Preface

One can only write a book if other people don’t. I am ashamed to admit that when I was younger I did not realize this, or perhaps just did not show. As an excuse I can adduce that the authorities that rule our universities for all their wisdom still do not know.

Setup

Some books have appendices at the back, but I have adhered to the elephant model and put the appendices in front.

Notation

Functions are often given by a transformation rule. If a function $f$ corresponds in this way to a transformation of $x$ into $M(x)$, we define it by

$$f: x \mapsto M(x).$$

Let $B(x)$ be a statement about some individual $x$. Then $\forall x B(x)$ is short for ‘for every $x$, $B(x)$’; and $\exists x B(x)$ abbreviates ‘there are $x$ such that $B(x)$’. For example, if $B(x)$ is ‘$x$ bothers me’, then $\forall x B(x)$ is ‘everything bothers me’, and $\exists x B(x)$ ‘something bothers me’.

Instead of the word ‘and’ we sometimes use the symbol $\&$, as in ‘it is raining & the wind is up’. We write $A \Rightarrow B$ for ‘if $A$, then $B$’, and $A \Leftrightarrow B$ for ‘$A$ if and only if $B$’.

If $P$ is a class, we write $x \in P$ for ‘$x$ belongs to $P$’. The $\in$-notation is sometimes combined with the quantifiers $\exists$ and $\forall$; then we write

(1) $\forall x \in P B(x)$

(2) $\exists x \in P B(x)$

if $B(x)$ holds for some elements of $P$. More elaborately (1) might be rendered as

(1a) $\forall x (x \in P \Rightarrow B(x))$,

an (2) as

(2a) $\exists x (x \in P \& B(x))$.

With $B$ as before and $P$ denoting the class of problems, (1) represents ‘every problem bothers me’, and (2) ‘there is a problem bothering me’.