Labor income and the Demand for Long-Term Bonds

Ralph Koijen, Theo Nijman, and Bas Werker

Tilburg University and Netspar

January 2006
Motivation

- Inflation-protected securities (inflation-linked bonds, annuities, ..) have been advocated for long-term investment purposes

- Markets for inflation-linked bonds are rather illiquid and in several countries even absent

- In these cases, nominal securities have been proposed to replicate inflation-linked securities

- However, in most countries labor income is indexed with inflation, which already implies a (non-tradable!) position in inflation-linked securities

- Impact of labor income on the optimal demand for real and nominal securities largely unexplored
Motivation: Long-term investment literature

- Long-term investment literature highlights the importance of
  
  A. Stochastic interest rates and inflation rates
  
  
  B. Time-varying prices of risk
  
  
- But, these papers do not account for
  
  - Labor income during the investment period
  - Portfolio constraints when risk premia are time-varying
Motivation: Life-cycle literature

- Recent life-cycle models consider the impact of labor income and omnipresent portfolio constraints
  
  Bodie, Merton, and Samuelson (1992), Campbell and Cocco (2003), Cocco, Gomes, and Maenhout (2005), Gomes and Michaelides (2005), Munk and Sorensen (2005), Viceira (2001)

- However, the financial markets considered are rather simple
  
  - Constant inflation rates
  - Constant bond risk premia

- The different role of nominal and inflation-linked bonds cannot be assessed

- Horizon effects, if any, driven by changes in human capital and time-variation in real rates
Main contributions of the paper

- This paper solves a long-term investment problem in which
  - The investor receives (stochastic) labor income during the investment period
  - Interest and inflation rates, as well as bond risk premia are time-varying

- We answer three main questions
  1. How does labor income affect the demand and economic value added of inflation-linked bonds (ILBs)?
  2. In absence of ILBs, what is the impact of (indexed) labor income on the demand for nominal bonds?
  3. What is the sensitivity of these results to time-variation in bond risk premia, idiosyncratic labor income risk, and correlations between labor income risk and financial risks
Incorporating labor income results in the following conclusions

1. Prominent role of ILBs substantially reduced, both in terms of portfolio choice and utility gains

2. But, ILBs remain an important asset class, in particular for conservative long-term investors

3. Duration optimal nominal bond portfolio lengthened

4. Correlations between labor income and expected inflation or stock returns are key parameters
Financial market, preferences, and labor income
Model Specification

- Real interest rate and expected inflation are modeled mean-reverting:
  \[ dr_t = \kappa_r (\delta_r - r_t) dt + \sigma'_r dZ_t \]
  \[ d\pi_t = \kappa_\pi (\delta_\pi - \pi_t) dt + \sigma'_\pi dZ_t \]

- Stock prices are modeled assuming that excess returns are i.i.d.:
  \[ dS_t / S_t = (\eta_S + R_t) dt + \sigma'_S dZ_t \]

- Inflation is modeled as an ARMA(1,1) process:
  \[ d\Pi_t / \Pi_t = \pi_t dt + \sigma'_\Pi dZ_t \]

- Prices of risk are affine in the real rate and expected inflation:
  \[ \Lambda_t(1) = \Lambda_{0(1)} + \Lambda_{1(1,1)} r_t \]
  \[ \Lambda_t(2) = \Lambda_{0(2)} + \Lambda_{1(2,2)} \pi_t \]
Model Specification

This structure of the model implies:

- Prices of nominal and inflation-linked bonds are exponentially affine in the real rate and expected inflation (Duffee, 2002)
- Real bond risk premia co-move with the real rate, while nominal bond risk premia co-move with both the real rate and expected inflation

Asset menus considered (cash available in all cases):

- Menu 1: 3Y nominal, 10Y nominal bonds, and 10Y ILBs
- Menu 2: 3Y nominal and 10Y nominal bonds
Preferences and labor income

- The investor derives utility from terminal real wealth:

\[ E_t \left( \frac{1}{1-\gamma} \left( \frac{W_T}{\Pi_T} \right)^{1-\gamma} \right) \]

- If the investor is endowed with labor income, the savings rate is set exogenously

- For instance, investors saving for retirement or a fixed savings rate induced by some form of precommitment

- Real labor income grows at a rate \( g \) and is subject to permanent shocks, in line with Viceira (2001):

\[ dY_t/Y_t = gdt + \sigma'\xi dZ_t^Y, \]

with \( Z^Y \) and \( Z \) possibly correlated
Implication of the estimation results
Implications of the estimation results

- Expected inflation far more persistent than the real rate
- Real rate and expected inflation are negatively correlated
- Implications for exposures of nominal and inflation-linked bonds to the real rate and expected inflation

![Chart showing real rate and expected inflation exposures](image)
## Implications of the estimation results

### Summary statistics

<table>
<thead>
<tr>
<th></th>
<th>Stocks</th>
<th>1Y</th>
<th>3Y</th>
<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk premium stocks</td>
<td>5.36%</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Risk premia nom. bonds</td>
<td>0.64%</td>
<td>1.25%</td>
<td>1.98%</td>
<td></td>
</tr>
<tr>
<td>Risk premia ILBs</td>
<td>0.42%</td>
<td>0.82%</td>
<td>0.97%</td>
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<tr>
<td>Inflation risk premium</td>
<td>22bp</td>
<td>43bp</td>
<td>101bp</td>
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<tr>
<td>Volatility stocks</td>
<td>15.98%</td>
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<tr>
<td>Volatility nom. bonds</td>
<td>1.76%</td>
<td>4.11%</td>
<td>11.71%</td>
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<tr>
<td>Volatility ILBs</td>
<td>1.70%</td>
<td>2.70%</td>
<td>3.08%</td>
<td></td>
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</tbody>
</table>
Simulation-based portfolio choice with labor income and portfolio constraints
Simulation-based portfolio choice

- Brandt, Goyal, Santa-Clara, and Stroud (2005, RFS), introduce a powerful simulation approach for solving portfolio problems, related to Longstaff and Schwartz (2001)

- BGSS05 propose
  1. Replace conditional expectations by projections on basis functions
     \[ \mathbb{E}(Y \mid X_t) = \zeta(x)' \cdot f(X_t), \]
     with \( X_t \) the value of the state variables at time \( t \) and \( f \) a set of basis functions
  2. Simulate paths of returns and state variables and estimate the projection coefficients, \( \zeta(x)' \), exploiting the cross-sectional information
  3. Optimize in all branches (time-consuming!)
Simulation-based portfolio choice

To accelerate the final step, an iterative procedure is proposed based on a fourth order expansion of the utility index.

However, convergence is not ensured (DeTemple, Garcia, and Rindisbacher, 2003, 2005).

We by-pass this problem by observing that the regression coefficients, \( \zeta(x) \), are smooth function of the portfolio weights, \( x \).

Hence, we parameterize the regression coefficients:

\[
\zeta(x) = \Psi' h(x),
\]

with \( h \) a set of basis functions.
Labor income and the demand for long-term bonds
The main effects of introducing labor income

If real labor income risk is uncorrelated with financial risks

- Labor income can be viewed upon as a mixture between a particular position in real bonds and an idiosyncratic risk component

- This first component tends to reduce the demand for real bonds since the investors evaluates the total portfolio

- But, the idiosyncratic risk component induces an effective increase in the risk aversion parameter, and hence an increase in the demand for real bonds

**Labor income and demand long-term bonds**

- **Impact of the coefficient of RRA \((T = 10)\) on asset allocation**

![Impact of RRA on Asset Allocation](chart)

- **Motivation**
  - Financial market, preferences, and labor income

- **Implication of the estimation results**

- **Simulation-based portfolio choice with labor income and portfolio constraints**

- **Labor income and the demand for long-term bonds**
  - The main effects of introducing labor income
  - Labor income and demand for long-term bonds
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  - Utility gains
  - What makes ILBs so valuable?
  - Impact of labor income: Utility gains

- **Correlated labor income risk**

- **Conclusions**
Labor income and demand long-term bonds

- Impact of the coefficient of RRA \( T = 10 \) on asset allocation

![Graph showing asset allocation for different RRA values with and without labor income (LI).](image-url)
Impact of the investment horizon ($\gamma = 5$) on asset allocation
Labor income and demand long-term bonds

- Impact of the investment horizon ($\gamma = 5$) on asset allocation

![Bar chart showing asset allocation for different investment horizons with and without labor income (LI).]

- T=5
- T=5, LI
- T=10
- T=10, LI
- T=30
- T=30, LI

Legend:
- R10Y
- N10Y
- Cash

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Impact on nominal bond demand for different risk attitudes ($T = 10$) on asset allocation

- Impact on nominal bond demand for different risk attitudes ($T = 10$) on asset allocation

![Bar chart showing asset allocation for different RRA values and bond maturities: N3Y, N10Y, and Cash.](image)
Labor income and demand long-term bonds

- Impact on nominal bond demand for different risk attitudes ($T = 10$) on asset allocation
Utility gains

Utility gains induced by access to ILBs without labor income

![Bar Chart](image.png)

- Utility gains induced by access to ILBs without labor income
- What makes ILBs so valuable?
- Impact of labor income: Utility gains

Correlated labor income risk

Conclusions
**What makes ILBs so valuable?**

- The (unconditional) price of real interest rate risk is higher than of expected inflation risk

- Aggressive investors exploit the attractive risk-return trade-off

- Conservative investors are unwilling to bear inflation and interest rate risk; long-term inflation-linked bonds provide a hedge against both

- Hence, both aggressive and conservative investors are better off by having access to inflation-linked bonds
Impact of labor income: Utility gains

- Reduction in utility gains from having access to ILBs with labor income

![Bar Chart]

- Impact of labor income: Utility gains
  - Reduction in utility gains from having access to ILBs with labor income

**Motivation**
- Financial market, preferences, and labor income

**Implication of the estimation results**
- Simulation-based portfolio choice with labor income and portfolio constraints
- Labor income and the demand for long-term bonds
  - The main effects of introducing labor income
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  - What makes ILBs so valuable?

**Correlated labor income risk**

**Conclusions**
Correlated labor income risk
Correlated labor income risk

- If real labor income risk is correlated with financial risks, we have in general
  - Assets can be used to offset unfavorable changes in labor income (*hedge effect*)
  - The (implicit) value of labor income changes if the financial risks are priced (*value effect*)

- The correlation with inflation rates may deteriorate or enhance the role of labor income as inflation hedge

- Correlations likely to be industry dependent
Correlation labor income risk and exp. inflation risk

-60% -40% -20% 0% 20% 40% 60%

-60% -40% -20% 0% 20% 40% 60%

R10Y N10Y Cash
Conclusions
Main conclusions

- We propose a realistic financial market model and incorporate labor income into the investment problem.

- We find that the introduction of labor income:
  1. Prominent role of ILBs substantially reduced, both in terms of portfolio choice and utility gains (like 30 – 40%)
  2. But, ILBs remain an important asset class, in particular for conservative long-term investors.
  3. Duration optimal nominal bond portfolio lengthened.
  4. Correlations between labor income and expected inflation or stock returns are key parameters.
Main conclusions

- In the presence of equities, the optimal portfolio is tilted substantially towards equities, but the stock-bond mix still decreasing in risk aversion.

- The conclusions are robust to time-variation in bond risk premia, the amount of idiosyncratic labor income risk, and correlations between labor income innovations and financial risks.

- Results have been derived by extending recently developed simulation-based approaches to solve portfolio choice problems.