Introduction to Modal Logic
Exercise class 6

October 11, 2018

Exercise 1. Sahlqvist algorithm.
Compute the standard translation and a first order corespondent of (some of) the following formulas. For (6)-(8) Exercise 2 could be helpful.

(1) $\Box\Box p \to \Box p$

(2) $\Box p \land p \to \Diamond\Diamond p$

(3) $\Diamond\Box p \to \Box\Diamond p$

(4) $\Diamond\Box p \to \Box\Diamond\Diamond p$

(5) $\Diamond p \land \Diamond q \to (\Diamond(p \land q) \lor \Diamond(p \land \Diamond q) \lor \Diamond(q \land \Diamond p))$

(6) $\Box((\Box p \land p) \to \Diamond\Diamond p)$

(7) $\Box((\Box p \to p) \lor (\Diamond p \to \Box\Box p))$

(8) $\Box(\Box p \to q) \lor \Box(\Box q \to p)$

(9) $q \to \Box_F \Box_P q$ (a temporal example; can you think how to do it?)

Exercise 2 (3.6.3 in Blackburn et al.)**.**

(1) Prove that if $\phi \to \psi$ and $\phi' \to \psi'$ are simple Sahlqvist formulas, then $(\phi \to \psi) \lor (\phi' \to \psi')$ is equivalent to a simple Sahlqvist formula.

(2) Show that if ϕ and $\alpha(x)$ locally correspond, so do $\Box\phi$ and $\forall y(Rxy \to \alpha(y))$.

(3) Prove that if ϕ (locally) corresponds to $\alpha(x)$ and ψ (locally) corresponds to $\beta(x)$, then $\phi \land \psi$ (locally) corresponds to $\alpha(x) \land \beta(x)$.

(4) (*) Show that if ϕ locally corresponds to $\alpha(x)$, ψ locally corresponds to $\beta(x)$, and ϕ and ψ have no proposition letters in common, then $\phi \lor \psi$ locally corresponds to $\alpha(x) \lor \beta(x)$.

(5) (*) Prove that (2) and (4) do not hold for global correspondence, and that the condition on the proposition letters in (4) is necessary as well. (Hint: for (2), think of the modal formula $\Diamond\Diamond p \to \Diamond p$ and the first-order formula $\forall xyz(Rxy \land Ryz \to Rxz)$.)