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Equivalence of transverse modes in Raman amplifiers and microchip lasers

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Abstract

It is shown that the transverse modes of a Raman amplifier with Gaussian pump are equivalent to those of a microchip laser with combined quadratic index guiding and Gaussian gain guiding. This equivalence allows for considerable cross-fertilization between these two hitherto separate fields. © 2001 Elsevier Science B.V. All rights reserved.

1. Introduction

The transverse mode structure of optical amplifiers and lasers is often of critical importance for their operation [1–4]. The most widely used sets of modal profiles are the so-called Hermite– and Laguerre–Gaussian (HG and LG) modes. These are solutions to the paraxial wave equation in free space, and are thus the "normal modes of free space" [1]. In addition, they are the transverse eigenmodes of stable-cavity laser resonators. Despite their widespread use, however, the HG and LG modes are often only crude approximations to the true transverse modes of practical optical amplifiers and lasers. Since it is desirable to maximize the overlap between the laser mode and the gain, it is common practice to make the width of the gain profile similar to (or even less than) that of the lowest-order mode. In this case, gain guiding and possibly also gain-related index guiding (where a change in the gain leads to a concomitant change in refractive index) will need to be taken into account in addition to the guiding mechanisms already present; this will generally lead to eigenmodes that differ from the standard HG and LG modes. The actual shape of the eigenmodes will typically depend on the details of the precise configuration, type of gain, and the transverse profiles of both the gain and losses that are present. This suggests that each particular laser or optical amplifier requires an analysis that is specific to that system.

In this paper we will show that two of these analyses, that previously have been treated completely separately, can in fact be directly mapped onto one another, and thus both lead to the same set of transverse eigenmodes. This set of modes

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can deviate significantly from the standard HG and LG modes. Specifically, we show that the well-established equations for the transverse modes of a Raman amplifier pumped by a Gaussian beam [3,5,6] are equivalent to the equations we have recently used to describe the transverse modes in a rare-earth microchip laser [7,8]. Both can be treated as a wave guide which combines parabolic index guiding with a Gaussian gain guide (see Fig. 1). This finding was inspired by the striking similarity between the results in [7] and those in [6] (e.g., compare [7, Fig. 2] with [6, Fig. 3]). In the following, we will show how these two problems can be directly mapped onto each other. We will first demonstrate the equivalence of the equations, and next discuss some of the insights that can be directly transferred from one case to the other.

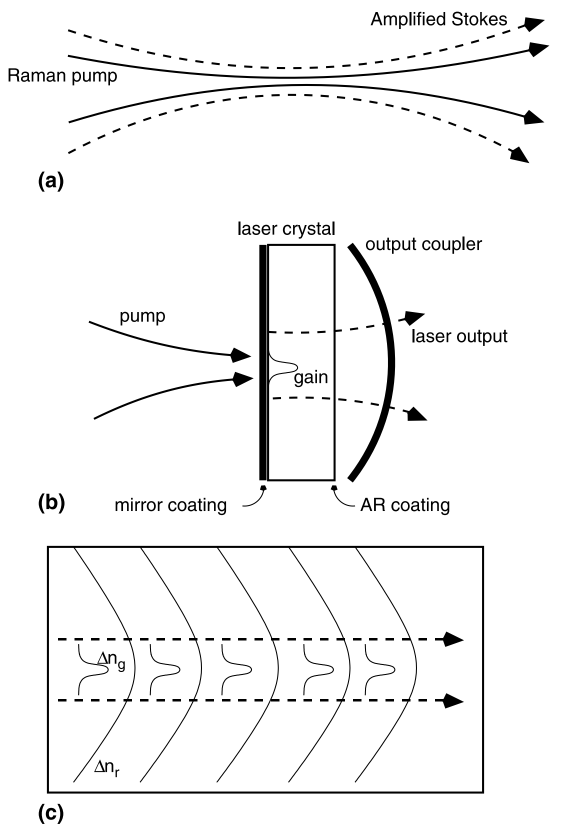

Fig. 1. The different systems we consider in this paper. The transverse modes of the experimental systems (a) and (b) are both equivalent to those of the hypothetical wave guide (c). The amplified eigenmode is indicated with dashed curves in all three cases. (a) Free-space Raman amplifier, pumped by a focused Gaussian beam; the spatially varying solutions for the amplified Stokes beam have the same Rayleigh range z_0 as the pump beam. (b) Microchip laser with localized gain and an effective index guide; the index guide may be caused by, e.g., the weakly curved output coupler, cf. [7,8]. (c) Longitudinally homogeneous wave guide with combined transverse quadratic index guide Δn_r and Gaussian gain guide Δn_g; the total guiding is $\Delta n_c = \Delta n_r + \Delta n_g$.

2. Demonstration of equivalence

We start with the case of Fig. 1(a), a free-space Raman amplifier pumped by a focused Gaussian pump beam. The aim is to describe the (spatially dependent) amplification at the Stokes wavelength. To this end, [5,6] start from the paraxial wave equation, including gain and index guiding. For an electric field

$$E(\mathbf{r},t) = \mathrm{Re}\{\exp[\mathrm{i}(\omega t - kz)]\mathscr{E}(\mathbf{r})\} \tag{1}$$

traveling in the $+z$ direction with wave vector $k = 2\pi/\lambda$, with $\mathscr{E}(\mathbf{r})$ the slowly varying amplitude, this may be written as

$$\left(\nabla_t^2 - 2\mathrm{i}k\frac{\partial}{\partial z}\right)\mathscr{E}(\mathbf{r}) = -\mathrm{i}kG(\mathbf{r})\mathscr{E}(\mathbf{r}) \tag{2}$$

with the transverse Laplacian

$$\nabla_t^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{3}$$

Perry et al. [5,6] studied Eq. (2) with the guiding $G(\mathbf{r})$ proportional to the intensity of a lowest-order Gaussian mode with Rayleigh range z_0. Expansion of the field $\mathscr{E}$ into the cylindrically symmetric LG modes (solutions of Eq. (2) for $G(\mathbf{r}) \equiv 0$) with the same Rayleigh range z_0 transforms the wave equation (2) into a set of ordinary linear coupled differential equations in z for the mode amplitudes [5,6]. Analogously, one could use the modes of free space with rectangular symmetry, the Hermite–Gaussian (HG) functions [1]. The disadvantage of these approaches is that the resulting first-order differential equations still depend on z. We now show that under a quite general condition for G, one may rewrite Eq. (2) so that the z-dependence factorizes out explicitly.

First, inspired by the approach in [5,6], we note that the solutions to the homogeneous ($G = 0$) part of wave equation (2), the LG and HG functions [1], all have the same form for a given Rayleigh range z_0. We use this as trial solution to Eq. (2) of the form

$$\mathscr{E}(\mathbf{r}) = \frac{1}{\sigma(z)} f(\tilde{\mathbf{r}}) \exp[\mathrm{i}\psi(\mathbf{r})], \tag{4}$$

where $\tilde{\mathbf{r}} = (\tilde{x}, \tilde{y}, \tilde{z})$ is a scaled coordinate, $\tilde{x} = x/\sigma(z)$, $\tilde{y} = y/\sigma(z)$, and $\tilde{z} = z_0 \arctan(z/z_0)$ with transverse scaling factor

$$\sigma(z) = \sqrt{1 + \left(\frac{z}{z_0}\right)^2} \tag{5}$$

and

$$\psi(\mathbf{r}) = -\frac{x^2 + y^2}{\sigma^2(z)} \frac{kz}{2z_0^2}. \tag{6}$$

The transverse scaling factor $\sigma(z)$ gives the z-dependence of the waist size $w(z) = w_0\sigma(z)$, with w_0 the focused waist size for wavelength λ and Rayleigh range z_0, i.e., $w_0^2 = \lambda z_0/\pi$ [1]. We leave f unspecified for now and insert the trial solution (4) into the left-hand side of Eq. (2). This yields after some manipulation

$$\left(\nabla_\mathrm{t}^2 - 2\mathrm{i}k\frac{\partial}{\partial z}\right)\mathscr{E}(\mathbf{r}) = \frac{1}{\sigma^3(z)} \exp[\mathrm{i}\psi(\mathbf{r})] \times \left[\tilde{\nabla}_\mathrm{t}^2 - \frac{k^2(\tilde{x}^2 + \tilde{y}^2)}{z_0^2} - 2\mathrm{i}k\frac{\partial}{\partial \tilde{z}}\right] f(\tilde{\mathbf{r}}) \tag{7}$$

with the scaled transverse Laplacian

$$\tilde{\nabla}_\mathrm{t}^2 = \frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{y}^2}. \tag{8}$$

Eq. (7) now becomes extremely useful when, as in [5,6], the gain and/or index term G can be written as

$$G(\mathbf{r}) = \frac{1}{\sigma^2(z)} \tilde{G}(\tilde{x}, \tilde{y}). \tag{9}$$

For such G the z-dependence of Eq. (2) drops out, by using Eq. (7), leading to

$$\left[\tilde{\nabla}_\mathrm{t}^2 - \frac{k^2(\tilde{x}^2 + \tilde{y}^2)}{z_0^2} + \mathrm{i}k\tilde{G}\right] f = 2\mathrm{i}k\frac{\partial f}{\partial \tilde{z}}. \tag{10}$$

As a result of the above exercise, the scaled equation for a free-space Raman amplifier with a focused Gaussian pump has become equivalent to the equation for propagation in a wave guide [1]. Writing the base refractive index of the wave guide as n_0 and the transverse (complex-valued) variation of the refractive index as Δn_c, the equivalence can be explicitly stated as $\Delta n_\mathrm{c}/n_0 = -(\tilde{x}^2 + \tilde{y}^2)/2z_0^2 + \mathrm{i}\,\tilde{G}/2k$. The resulting guiding is now completely independent of z, and the problem of solving the three-dimensional (3D) paraxial wave equation (2) is reduced to finding the eigenmodes of the left-hand side of Eq. (10), a 2D problem. In principle, our approach is not different from the approach in [5,6], where the z-dependence is also effectively eliminated by a coordinate transformation. However, the above treatment shows *explicitly* how the free-space Raman guiding problem is directly equivalent to a longitudinally invariant wave guide. The coordinate transformation replaces the z-dependence with a parabolic index guide, and replaces G with $\tilde{G}$. Note that traveling from $z = -\infty$ to $z = +\infty$ in Eq. (2) corresponds to traveling a distance $\Delta\tilde{z} = \pi z_0$ in the wave guide of Eq. (9). Note in addition that Eq. (9) is very general: it holds when G is proportional to the intensity of *any* LG or HG mode with Rayleigh range z_0 (cf. Eq. (4)), and also for any sum of intensities of such modes.

The transverse eigenmodes of the Raman amplifier are now determined by the left-hand side of Eq. (10). This is exactly the eigenmode problem we studied in the case of microchip lasers around threshold (see Fig. 1(b) and [7,8]). There, Eq. (10) was derived via a completely different route, however. In the theory of transverse modes in microchip lasers, one commonly assumes that the mode profile hardly changes upon a single round trip through the cavity. This then allows averaging the guiding effects over the cavity length, eliminating the dependence on the longitudinal (z) coordinate. In this way one also arrives at an equivalent wave guide, as in Eq. (10). In the case of a microchip laser, the parabolic index guide effectively arises from weak thermal lensing and/or from curvature of the mirrors [9], and the source of the gain is a longitudinal pumping beam with a transverse profile $\tilde{G}$.

Before discussing the correspondence between the two systems, it is useful to first simplify Eq. (10) further. Because of the cylindrical symmetry of the effective index guide that appears in Eq. (10) besides $\tilde{G}$, it is sensible to switch to cylindrical coordinates (ρ, ϕ). In addition, we scale the transverse coordinates to the waist size w_0 of the parabolic index guide, i.e., we use $\tilde{x} = \rho w_0 \cos\phi$ and $\tilde{y} = \rho w_0 \sin\phi$. This converts Eq. (10) into

$$\left[-\frac{1}{2}\nabla_\rho^2 + 2\rho^2 - 2\mathrm{i}\,\tilde{g}(\rho,\phi)\right] f(\rho,\phi) = -2\mathrm{i}\frac{\partial}{\partial\theta} f(\rho,\phi), \tag{11}$$

where we have defined $\tilde{g}(\rho,\phi) = z_0\tilde{G}(\tilde{x},\tilde{y})/2$, $\theta = \tilde{z}/z_0$, and

$$\nabla_\rho^2 = w_0^2\tilde{\nabla}_\mathrm{t}^2 = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}. \tag{12}$$

Note that with this scaling no free parameters remain: once $\tilde{g}$ is given, the eigenvalues and the corresponding eigenmodes f of the left-hand side of Eq. (11) are fixed. This is essentially identical to the left-hand side of [8, Eq. (7)]. The only remaining differences are notational, with $2\mathrm{i}\,\tilde{g}$ replacing $z_0 g$ of [8]. Note that the sign convention for the phase adopted here is the same as that of [5,6], while in our previous work the opposite sign was used (compare [8, Eq. (1)–(3)]). We here define the eigenvalues η via $\partial f/\partial\theta = \mathrm{i}\eta f$. For $\tilde{g} \equiv 0$ the resulting η are real-valued positive integers and the eigenmodes f correspond to scaled cross-sections of LG and HG modes.

3. Discussion

Both the initial theoretical work on Raman amplifiers [5,6] and our own initial theoretical work on microchip lasers [7], were limited to gain functions g that corresponded to pumping by the lowest-order LG mode (identical to the lowest-order HG mode)

$$\tilde{g} = \tilde{G}_\mathrm{p}\exp(-2\rho^2/\rho_\mathrm{g}^2) \tag{13}$$

and to pure gain (i.e., no gain-related index guiding) so that $\tilde{g}$ is purely real-valued $\tilde{G}_\mathrm{p} = G_\mathrm{p}$. More recently, the effect of detuning the amplified field $\mathscr{E}$ in frequency from maximum gain has been investigated. For a homogeneously broadened gain transition around frequency ω_g, with a width (FWHM) 2Γ, this leads to

$$\tilde{G}_\mathrm{p} = G_\mathrm{p}(1 - \mathrm{i}\delta)/(1 + \delta^2) \tag{14}$$

in Eq. (13) with $\delta = (\omega - \omega_\mathrm{g})/\Gamma$ the normalized detuning. For Raman amplifiers, this extension was studied both theoretically and experimentally in [3,10]. For microchip lasers this extension was studied in [8].

Given the correspondence demonstrated above, it is not surprising that identical results have been obtained for Raman amplifiers and microchip lasers. For instance, both in [3,10] and in [8] it was found that maximum modal gain occurs at a detuning towards higher frequencies, because there the gain-related index guiding helped in confining the mode transversely within the high-gain region.

In addition, there are quite a number of results of either system that have not been found for the other yet. Thus, the above correspondence can be used to carry over the results in the extensive literature on Raman amplifiers (e.g., [3,5,6,10,11] and references therein) to the relatively new field of microchip lasers, leading to some insights that are new in the latter context. For instance, for small gain (with $\tilde{g}$ as in Eq. (13)) first-order perturbation theory shows that the lowest-order mode $f_\mathrm{L} = \exp(-\rho^2)$ has a gain $\mathrm{Im}\eta = \mu G_\mathrm{p}$, with $\mu = \rho_\mathrm{g}^2/(1 + \rho_\mathrm{g}^2)$ (cf. [6, Eq. (21)]). In addition, [5,6] found that the cross-over between small $\tilde{g}$ (where the parabolic index guide dominates) and large $\tilde{g}$ (where the gain-related guiding will dominate) occurs at $\mu G_\mathrm{p} \approx 1$. Physically, this corresponds to the point where the modal gain per $\Delta\tilde{z} = z_0/2$ equals unity. In other words, the point where the modal gain can compensate for diffraction losses.

On the other hand, the interesting variety of possible mode profiles found for microchip lasers in [8] has not been discussed for Raman amplifiers yet. In addition, we have recently demonstrated theoretically that Eq. (11) can have strongly nonorthogonal eigenmodes, leading to large and resonant excess noise factors K [12]. Although the nonorthogonality of the eigenmodes of Eq. (2) has been recognized for Raman amplifiers [11,13,14], the possibility of resonant K factors had not been uncovered yet.

Other work on microchip lasers has mainly considered gain-related guiding and index guiding [9] separately. For gain-related guiding, the effects of saturation have been included [2], analytical solutions have been found for certain transverse pump profiles [15], and the interplay between longitudinal and transverse modes (in particular the role of detuning) has been investigated [16]. Recently, one has investigated the effects of additional gain-related guiding in a regime where the thermally induced index guide dominates [17]. It seems likely that some of this work will be useful in the context of Raman amplifiers as well.

Of course there are many differences between Raman amplifiers and microchip lasers. One such issue, directly related to the use of Eq. (11), is the following: for the case of Raman amplifiers, the scaled gain waist size ρ_g is fixed by the requirement that the Rayleigh range of gains equals z_0, i.e. $\rho_g = w_g/w_0 = \lambda_g/\lambda$, with λ_g the pump wavelength, and w_g the waist size of the pump. In contrast, for microchip lasers the gain waist size ρ_g can be arbitrarily chosen.

4. Conclusion

In conclusion, we have shown that the eigenmodes of free-space Raman amplifiers with focused Gaussian gain are equivalent to those of a rare-earth-doped microchip laser. This should lead to considerable cross-fertilization between these two hitherto separate fields.

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