TO MODAL LOGIC.
MIDTERM EXAM

24 OCTOBER 2017
UNIVERSITY OF AMSTERDAM

Name:

UvA Student ID:

General comments.

1. The time for this exam is 3 hours (180 minutes).
2. There are 100 points in the exam.
3. Make sure that you have your name and student ID on each of the sheets you are handing in.
4. If you have any questions, please indicate this silently and someone will come to you. Answers to questions that are relevant for everyone will be announced publicly.
5. No talking during the exam.
6. Cell phones must be switched off and stowed.

	Maximal Points	Your points
Exercise 1	20	
Exercise 2	20	
Exercise 3	20	
Exercise 4	20	
Exercise 5	20	
Total	100	

(1) (20pt) Consider the basic modal language. Show that the following properties on frames are not modally definable.

(a) The frame has more than 20 elements.
(b) The frame has less than 20 elements.

(2) (20pt)

(a) Show that any filtration of a model $\mathbb{M}$ satisfying

$$\forall x \exists y (Rxy \wedge Ryy)$$

is again a model satisfying this property.

(b) Show that not every filtration of a model $\mathbb{M}$ satisfying

$$\forall x \forall y \forall z ((Rxy \wedge Rxz) \rightarrow (y = z))$$

is a model satisfying this property.

Justify your solution.

(3) (20pt) Let $\Box_1$ and $\Box_2$ be the modalities with the following semantics

$\mathbb{M}, w \models \Box_1 \varphi$ iff $\forall v \in W(\mathbb{M}, v \models \varphi \iff wRv)$,
$\mathbb{M}, w \models \Box_2 \varphi$ iff $\forall v \in W(\neg(wRv) \implies \mathbb{M}, v \models \varphi)$.

(a) Show that $\Box_1$ is not expressible in the language of basic modal logic. (Hint: consider disjoint union of appropriate models.)

(b) Show that the modality $\Box_1$ is expressible in terms of $\Box_2$ and $\Box$ of the basic modal logic, i.e., show that the formula $\Box_1 \varphi$ is equivalent on models to a formula containing only the modalities $\Box_2$ and $\Box$.

(4) (20pt) Use the Sahlqvist algorithm to compute the first-order correspondents of the formulas:

(a) $\Diamond p \rightarrow \Box p$,

(b) $\Diamond \Box p \rightarrow \Box \Diamond p$.

(5) (20pt) Show that, in general, first-order formulas are not preserved under ultrafilter extensions. That is, give a model $\mathbb{M}$, a state w and a first-order formula $\alpha(x)$ such that $\mathbb{M} \models \alpha(x)[w]$, but $\mathbb{M}^* \not\models \alpha(x)[\pi_w]$, where π_w is the principle ultrafilter generated by w.

(Hint: consider a model based on the frame $(\mathbb{N}, <)$.)