Introduction to Modal Logic. Exercise class 3

20 September 2018

Exercise 1. Which of the following frame properties are preserved (reflected) by the operations of forming generated subframes, p-morphic images, disjoint unions?

(1) reflexivity;

(2) transitivity;

(3) irreflexivity;

(4) converse seriality ($\forall x \exists y\, Ryx$);

(5) having cardinality at least n, for some natural number n;

(6) having cardinality at most n, for some natural number n.

Exercise 2. Show that the following frame properties cannot be defined in the basic modal language:

(1) converse seriality;

(2) having cardinality at least n, for some natural number n;

(3) having cardinality at most n, for some natural number n;

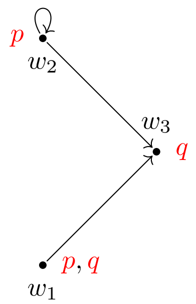
(4) acyclicity: 'there is no finite path (of non-zero length) from any point to itself'.

Exercise 3. Let $\mathbb{M} = (W, R, V)$ be a model and Σ a set of formulas closed under subformulas. Show that the filtrations $\mathbb{M}^s = (W_\Sigma, R^s, V^f)$ and $\mathbb{M}^l = (W_\Sigma, R^l, V^f)$ are indeed the smallest and the largest filtrations, respectively.[1]

Exercise 4. Compute both the largest and the smallest filtration of the model below through each of the following sets

$$\Sigma_1 \coloneqq \{p, q\} \quad \Sigma_2 \coloneqq \{p, q, \Diamond p\} \quad \Sigma_3 \coloneqq \{q\}.$$

[1] At the lecture W_Σ was denoted by W^f.



Exercise 5. Which of the following properties of frames are preserved by taking suitable filtrations of Kripke models?

(1) reflexivity

(2) symmetry

(3) seriality

(4) directedness: $\forall x_0 x_1 x_2\, ((Rx_0x_1 \wedge Rx_0x_2) \to \exists y(Rx_1y \wedge Rx_2y))$

(5) density: $\forall x_1 x_2 (x_1 R x_2 \to \exists y\, (Rx_1y \wedge Ryx_2))$

(6) every world sees a reflexive world: $\forall x \exists y\, (Rxy \wedge Rxy)$

Exercise 6. $(*)$ Let $\mathbb{M} = (W, R, V)$ be a model with R a transitive relation and Σ a subformula closed set. Let $(R^s)^*$ be the transitive closure of smallest filtration relation R^s. Show that $(W_\Sigma, (R^s)^*, V^f)$ is a transitive filtration of $\mathbb{M}$.