Photon Statistics of a Laser with Slow Inversion

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We have measured the photon number probability distribution of a laser in which the inversion is not slaved to the field. For the experiments, we have used a Nd^{3+}:YVO_4 laser which has a sufficiently slow inversion to allow measurement of the photon fluctuations at a time scale much shorter than that of the relaxation oscillations. The photon distribution function becomes highly nonstandard (i.e., non-Poissonian) in such a laser; this is consistent with available theoretical work. We point out the relevance of our results for the case of the semiconductor microlaser.

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The photon statistics of a single-mode laser has been studied for more than 30 years [1–5]. In those days the photon statistics were obtained by adiabatically eliminating the dynamic variables of the gain medium. Over the years, this approximation has given excellent agreement with experimental results. Incorporation of the variables of the gain medium has been considered in some theoretical papers [6,7] but this work received little attention. Recently, it has been stressed that the validity of adiabatically eliminating the gain medium depends on the size of the laser; conventional laser theory is expected to break down as the laser gets smaller and smaller [8,9]. This applies when the inversion is slow enough to fulfil the condition $\gamma_{\parallel} < \Gamma_C$, where $\gamma_{\parallel}$ and Γ_C are the inversion and cavity decay rates, respectively. Dramatic deviations from "standard" photon statistics have been predicted for the case $\Lambda\beta \gtrsim 1$, where $\Lambda \equiv \Gamma_C/\gamma_{\parallel}$ and β is the fraction of spontaneous emission going into the lasing mode [10]. Since β is roughly proportional to the inverse of the laser mode volume [11], the condition $\Lambda\beta \gtrsim 1$ is easier fulfilled the smaller the laser is. In view of the present trend of laser miniaturization, in particular, for semiconductor lasers, this deviation from standard photon statistics is a highly relevant issue; it is in fact the theme of our Letter.

We report experimental observation of highly nonstandard photon statistics of a Nd^{3+}:YVO_4 microchip laser operating under the condition $\Lambda\beta \approx 1$, and we interpret the result in the context of available theories [6,7]. The good agreement allows us to predict similar nonstandard photon statistics for semiconductor lasers operating under the condition $\Lambda\beta \gtrsim 1$; such lasers are already available on a prototype basis [12–14]. Our experimental validation of the generalized theories on the photon statistics [6,7] is all the more important in view of the fact that standard semiconductor lasers will soon operate in the regime $\Lambda\beta \gtrsim 1$.

In technical terms, if we make the usual assumption that the polarization of the gain medium can be adiabatically eliminated, a laser with $\Lambda < 1$ is a class-A laser whereas $\Lambda > 1$ corresponds to class B [7]. One may wonder why the photon statistics of a class-B laser have not been addressed so far experimentally, in particular, for semiconductor lasers for which so much noise data are available. The problem in measuring photon statistics is that one requires high quality time-domain data for faithful sampling of the relaxation oscillations. For semiconductor lasers this implies a time resolution of 100 ps, which is at the border of the present technical possibilities. For this reason we have used a Nd^{3+}:YVO_4 microchip laser as an experimental model system; this laser offers a relatively low relaxation oscillation frequency ($\omega_{ro}/2\pi \approx 10$ MHz) and an extreme class-B character ($\Lambda \approx 10^6$). Recently we have studied the second-order moment of the photon distribution of this laser within the context of a linearized theory [8]; this approximation fails when studying the shape of the photon number distribution since linearization leads by necessity to a Gaussian distribution. It is the shape of the photon distribution of a class-B laser that we address in this Letter: this distribution is very different from the predictions of the linearized model, and as we see below, the distributions also show substantial deviations from a class-A distribution.

As a reminder, the class-A laser, defined by $\Lambda \ll 1$, has a photon probability distribution $P(n)$ given by the generalized Poissonian distribution [15]

$$P(n) = \frac{(p + \bar{n})^{(p+n)} \exp(-p - \bar{n})}{(p + n)!}, \qquad (1)$$

where n is the intracavity photon number and p is interpreted as the number of modes available for spontaneous emission. Equivalently, $p = 1/\beta$, where β is the fraction of spontaneous emission going into the lasing mode. Above threshold, the parameter $\bar{n} = p(M - 1)$, where M is the pump parameter, approximates the average photon number [15]. For high-intensity beams, where effects due to reflected vacuum fluctuations can be neglected [16] and a semiclassical description suffices, n is related to the output intensity I by $I = n\Gamma_C h\nu$.

Because of the nonlinear nature of the full class-B coupled rate equations, solutions are far from trivial. Both Ogawa [6] and Paoli *et al.* [7] have put forward theoretical predictions for the photon probability distribution in class-B lasers. The two approaches have some common traits but differ at crucial points; however, their results can be represented by the same generic equation, which

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we will give first, before discussing the differences in derivation:

$$P(n) = C(a,b)n^a e^{-bn}, \qquad (2)$$

where the normalization constant $C(a,b) = b^{(1+a)}/a!$. The first and second moments of this distribution are given by the parameters a and b via $\bar{n} = (a+1)/b$ and $g_2(0) - 1 \approx \Delta n^2/n^2 = 1/(a+1)$. The function $P(n)$ is thus entirely defined by the two experimentally accessible variables $\bar{n}$ and $g_2(0)$.

Paoli *et al.* [7] use the concept of "pseudo energy" to solve the laser dynamics. The inversion and the intensity are reexpressed in terms of one fast and one slow variable, which are then separated. These variables are the coordinate $q = \ln(n/n_0)$, with n_0 as the equilibrium photon number, and the pseudo energy W, defined by

$$W \equiv \frac{1}{2(M-1)\Gamma_c \gamma_\parallel}\left(\frac{dq}{dt}\right)^2 + V(q), \qquad (3)$$

where M is the normalized pump parameter and $V(q) = e^q - q - 1$ is the Toda potential [7]. The pseudo energy quantifies the strength of the intensity fluctuations; for low noise levels its time average is in fact equal to $g_2(0) - 1$. The advantage of the pseudo energy over the true energy is that, in the limit of weak excitation and damping of the noise, the pseudo energy is conserved, whereas the true energy of the system is not. Interestingly, Paoli *et al.* [7] (and also Ogawa [6]) predict that the photon probability goes to zero for $n/n_0 \to 0$, since $V(q)$ [and thus $W(q)$] diverges in that limit. To obtain an expression for the photon probability distribution, Paoli *et al.* find themselves forced to expand the probability distribution in terms of the pseudo energy, W, taking only terms up to the second order. The distribution becomes non-normalizable and remains in principle valid only for small values of W, i.e., relatively weak relaxation oscillations, a condition that is *not* fulfilled in our experiments.

Also Ogawa [6] makes use of the pseudo energy concept, but he does not invoke the separation of time scales. Moreover, his model corresponds to what Siegman [17] calls an ideal three-level laser [18]. This model leads to approximately equal population in the two laser levels, whereas a four-level laser [18] such as our Nd^{3+}:YVO_4 laser has an almost negligible population in the lower level [19]. This could conceivably lead to different noise properties from what is expected in a Nd^{3+}:YVO_4 laser. Nevertheless, both Ogawa and Paoli *et al.* arrive at the same final result, namely, Eq. (2). This suggests that Eq. (2) has generic validity beyond the stringent conditions used in the two derivations [6,7]. As we show, our experimental results confirm this hypothesis.

Our experimental setup is shown in Fig. 1. A Nd^{3+}:YVO_4 chip with a thickness of 0.1 mm and a doping of 1% atomic Nd^{3+} was put as close as possible to a concave mirror with a radius of curvature of 25 mm and a reflectivity of 80%. The mirror has a diameter of only

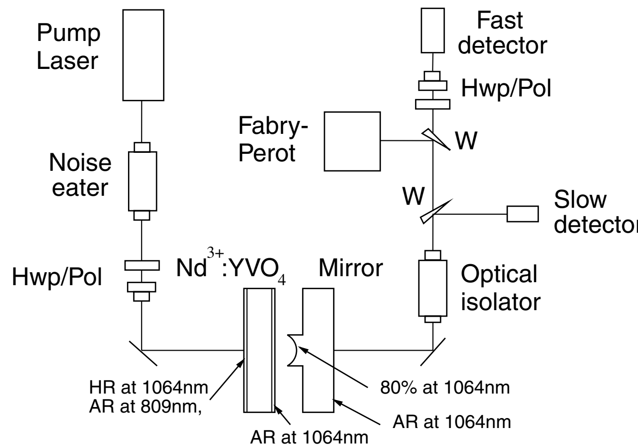

FIG. 1. The setup, with the laser cavity depicted in the bottom of the figure. The cavity, which is not drawn to scale, is shown with the remodeled concave mirror. W indicates a Wedge, and Hwp/Pol is a half-wave-plate and polarizer combination used for the adjustment of the light intensity. AR and HR denote an antireflection and a high-reflection ($\approx$100%) coating, respectively.

$\approx$1 mm; it is basically a small platform made by the careful removal of the surrounding mirror through grinding. The small size was needed to keep the cavity length small and β relatively large. The crystal was optically pumped with an intensity-stabilized titanium-sapphire laser at 809 nm, with relative noise below 0.1% rms. By using a Fabry-Perot we confirmed that the Nd^{3+}:YVO_4 laser was oscillating in a single mode only. The Nd^{3+} fluorescence, at 1064 nm, had an almost Lorentzian spectrum with a width (FWHM) of $\gamma_\perp/\pi = 0.22$ THz; this large value allows for adiabatic elimination of the polarization of the gain medium. The values of β and Γ_C were found directly from the experimental data themselves. Plotting the output as a function of pump parameter and using the relation $n = (M-1)/\beta$ yielded $\beta = 1.8 \times 10^{-5}$. Furthermore, the value of Γ_C was deduced from the relaxation oscillation frequency $\omega_{ro} = \sqrt{\Gamma_C \gamma_\parallel (M-1)}$, leading to $\Gamma_C = 1.05 \times 10^{11}$ s^{-1}. The upper level decay rate was measured to be $\gamma_\parallel = 1.3(1) \times 10^4$ s^{-1}. Our laser is an extreme class-B laser since $\Lambda = \Gamma_C/\gamma_\parallel = 8 \times 10^6 \gg 1$ [10]; this facilitates the differentiation of the class-B from the class-A photon probability distribution.

The photon number probability distribution was determined by direct binning of the intensity values observed in an intensity-time trace. With a typical output power of 1 mW, there was no need to use a photon counter; instead we used a dc-coupled 125 MHz photodetector (NewFocus 1811) which has a much larger dynamic range than a photomultiplier. Nevertheless, we refer to intensities in terms of the corresponding intracavity photon numbers. To obtain high-quality data, we took special care to minimize the background signal since this produces a smearing of the probability distribution through its noise. Therefore, to ensure that the signal was maximized without saturating the detector, the intensity was adjusted for each measurement using a half-wave-plate and polarizer combination (Fig. 1). This made it impractical to employ this detector to measure the absolute intensity, for which another, slower Si photodiode with an adjustable current amplifier was used. The oscilloscope, a LeCroy 9304A, had a nonideal flash-type analog-to-digital converter, which introduced extra noise. Some of the intensity bins of the converter had a larger probability to be filled than others and thus, for each pump value, ten measurements at various oscilloscope offsets were conducted in order to average out this effect.

The photon number probability distribution is shown in Fig. 2, where the experimentally obtained data are compared to the theoretical curves according to Eq. (2). Since the variables a and b [Eq. (2)] are already set by the measured values of $\bar{n}$ and $g_2(0)$, the comparison is a test of the shape of the curves. Figure 3 shows that the experimental data are in excellent agreement with theory. Smearing of $P(n)$ due to the noisy dark signal can be seen only in Fig. 2a, where the sharp drop in probability at very small photon numbers is somewhat diffuse in the experimental data (see inset).

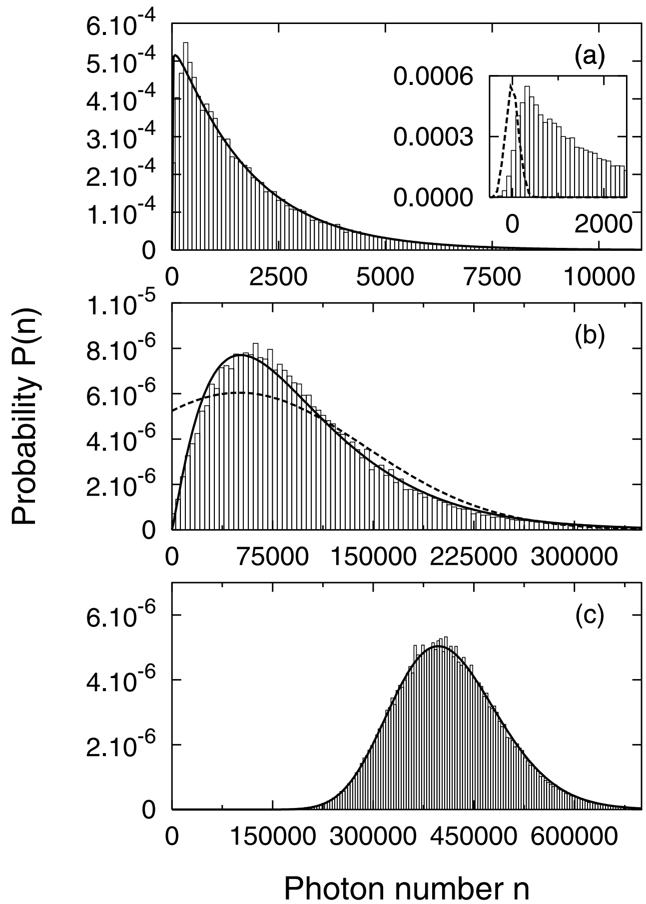

FIG. 2. The photon probability distribution for three different pump values: (a) $M = 1.04$, (b) $M = 2.85$, (c) $M = 7.28$ with the experimental data shown as bar graphs. The full curves represent the theoretical predictions based on the measurement of $g_2(0)$ and $\bar{n}$, and the dashed curve in (b) shows a class-A distribution with the same value of $g_2(0)$ and $\bar{n}$ as the experimental data. The inset in (a) shows the distribution of the background (dark) signal as a dashed curve. The characteristic values for these graphs are (a) $g_2(0) = 1.96$, $\bar{n} = 1750$; (b) $g_2(0) = 1.47$, $\bar{n} = 96\,600$; (c) $g_2(0) = 1.038$, $\bar{n} = 413\,000$.

It is instructive to compare the shapes of the $P(n)$ curves in Fig. 2 with those for a class-A laser. Figure 2a may remind the reader of the thermal photon statistics of a class-A laser below threshold; however, it is obtained here *above* threshold ($M = 1.04$); its nature is due to the relaxation oscillation enhanced spontaneous emission noise [8].

Jumping now to Fig. 2c, we observe at $M = 7.28$ a curve that is approximately Gaussian, as applies to a class-A laser sufficiently far above threshold, *but* with an anomalously large width (at this point a class-A distribution would have a standard deviation $\sigma_n \approx \bar{n}^{1/2} = 642$, while the measured standard deviation is $\sigma_n = 80\,500$, i.e., 125 times larger). This is due to the extreme class-B behavior of our laser; for lasers that are only marginally class B, the distribution is narrower; furthermore, lasers with marginal class-B properties retain their Gaussian shape (such as depicted in Fig. 2c) till closer towards threshold. Apart from a somewhat larger width, these marginal class-B lasers have photon probability distributions that are indistinguishable from those of class-A lasers.

Figure 2b shows an intermediate case, which for a class-A laser would correspond to a truncated Gaussian as shown by the dashed curve [20]. As can be seen, the class-B distribution deviates strongly from this truncated Gaussian; this is shown in more detail in Fig. 3 where two sections of Fig. 2b have been enlarged. Whereas there is a finite probability of having zero photons in the cavity in the class-A case, class-B theory predicts that there will be zero probability of having no photons in the

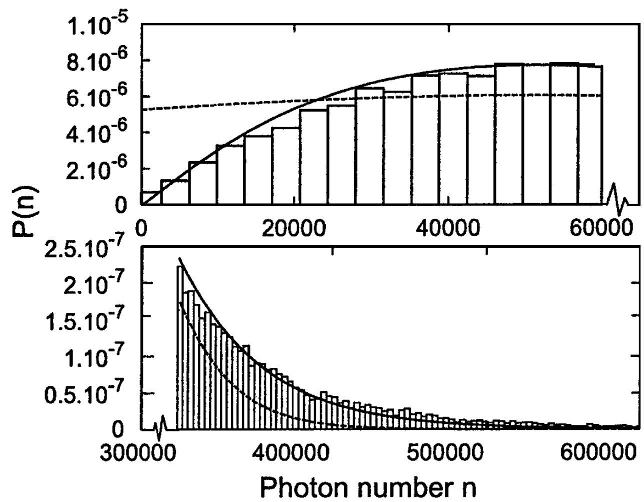

FIG. 3. Enlargement of two sections of Fig. 2c: again a comparison of the experimental data with the theoretical curves for the class-B model (solid line) and the class-A model (dashed line). The two curves have the same values for $g_2(0)$ and $\bar{n}$ as the experimental data.

cavity (this holds true, even very close to threshold such as in Fig. 2a, where the probability drop is very abrupt just above zero photons). This prediction is confirmed by our experimental results. In Fig. 3 the dashed curves represent the Gaussian photon probability distribution of a class-A laser, chosen with $\bar{n}$ and $g_2(0)$ equal to those of the class-B distribution, here shown by drawn curves. Note also how the upper tail extends much farther for the class-B distribution; this is due to the nonlinearizable nature of the class-B laser dynamics.

Despite the *a priori* weaknesses of the theories [6,7] discussed above, they are highly successful in predicting the probability distribution. The use of a three-level system by Ogawa [6,18] is apparently appropriate for a four-level laser. This surprising result is consistent with the finding of Levien *et al.* [21] that the differences in the noise of a three-level system and a four-level system is sometimes smaller than anticipated [18]. The approximation of Paoli *et al.* [7] that W remains relatively small also seems better than expected. In the framework of pseudo energy this could be explained by a spurious drift term [22], which leads to an extra damping of the intensity fluctuations and a reduction in W, thus effectively increasing the region of validity for the low W approximation.

In conclusion, our experiment confirms the theoretical predictions [6,7] for a class-B laser and shows that these theories apply surprisingly far beyond the parameter ranges of nominal validity. Our findings are significant for all class-B lasers, in particular, for semiconductor microlasers with $\Lambda\beta \gtrsim 1$, where we expect to see photon number statistics that are equivalent to those described here [8,9]. In pioneering work, microdisk and microring semiconductor lasers have already crossed the limit $\Lambda\beta \gtrsim 1$ [12–14]: Reference [12] has a value of $\Lambda\beta$ that equals $\approx 2 \times 10^2$. It would therefore be very interesting to study the intensity characteristics of these devices; so far, only dc properties have been reported. For more common semiconductor lasers (edge-emitting and vertically emitting devices), the $\Lambda\beta \gtrsim 1$ criterion has not yet been satisfied since typically $\Gamma_C \approx 300\ \text{ns}^{-1}$, $\gamma_\parallel \approx 3\ \text{ns}^{-1}$, $\beta \approx 10^{-4}$ so that $\Lambda\beta \approx 10^{-2}$. However, since the dominance of class-B properties is mainly a question of laser size [8,9], the nonstandard photon statistics emphasized in this Letter will become obvious once these lasers are made an order of magnitude smaller. Especially close to threshold, deviations from class-A photon statistics should become visible even *before* $\Lambda\beta \gtrsim 1$ is reached as an onset towards extreme class-B behavior.

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