Critical Petermann K Factor for Intensity Noise Squeezing

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(Received 12 April 2000)

We investigate the impact of the Petermann-excess-noise factor $K \geq 1$ on the possibility of intensity noise squeezing of laser light below the standard quantum limit. Using an N-mode model, we show that squeezing is limited to a floor level of $2(K - 1)$ times the shot noise limit. Thus, even a modest Petermann factor significantly impedes squeezing, which becomes impossible when $K \geq 1.5$. This appears as a serious limitation for obtaining sub-shot-noise light from practical semiconductor lasers. We present experimental evidence for our theory.

PACS numbers: 42.50.Lc, 42.50.Dv, 42.60.Da

Semiconductor lasers can generate sub-shot-noise light when they are operated far above threshold and driven by a quiet pump, i.e., a constant current source. This squeezing of the intensity noise was first demonstrated by Yamamoto and co-workers [1]. However, subsequent experiments on different types of semiconductor lasers revealed that relatively few quietly pumped semiconductor lasers exhibit intensity squeezing, and if they do, they often stay above the theoretically expected squeezed noise level [2]. For a long time, the chief suspect for the discrepancy has been the influence of *intensity* multimode effects (mode partition noise) [3,4]. However, the modal intensity partition noise can in principle be eliminated by using very strong side-mode rejection, so that the intensity noise is negligible in all modes except one [3]. But even in that case, the measured noise is often higher than would be expected from the laser quantum efficiency [2,5,6]. The observed limitations to noise reduction in semiconductor lasers have thus remained largely unexplained, frustrating further progress.

In this Letter, we show that a slight nonorthogonality of the cavity eigenmodes might be an explanation for the bottleneck of the observed squeezing. In practical "single mode" lasers, other modes with negligible intensity and intensity noise still contain some spontaneous emission noise. Because of the nonorthogonality of the eigenmodes, this noise from other modes is homodyned into the lasing mode, leading to excess noise into the laser light. In this excess noise mechanism *homodyning* is the key effect, which makes that in contrast to mode partition noise, the *field* fluctuations of the side modes are important, not their *intensity* fluctuations. Thus, even when laser side mode intensities are negligibly small, this field multimode effect can still impede intensity squeezing.

Mode nonorthogonality leads generally to an increase of the quantum noise by the so-called Petermann excess noise factor K [7,8]. The consequence of this increase in quantum noise on laser intensity noise has not been investigated for a laser high above threshold. We will show theoretically that above a critical value of K ($K_{\mathrm{crit}} = 1.5$), the laser intensity noise can no longer be brought below the standard quantum limit (SQL). This result is confirmed by experimental data, and K values in this range may easily occur in practical semiconductor lasers.

The semiclassical theory of the Petermann excess-noise factor is based upon nonorthogonal cavity eigenmodes, and it has the advantage of giving a very simple "geometrical" expression for the amount of excess quantum noise: $K = \langle v_i|v_i\rangle\langle e_i|e_i\rangle/|\langle v_i|e_i\rangle|^2$, where $|e_i\rangle$ is the eigenmode and $|v_i\rangle$ is its associated adjoint mode [9]. However, the complex amplitudes of a set of classical nonorthogonal modes cannot be turned into a set of operators obeying standard canonical commutation relations [10], and cannot yield the simple quantum picture which is required for our purpose. In several recent papers [10,11], two of us showed that this difficulty can be solved by introducing appropriate "vacuum modes" that allow one to recover the unitarity of the input-output scattering matrix. Moreover, we showed that instead of working in the nonorthogonal eigenbasis, it is more convenient to use an orthogonal basis, that consists of the lasing mode and modes orthogonal to it, constructed in a Gramm-Schmidt fashion from the nonorthogonal eigenbasis [11]. The time-evolution equation for a roundtrip through the cavity then takes the specific form

$$\frac{d}{dt}\begin{pmatrix}\hat{a}_0\\ \hat{a}_1\\ \vdots\\ \hat{a}_n\end{pmatrix} = \begin{pmatrix}\lambda_0 & \kappa_{0,1} & \cdots & \kappa_{0,n}\\ 0 & \lambda_1 & \cdots & \kappa_{1,n}\\ \vdots & \ddots & \ddots & \vdots\\ 0 & \cdots & 0 & \lambda_n\end{pmatrix}\begin{pmatrix}\hat{a}_0\\ \hat{a}_1\\ \vdots\\ \hat{a}_n\end{pmatrix} + \begin{pmatrix}\hat{F}_0\\ \hat{F}_1\\ \vdots\\ \hat{F}_n\end{pmatrix}, \tag{1}$$

where $\hat{a}_0$ is the photon annihilation operator for the lasing mode (the eigenmode with the lowest loss), $\hat{a}_1 \ldots \hat{a}_n$ are

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the corresponding operators for the orthogonal subthreshold modes, and $\hat{F}_0 \ldots \hat{F}_n$ are noise operators required for quantum consistency [11]. The matrix elements λ_i are the eigenvalues of the corresponding eigenmodes from which mode i is constructed, and $\kappa_{i,j}$ describes the scattering of the j mode into the i mode. The upper off-diagonal coupling terms cannot be eliminated through a redefinition of the orthogonal basis, because they are caused by loss modes which are not part of the set of laser modes. This effect was termed "loss-induced coupling" [12].

The Petermann-excess-noise contribution originates from homodyning. Because of the upper triangular structure of Eq. (1), noise in the orthogonal modes leaks into the lasing mode, where it beats with the strong lasing field to produce an intensity fluctuation that is first order in the fluctuating field. The extra noise introduced by this homodyning depends both on the amount of mode mixing, related to the $\kappa_{0,j}$ coefficients, and on the strength of the nonlasing modes which in turn is related to λ_j. These noise sources will have a colored spectrum, which is a general feature of the Petermann excess noise [13], stemming from the dynamics of the different subthreshold modes. As far as the laser linewidth is concerned, it was shown in [11] that this approach recovers the semiclassical result for K, under the same hypotheses.

Now we can formulate general equations for the intensity noise of a laser with N modes, as schematically depicted in Fig. 1. We will assume that the amplitude of the subthreshold modes is so small that their intensity can be neglected. The relevant equations for the intensity noise are then the following equations for the amplitude quadrature and inversion:

$$\frac{d}{dt}\hat{P}_0 = \frac{1}{2}(A\hat{N} - \Gamma_i - \Gamma_m)\hat{P}_0 + \hat{f}_i + \hat{f}_m + \hat{f}_g + \hat{\mathcal{L}} + \hat{\mathcal{L}}^\dagger, \tag{2}$$

$$\frac{d}{dt}\hat{N} = \hat{\Lambda} - \gamma_0\hat{N} - A\hat{N}\hat{n}_0 + \hat{f}_P + \hat{f}_{sp} + \hat{f}_{st},$$

where $\hat{P}_0$ is the amplitude quadrature operator, $\hat{P}_0 = \hat{a}_0 + \hat{a}_0^\dagger$, Γ_i is the internal loss of the lasing mode, Γ_m is the mirror outcoupling loss, A is the normal spontaneous emission rate into the lasing mode, and $\hat{N}$ is the inversion operator. The second equation describes the evolution of the inversion for an ideal four-level laser, with Λ the pump rate, γ_0 the decay rate of the inversion, and $\hat{n}_0$ the photon number operator. The standard noise sources are represented by δ correlated Langevin noise operators $\hat{f}$. In the amplitude equation the noise operator $\hat{f}_i$ corresponds to internal losses, $\hat{f}_m$ to outcoupling loss, and $\hat{f}_g$ to noise due to the stimulated emission gain. Their diffusion constants are, respectively, $D_{i,i} = \Gamma_i$, $D_{m,m} = \Gamma_m$, and $D_{g,g} = A\langle\hat{N}\rangle$. In the inversion equation the Langevin noise operator $\hat{f}_P$ corresponds to the noise due to the pumping process, $\hat{f}_{sp}$ to spontaneous emission noise, and $\hat{f}_{st}$ to stimulated emission noise. Their respective diffusion constants are $D_{P,P} = \epsilon\langle\Lambda\rangle$, where $\epsilon = 0$ for a noiseless pump and $\epsilon = 1$ for a Poissonian pump, $D_{sp,sp} = \gamma_0\langle\hat{N}\rangle$, and $D_{st,st} = A\langle\hat{N}\rangle\langle\hat{n}_0\rangle$. Finally, due to their same physical origin the noise terms associated with the stimulated emission and the inversion are perfectly anticorrelated, and their cross correlation is $D_{g,st} = -A\langle\hat{N}\rangle\sqrt{\langle n_0\rangle}$.

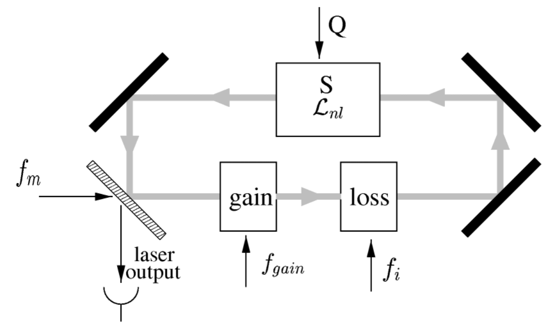

FIG. 1. Sketch of laser resonator. In a cavity roundtrip the laser mode encounters: isotropic gain, isotropic loss, a mode mixing element S, and the output coupling mirror. The isotropic gain, isotropic loss and output coupling introduce vacuum noise into each lasing mode. The mode mixing element S couples the orthogonal modes via the "loss modes" Q, thus leading to the Petermann-excess-noise contribution in the lasing mode ($\mathcal{L}_{nl}$).

These equations are similar to the standard one for the intensity noise of a single mode laser [14], apart from the effective extra noise source $\hat{\mathcal{L}} = \sum_{i>0}^{N}\kappa_{0,i}\hat{a}_i$, that represents the Petermann-excess-noise contribution. The physically interesting part of the new excess noise term corresponds with the normally ordered correlation function

$$\langle\mathcal{L}^\dagger(0)\mathcal{L}(\tau)\rangle = \left\langle\sum_{i,j>0}^{N}\kappa_{0,i}^*\kappa_{0,j}a_i^\dagger(0)a_j(\tau)\right\rangle = (K - 1)\Gamma_c g(\tau), \tag{3}$$

where Γ_c is the total cavity loss rate of the lasing mode. The normalized correlation function $g(\tau)$, with $\int_{-\infty}^{\infty} g(\tau)\,d\tau = 1$, generally has a very complicated form, but its short-time and long-time limits are easily found [11,13]. For the low-frequency fluctuations, which will be considered here (corresponding to long time scales), the spectral variance of the Petermann-excess-noise source simply reduces to $(K - 1)\Gamma_c$ as expected.

After linearization around the mean field and mean inversion, the intensity noise at low frequency measured outside the laser can be obtained using the input-output formalism, which writes here $\hat{P}_{0,\text{out}} = \Gamma_m^{1/2}\hat{P}_0 - \Gamma_m^{-1/2}\hat{f}_m$. The result can be written in a convenient form by introducing $S = \langle\delta P_{0,\text{out}}^2(\omega = 0)\rangle$, $x = \gamma_0/(A\langle\hat{n}_0\rangle)$, and $\eta = \Gamma_m/\Gamma_c$, leading to

$$S = 1 + \eta(1 + x)(\epsilon - 1) + 2\eta(1 + x)[K(1 + x) - 1], \tag{4}$$

normalized with respect to the SQL. Equation (4) reduces to the standard single-mode result of the amplitude noise in the lasing mode when $K = 1$ [14]. It also gives the expected limit close to threshold ($x \gg 1$): the noise will be much larger than shot noise and the quantum noise observed in the lasing mode is enhanced with the Petermann-excess-noise factor K. In the general case, Eq. (4) shows that the excess quantum noise caused by the nonorthogonality of the cavity eigenmodes can cancel squeezing. Assuming ideal noiseless pumping ($\epsilon = 0$), squeezing becomes impossible as soon as $K > 3/[2(1 + x)]$. This condition has two consequences: first, squeezing never occurs at any current when $K > 1.5$. Second, if $1 < K < 1.5$, squeezing requires a smaller x, and therefore a higher pump current, with respect to what would be expected from the single-mode theory.

In the limit of high pump current, where $x \to 0$, Eq. (4) simplifies into

$$S = 1 + \eta[(\epsilon - 1) + 2(K - 1)]. \tag{5}$$

For quiet pumping ($\epsilon = 0$), one obtains $S = 1 + \eta(2K - 3)$, showing again that squeezing vanishes for $K > 1.5$. The appearance of a critical Petermann factor of 1.5 is clearly due to the excess noise contribution $2(K - 1)$, which originates from the $\mathcal{L}$ and $\mathcal{L}^\dagger$ terms in Eq. (3). The factor of 2 is related to the (linear) phase insensitive amplification of the nonlasing modes, giving the usual 3 dB enhanced noise above the shot-noise limit. Another interesting limit is obtained for a laser with a Poissonian pump ($\epsilon = 1$) far above threshold, where $S = 1 + 2\eta(K - 1)$: The intensity noise depends not only on the Petermann excess-noise factor, but also on the outcoupling efficiency. This prediction is different from the semiclassical result of Ref. [9], and only when $\eta = 1/2$ is the intensity noise K times larger than shot noise.

The above analysis is highly relevant for practical semiconductor lasers. Petermann introduced the K factor to calculate the enhanced spontaneous emission rate in the lasing mode of semiconductor lasers with quadratic gain and index guiding [7]. For a purely gain-guided laser in 1D (edge emitters), this yields $K = \sqrt{2}$, and for 2D (vertical-cavity surface emitting lasers, VCSELs) it yields $K = 2$, seriously impeding squeezing. In real experimental situations the gain and index guiding are not given by a simple quadratic profile, generally leading to much larger K factors. For lasers which are mainly gain guided, K factors of 15–25 have been reported [15]. In contrast, lasers which are almost purely index guided are expected to have a K factor which is very close to 1. However, a certain amount of gain guiding is unavoidable in any efficient semiconductor laser since the gain must be localized, i.e., must not extend too much beyond the volume occupied by the lasing mode. The exact guiding properties and transverse mode structure of a real device represent a very complicated problem, and they are usually designed as a "best compromise." Since slight deviations of K from 1 can pose a stringent limit to the maximally possible squeezing, this aspect of the laser design should be considered to explain the difference in observed squeezing for different types of semiconductor lasers. To strengthen this claim we mention that among VCSELs squeezing has been observed only in oxide-confined devices, which have very strong index guiding [16]. Also the TJS lasers that have recently been studied and yielded a large reproducible amount of squeezing are lasers with a strong index guide [17].

We have made various experimental observations that confirm the above analysis. Using a semiconductor laser, we have measured both the intensity noise of the lasing mode, and the spontaneous emission noise in the next nonlasing spatial mode (in a two-mode approximation), as explained in detail in Ref. [18]. It can then be shown that these two noises are correlated, which is a generic feature of the Petermann excess noise [19,20]. In Fig. 2 we have plotted the measured intensity noise of the laser as a function of the inferred value of the K factor; the different points are obtained for different values of the driving current (see [18]). Though the value of K is inferred from our model and not directly measured, this curve clearly illustrates the relationship between the intensity noise and the Petermann K factor.

Another experimental approach uses a HeXe laser, which has the advantage of adjustable nonorthogonality between the polarization modes [13]. The disadvantage is that sub-shot-noise operation cannot be achieved, due to the Poisson pump statistics and incomplete inversion [21]. However, one may still test the prediction of Eq. (4) by measuring the experimental enhancement $K' \equiv S(K)/S(K{=}1)$ of the intensity noise at low frequencies, as a function of the applied nonorthogonality K. Close to threshold we expect $K' = K$, whereas far above threshold $K' \to S$ and Eq. (5) predicts a dependence on the outcoupling efficiency η. A typical measurement of K' as a function of K, done far above threshold, is plotted in the inset of Fig. 3. A linear fit to the data yields the value of $(K' - 1)/(K - 1)$. Such measurements were

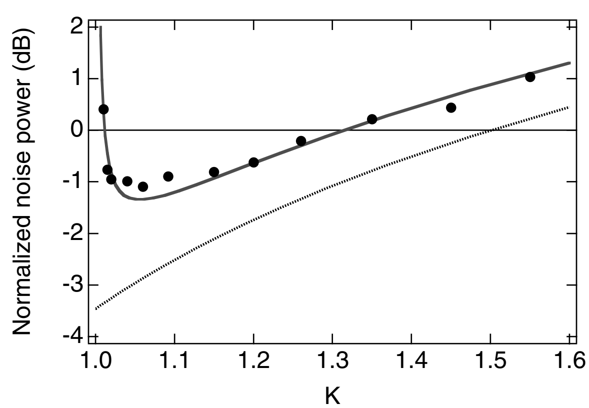

FIG. 2. Intensity noise relative to shot noise as a function of the inferred K factor for the experiment of Ref. [18]. The experimental points are obtained for different values of the driving current. The full line is the theoretical fit for the actual current, while the dashed line is the lowest possible noise level obtained very far above threshold ($x = 0$).

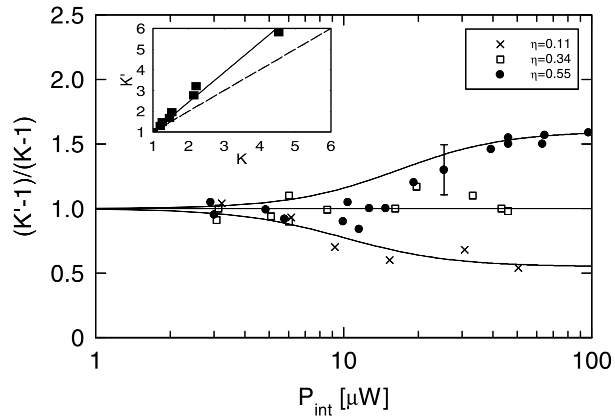

FIG. 3. Experimental data, which show the ratio of observed excess noise $K' - 1$ and Petermann excess noise $K - 1$ as a function of P_{int} (as determined from P_{out} and mirror reflectivity) for three different η (as determined from the experimentally known internal and mirror loss rates). The solid lines are guides to the eye. It is clearly seen that $(K' - 1)/(K - 1)$ starts at 1 close to threshold while it changes far above threshold to a limit that depends on η. The inset shows a measurement of K' vs K, for high P_{int} and $\eta = 0.55$. The dashed curve is the line $K' = K$ and the solid curve is the fit to the data to determine the ratio $(K' - 1)/(K - 1)$.

performed for a range of internal cavity powers P_{int}, and outcoupling efficiencies η. The data are summarized on the main graph of Fig. 3. We indeed find a clear qualitative agreement with the above predictions. For small P_{int}, the experimental data show that $K' \approx K$. For large P_{int}, i.e., far above threshold, the value of $(K' - 1)/(K - 1)$ goes to a limit that depends on η. For small η this limit is smaller than 1, while for large η it actually becomes larger than 1 ($K' > K$). This again dramatically illustrates the importance of mode nonorthogonality for the intensity noise when it approaches the shot-noise limit. Quantitatively the observed limiting values of $(K' - 1)/(K - 1)$ are about a factor of 2 higher than expected for an ideal four-level laser with the same η. This deviation can be explained as a consequence of incompleteness of the inversion of the He-Xe laser ($N_{sp} \approx 3$).

In conclusion, we have shown that the multitransverse mode structure of a laser cavity plays a crucial role in both the intensity and phase fluctuations of the single lasing mode. Intensity squeezing can be obtained only if the excess noise factor K is smaller than 1.5, which eliminates *de facto* many possible cavity structures, concerning in particular fully or partly gain-guided semiconductor lasers.

The authors from Leiden thank H. Assadian and G. Kütük for experimental assistance, and acknowledge support by FOM. The research of NJvD was made possible by the KNAW. We acknowledge support from the EC through the ESPRIT program 20029 "ACQUIRE" and the TMR network "Microlasers and cavity QED."

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