INTRODUCTION TO MODAL LOGIC 2018
HOMEWORK 4

- Deadline: November 13 — at the **beginning** of class.
- Please staple and hand in your homework. Submit electronically only (!) in case of emergency.
- Grading is from 0 to 100 points.
- Good luck!

(1) (30pt) Recall that for each frame $\mathfrak{F}$ we let

$$Log(\mathfrak{F}) = \{\varphi : \mathfrak{F} \Vdash \varphi\}.$$

(a) Show that if a frame $\mathfrak{F}$ is a bounded morphic image of a frame $\mathfrak{G}$, then

$$Log(\mathfrak{G}) \subseteq Log(\mathfrak{F}).$$

(b) Show that if a frame $\mathfrak{F}$ is a generated subframe of a frame $\mathfrak{G}$, then

$$Log(\mathfrak{G}) \subseteq Log(\mathfrak{F}).$$

(c) Let $\mathcal{C}$ be a non-empty class of frames. Use (a) and (b) to show that $Log(\mathcal{C})$ is contained in the logic of a single reflexive point or $Log(\mathcal{C})$ is contained in the logic of a single irreflexive point.

(2) (30pt) Let

$$\mathbf{K5} = \mathbf{K} + (\Diamond\Box p \to \Box p).$$

(a) Show that **K5** is sound and complete with respect to frames satisfying

$$\forall x \forall y \forall z((Rxy \land Rxz) \to Rzy)).$$

(b) Show that $\vdash_{\mathbf{K5}} \Box(\Box p \to p)$.

(c) Show that $\nvdash_{\mathbf{K5}} \Box p \to p$.

(3) (40pt) Recall that $\mathbf{S4.2} = \mathbf{S4} + (\Diamond\Box p \to \Box\Diamond p)$.

Show that **S4.2** is sound and complete with respect to reflexive, transitive and directed frames (a frame is *directed* if $\forall x \forall y \forall z((Rxy \land Rxz) \to \exists u(Ryu \land Rzu)))$.

You may use the fact that **S4** is sound and complete with respect to reflexive and transitive frames.

You are not allowed to use Sahlqvist's completeness theorem in any of these exercises.