Rydberg dressing of a one-dimensional Bose-Einstein condensate

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We study the influence of Rydberg dressed interactions in a one-dimensional (1D) Bose-Einstein Condensate (BEC). We show that 1D is advantageous over 3D for observing BEC Rydberg dressing. The effects of dressing are studied by investigating collective BEC dynamics after a rapid switch-off of the Rydberg dressing interaction. The results can be interpreted as an effective modification of the s-wave scattering length. We include this modification in an analytical model for the 1D BEC, and compare it to numerical calculations of Rydberg dressing under realistic experimental conditions.

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Ultracold quantum gas experiments allow for an extremely precise control of interatomic interactions. Strong interactions at the atomic level enable in principle the creation of strongly correlated many-body systems, where the tunability gives them an important advantage over their solid state equivalents. The short-range interactions between ground-state atoms can be controlled by Feshbach resonances [1–3], which resulted, *e.g.*, in the demonstration of the BCS-BEC type superfluid crossover [4–9]. However, while the interactions can be made very strong by going to the unitarity regime, it is still under debate whether this quantum gas can be considered as a strongly correlated system [10]. Strong correlations will be evident when the interaction is both strong and long range, *i.e.*, the range of the interaction exceeds the average interparticle separation. Rydberg atoms [11] take a central role in the broad spectrum of systems that can be categorized from short range to long range, since Rydberg atoms can be classified as intermediate range. Their mutual interaction is generally of van der Waals nature, which is neither long-range or short-range: the van der Waals coefficient $C_6 \sim n^{11}$ scales rapidly with the principal quantum number n, allowing for a range larger than the interparticle separation.

Rydberg atoms in the context of ultracold atomic gases [12–23] open up a whole new direction of strongly correlated many body physics with a focus on quantum computation and quantum simulations [24–31]. Most of the applications have their origin in the ability to manipulate these Rydberg atoms coherently on timescales below their radiative lifetime. When the atoms are cold enough, they remain spatially frozen at those timescales. Off-resonant coupling to Rydberg states, referred to as Rydberg dressing [21–23], allows experimentalists to achieve timescales longer than those required to stay in the frozen gas limit. These timescales enable BEC dynamics with long-range interactions, which is predicted to give rise to novel exotic many-body physics such as supersolidity [32–38].

Rydberg dressing of individual atoms trapped in optical tweezers [39] and in optical lattices [40] has been observed experimentally. Furthermore, modification of electromagnetically induced transparency (EIT) via resonant Rydberg dressing has also been observed [41]. However, observation of Rydberg dressing of a regular BEC has proven elusive so far. Rydberg dressing of a BEC in 3D has been theoretically studied [23, 42, 43] where at relatively low density the influence of dressing could be interpreted as a modification of the s-wave scattering length. In Ref. [43], the authors conclude that experimental observation of a dressed BEC in 3D is very difficult due to a strong reduction in the amount of Rydberg atoms in the BEC caused by the Rydberg blockade mechanism at higher densities. Moreover, the interpretation as an effective modification of the scattering length is not obvious under these conditions.

In this paper, we show that a one-dimensional geometry is preferential for the observation of BEC dressing. We show that a sudden switch-off of the dressing interaction (via switch-off of the dressing laser) results in collective BEC dynamics, which strongly suggests an interpretation in terms of an effective change of the s-wave scattering length. We show that the BEC breathing mode is a tenable experimental signature to observe BEC Rydberg dressing.

For the purpose of this paper we limit ourselves to isotropic Rydberg dressing, by coupling to a Rydberg S state with principal quantum number n. The key parameters are now the detuning $\varDelta$, the Rabi frequency $\varOmega$ of the coupling laser, and in particular their ratio $\beta = \varOmega/2|\varDelta|$ [21–23]. The Rydberg population fraction is then given by β^2. In the regime of large detuning $|\varDelta| \gg \varOmega$ the ground state $|g\rangle$ is weakly dressed by the Rydberg state $|r\rangle$ [23]:

$$|\psi\rangle = |g\rangle + \beta|r\rangle. \quad (1)$$

The van der Waals coeffient $C_6 > 0$ [44] of the Rydberg state gives rise to a repulsive interaction and determines the Rydberg blockade radius $R_B = \left(C_6/2\hbar\sqrt{\varDelta^2+\varOmega^2}\right)^{1/6} \approx \left(C_6/2\hbar|\varDelta|\right)^{1/6}$. Combined with negative (red) detuning $\varDelta < 0$, this results in an effective two-body interaction between Rydberg-dressed states at distance r [21]

$$W(r) = \beta^4 \frac{C_6}{R_B^6 + r^6}. \quad (2)$$

To show why one-dimensional systems can be more suitable for observing BEC Rydberg dressing we follow the treatment of [43], where the authors consider an additional energy from Rydberg dressing on an ensemble of N atoms by calculation of the energy difference Θ between N interacting atoms confined in a Rydberg blockade volume V_B with radius R_B. By expressing Θ in terms of atomic density ρ one can obtain the additional energy from Rydberg dressing of an N atom ensemble. The additional energy saturates above the critical density $\rho_B = 1/\beta^2 V_B$, which is a density with one excited Rydberg atom per Rydberg blockade volume V_B. This leads to an overall offset of the chemical potential of the BEC, but only to a small modification to the shape compared to no Rydberg dressing. For a given Rydberg dressing coupling β, only for low relative density $\rho < \rho_B$ there is a significant alteration to the BEC shape due to dressing. In 3D, as considered in [43], one has $V_B = \frac{4}{3}\pi R_B^3$ and for typical parameters ($R_B = 3$ μm, $\Delta/2\pi = 170$ kHz, $\Omega/2\pi = 10$ kHz, such that $\beta = 0.03$ — a value that induces a noticeable modification to the BEC, as will be shown later), this results in $\rho_{B,3D} \approx 10^{13}$ cm^{-3}, which is lower than typical 3D BEC densities, which are rather in the 10^{14} cm^{-3} range. In 1D, the case we consider here, one has a 1D blockade volume $V_B = 2R_B$, leading to a different scaling of ρ_B. For the same parameters (R_B, Δ and Ω as above) the linear blockade density $\rho_{B,1D} \approx 200$ μm^{-1}, which is *higher* than typical linear densities of $\lesssim 50$ μm^{-1} achieved, for example, with 1D BECs in atom chip experiments [45–47]. The argument derived in [43] is for a cloud of ground-state atoms, containing one Rydberg atom only. As we wish to consider the case of multiple Rydberg excitations we derive a mean field energy starting from the full quantum many-body description.

We assume a cigar-shaped BEC, with a radial trapping frequency $\omega_\perp$ much larger than the axial trapping frequency ω_0 and the chemical potential, such that it satisfies the criteria of a 1D BEC. Assuming quantum oscillator units $E_0 = \hbar\omega_0$, $l_0 = \sqrt{\hbar/m\omega_0}$, $t_0 = \omega_0^{-1}$ as units for energy, length and time, respectively, we start from the 1D Hamiltonian for N atoms with long-range interaction W:

$$\hat{H} = \int \hat{\psi}^\dagger(x)\left[-\frac{1}{2}\partial_x^2 + \frac{g_0 N}{2}\hat{\psi}^\dagger(x)\hat{\psi}(x) + \frac{1}{2}x^2\right]\hat{\psi}(x)dx + \frac{N}{2}\int\int \hat{\psi}^\dagger(x)\hat{\psi}^\dagger(x')W(|x-x'|)\hat{\psi}(x')\hat{\psi}(x)dxdx'. \quad (3)$$

It might not be directly obvious that the long range interaction in Eq. (2) can be directly incorporated in the Gross-Pitaevskii (GP) equation, therefore, as a standard procedure [48], we formally derive the generalized GP equation by first expressing the order parameter as the mean of the field operator $\psi(x) = \langle\hat{\psi}\rangle$. From this, we calculate the energy functional of a BEC as

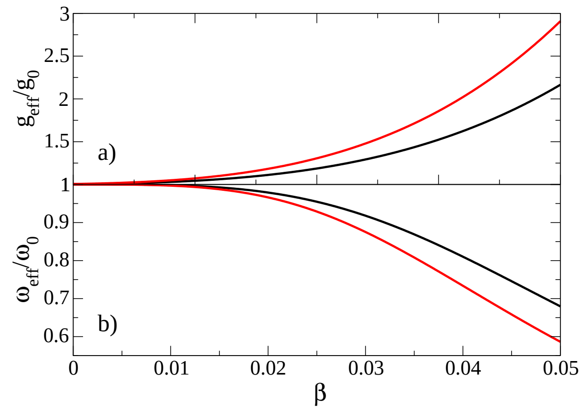

Figure 1. (a) Relative change of the mean-field coupling $g_{\rm eff}$ and (b) relative change of the effective trapping frequency $\omega_{\rm eff}$ as function of the Rydberg dressing parameter β, for Rydberg states $35S$ (black line) and $50S$ (red line), in ^{87}Rb.

$$E[\psi] = \int\left[-\frac{1}{2}|\partial_x^2\psi(x)|^2 + \frac{g_0 N}{2}|\psi(x)|^4 + \frac{1}{2}x^2|\psi(x)|^2\right] + N\int\left[\frac{1}{2}|\psi(x)|^2\int W(|x-x'|)|\psi(x')|^2dx'\right]dx. \quad (4)$$

From a variational argument, the generalized 1D GP equation can be derived as

$$i\partial_t\psi(x) = \left[-\frac{1}{2}\partial_x^2 + g_0 N|\psi(x)|^2 + \frac{1}{2}x^2 + V_{MF}\right]\psi(x), \quad (5)$$

where N is the particle number, and g_0 is the one-dimensional mean-field coupling parameter proportional to the s-wave scattering length a_s. In physical units $g_0 = 2a_s\hbar\omega_\perp$ [49], which corresponds to $g_0 = 2\frac{a_s}{l_0}\frac{\omega_\perp}{\omega_0}$ in our dimensionless units. The long-range dressed interaction appears in the equation as an energy dressing V_{MF} in the mean-field regime

$$V_{MF} = N\int W(x-x')|\psi(x')|^2dx', \quad (6)$$

which is treated in the same way as the long-range dipole-dipole interaction in dipolar BECs [50–52].

Let us first assume a BEC in the ground state ψ_0 of Eq. (5) without the V_{MF} term. The BEC has approximately a Thomas-Fermi profile with radius R_{TF}. We show in the following that under weak Rydberg dressing, indicated by $\beta \ll 1$, the BEC ground state is only slightly modified by the presence of V_{MF}. Such a modification can be interpreted as an effective change of the mean-field coupling strength g_0. For the case of a large particle number N (or low Rydberg principal quantum number n) we can assume that the Thomas-Fermi radius R_{TF} of a 1D BEC is much larger than the Rydberg blockade radius R_B, which allows us to approximate the V_{MF} term as $V_{MF} \approx N|\psi(x)|^2 \int W(x')dx'$. Contrary to the dipole-dipole interaction or bare van der Waals interaction, the combination of short-range saturation and long-range $1/r^6$ tail of Eq. (2) assures that the above integral is finite, and therefore gives a correction to the mean-field coupling g_0 (in physical units) of

$$g_{\rm eff} = g_0 + \frac{2}{3}\pi \frac{C_6}{R_B^5}\beta^4 = g_0 + \frac{\pi}{12}\frac{\hbar\Omega^4}{|\Delta|^3}R_B. \qquad (7)$$

This results in a direct interpretation of Rydberg dressing of a BEC as an effective tuning of the s-wave scattering length. Note that when these conditions are applied to a 3D BEC, we similarly find

$$g_{\rm eff}^{3D} = g_0^{3D} + \frac{2}{3}\pi^2 \frac{C_6}{R_B^3}\beta^4 = g_0^{3D} + \frac{\pi^2}{12}\frac{\hbar\Omega^4}{|\Delta|^3}R_B^3, \qquad (8)$$

which already had been derived in Refs. [23, 42, 43]. Also note that an effective change of the scattering length is not obvious from Ref. [43]: such an interpretation is possible only from an energy proportional to the density, which is not the case in this treatment.

A complementary interpretation to the change of the mean-field coupling constant would be an effective change in the trapping frequency:

$$\frac{\omega_{\rm eff}}{\omega_0} = \sqrt{\frac{g_0}{g_{\rm eff}}} = \frac{1}{\sqrt{1+\frac{2}{3}\pi\frac{C_6}{R_B^5 g_0}\beta^4}} = \frac{1}{\sqrt{1+\frac{\pi}{12}\frac{\hbar\Omega^4}{|\Delta|^3}\frac{R_B}{g_0}}}. \qquad (9)$$

Figure 1 shows both the relative change in the mean field interaction and in the effective trapping frequency, for two different Rydberg states, and for experimentally realisable values of the dressing coupling parameter. On the other hand, for small particle number N where $R_{TF} \lesssim R_B$, the two-body interaction $W(r)$ in Eq. (2) is basically constant over the system size, and therefore the dressing energy term V_{MF} has a constant value: $V_{MF} \approx N\beta^4 C_6/R_B^6$, which gives only an overall energy level shift of the system which we do not investigate further since it does not impact the dynamics.

A sudden switch-off of the dressing laser, which should take place on a timescale much faster than other timescales in the system, results in a sudden change of the harmonic trapping frequency from $\omega_{\rm eff}$ to ω_0. This non-adiabatic change results in the excitation of a BEC breathing mode [53–57]. The dynamics of a BEC can be described by the collective motion of atoms with time-dependent density $\rho(x,t) \propto \frac{1}{\lambda(t)}\rho_0(x/\lambda(t))$, where $\rho_0(x)$ is the initial BEC density at time $t=0$, and the scaling parameter $\lambda(t)$ obeys

$$\frac{d^2\lambda}{dt^2} = \frac{\omega_{\rm eff}^2}{\lambda^2} - \omega_0^2\lambda. \qquad (10)$$

A solution of this equation is periodic, with an amplitude dependening on $\omega_{\rm eff}$, and a frequency $\omega_b \simeq \sqrt{3}\omega_0$ [58].

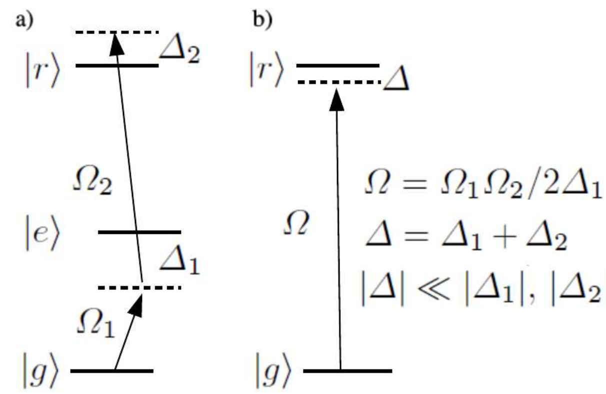

Figure 2. a) Ground state $|g\rangle$ dressed with the Rydberg level $|r\rangle$ via intermediate state $|e\rangle$ with the corresponding Rabi frequencies and detunings. b) Two level system with effective Rabi frequency and detuning.

In the regime of weak dressing, $\beta \ll 1$, an approximate analytical expression for λ has the form

$$\lambda(t) = \frac{1}{1+A}(\cos(\sqrt{3}\omega_0 t) + A), \qquad (11)$$

where $A = (1+\lambda_{\rm min})/(1-\lambda_{\rm min})$ and $\lambda_{\rm min}$ is an experimental observable that corresponds to the minimal radius of the BEC, which can be directly compared to the solution of Eq. (10).

We also consider another scenario, namely a simultaneous switch-off of the Rydberg dressing and the axial harmonic trap (while maintaining the radial confinement). This corresponds to an axial expansion of the BEC with scaling parameter λ described by

$$\frac{d^2\lambda}{dt^2} = \frac{\omega_{\rm eff}^2}{\lambda^2}. \qquad (12)$$

After a sufficiently long time, when the interactions have become negligible, the BEC expands ballistically with velocity $v_{\rm eff} = \sqrt{2}\omega_{\rm eff}$ and scaling parameter $\lambda(t) \simeq v_{\rm eff}t$.

We now compare the predictions above with the exact numerical solutions of Eq. (5), and include realistic experimental parameters. In practice the atomic system is a three-level system (Fig. 2a) with atomic ground state $|g\rangle$ coupled to Rydberg S-state $|r\rangle$ via the intermediate state $|e\rangle$. The Rabi frequency and detuning for the transition from $|g\rangle$ to $|e\rangle$ are Ω_1 and Δ_1, while for the similar transition from the intermediate state $|e\rangle$ to the Rydberg state $|r\rangle$ they are denoted as Ω_2 and Δ_2. The intermediate level $|e\rangle$ is far detuned, $|\Delta_1|, |\Delta_2| \gg \Omega_1, \Omega_2$ and can be adiabatically eliminated which effectively reduces the three-level system to a two-level system (Fig. 2b) with two-photon Rabi frequency $\Omega = \Omega_1\Omega_2/2\Delta_1$ and total detuning $\Delta = \Delta_1 + \Delta_2$, with $|\Delta| \ll |\Delta_1|, |\Delta_2|$. We assume $N = 2000$ ^{87}Rb atoms with an s-wave scattering length $a_s = 99a_0$ (where a_0 is the Bohr radius) [59] confined in an asymmetric harmonic trap with radial frequency $\omega_\perp = 2\pi \times 3000$ Hz and axial frequency $\omega_0 = 2\pi \times 30$

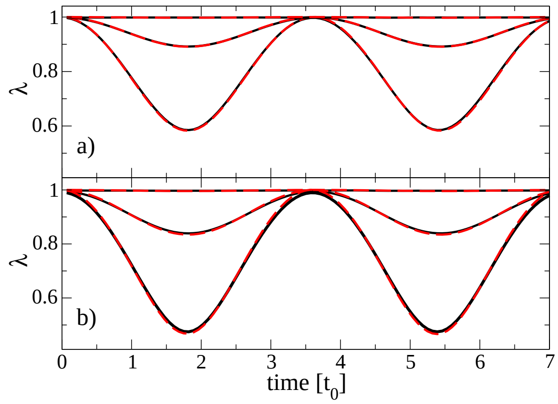

Figure 3. Time evolution of the BEC scaling parameter λ in an external harmonic trap, after a sudden switch-off of the Rydberg dressing. The initial state corresponds to a BEC ground state coupled to (a) the $35S$ and (b) $50S$ Rydberg states for different coupling $\beta = (0.01, 0.03, 0.05)$, giving rise to increasing amplitude in the figures. Black lines correspond to numerical results while red dashed lines correspond to analytical predictions.

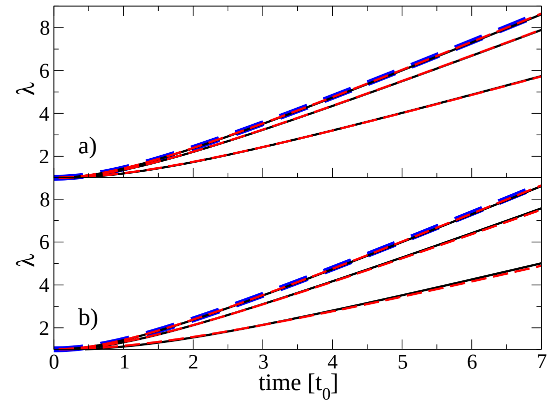

Figure 4. Time evolution of the BEC scaling parameter λ after a simultaneous switch-off of the Rydberg dressing and the external harmonic trap. The initial state corresponds to a BEC ground state coupled to (a) the $35S$ and (b) $50S$ Rydberg states for different coupling $\beta = (0.01, 0.03, 0.05)$ (from top to bottom). Black lines correspond to numerical results while red dashed lines correspond to analytical predictions. The blue dashed line corresponds to free evolution without initial dressing, *i.e.* $\beta = 0$.

Hz. These parameters correspond to a mean-field coupling strength $g_0 = 2.08 \times 10^{-34}$ Jm $= 0.53\hbar\omega_0 l_0$, a BEC radius of the undressed BEC $R_{TF} = 22.3\ \mu$m $= 11.4 l_0$, with length unit $l_0 = 1.96\ \mu$m and time unit $t_0 = 53$ ms. The C_6 coefficient for the $35S$ state is 1.2531×10^{-61} Jm^6, and for the $50S$ state it is 1.0262×10^{-59} Jm^6 [44].

First, we consider the breathing mode of the BEC. We integrate the time-dependent GP equation (5) with the initial condition being the Rydberg-dressed BEC ground state. At time $t = 0$ we suddenly switch off the Rydberg dressing and allow this equation to evolve without the V_{MF} term. The BEC is therefore no longer in its ground state and the time-evolution reveals collective dynamics. We numerically calculate λ defined as $\lambda(t) = R(t)/R_0$, where $R(t)$ is the radius of a BEC at time t while R_0 is the initial BEC radius. In Fig. 3 we present a comparison between the (numerical) solution of the GP equation and the (analytical) prediction of Eq. (10) for the evolution of the scaling parameter λ. The agreement between both models is excellent. The periodic behavior of λ indicates a BEC breathing mode with the expected breathing frequency $\omega_b = \sqrt{3}\omega_0$ [58] and an amplitude dependent on $\omega_{\rm eff}$, which depends on β. In Fig. 4 we present a comparison between numerical results and analytical predictions for the free axial expansion of a BEC. Similar to the previous case, the agreement between the analytical and numerical models is excellent.

The lifetime of the Rydberg-dressed state should be much longer than the timescale of the trap dynamics, which is 5 ms for the parameters considered here. We calculate this lifetime by considering a weak admixture to the intermediate state $|e\rangle = |5P_{3/2}\rangle$ with a decay rate $\Gamma_{5P_{3/2}}$, and an admixture of the Rydberg state $|r\rangle = |nS\rangle$ with a decay rate $\Gamma_{|nS\rangle}$, which results in an effective decay rate [23]

$$\Gamma_{\rm eff} = \left(\frac{\Omega}{2\Delta}\right)^2 \Gamma_{|nS\rangle} + \left(\frac{\Omega_1}{2\Delta_1}\right)^2 \Gamma_{5P_{3/2}}. \qquad (13)$$

The decay rates and lifetimes for Rb Rydberg states can be expressed as $\Gamma_{|nS\rangle} = \Gamma_s/n^\epsilon$ and $\tau_{|nS\rangle} = \tau_s n^\epsilon$, where $\epsilon = 3.0008$, $\tau_S = 1.368$ ns and $\Gamma_s/2\pi = 116$ MHz [60]. The intermediate state decay rate is $\Gamma_{5P_{3/2}}/2\pi = 6.1$ MHz. For the considered dressing-coupling parameters β equal to 0.01, 0.03 and 0.05, with $\Omega_1/2\Delta_1 = 4 \times 10^{-4}$, the corresponding lifetimes are found to be 140 ms, 47 ms, and 21 ms for $35S$, while these are 150 ms, 88 ms and 48 ms for $50S$. These lifetimes are sufficiently long to allow the BEC to equilibrate to the new ground state when Rydberg dressing is switched on.

The Rydberg-mediated control over the interactions we have discussed here offers an important alternative to previously considered schemes. For instance, Feshbach resonances allow for time-dependent non-linear dynamics as they can be utilized for a periodic modification of the mean field coupling. This was proposed by Saito and Ueda [61], who considered a sinusoidal time-dependent modulation of the coupling constant, and by Kevrekidis [62], who considered a block-type of periodic modulation of the mean-field coupling constant. This type of Feshbach-mediated manipulation is rather slow, not allowing for rapid switch-on and switch-off of the coupling constant. With the help of Rydberg dressed interactions, now delta-function type mean-field kicks of the condensate are possible. This opens up the experimental study of a new type of kicked-quantum rotor, where in contrast to the regular kicked BEC the mean field interaction is kicked and not the external trapping potential.

In conclusion, we have demonstrated that one-dimensional systems are preferable to three-dimensional systems for utilizing BEC Rydberg dressing, based on a simple dimensional scaling of the Rydberg blockade volume. We have shown how Rydberg dressing effectively changes the s-wave scattering length in the BEC coupling parameter, and how dressing can be observed experimentally from the collective dynamics of the condensate. The presented results correspond to realistic experimental parameters and lifetimes.

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