The market price of one-touch options in foreign exchange markets

What is a one-touch option?

Internationally active market participants are always subject to changing foreign exchange rates and hedge their exposure by trading an immense variety of options worldwide. EUR/USD is considered the most liquid traded currency pair in the foreign exchange market.

Besides vanilla (European style put and call) options, the so-called *first generation exotics* have become standard derivative instruments. We distinguish between

(a) vanilla options that knock in or out if the underlying hits a barrier (or one of two barriers) and
(b) all kind of *touch options* which include one-touch/no-touch options and pay a fixed amount of either USD or EUR if the spot ever/never trades at or beyond the touch-level and zero if it doesn't. Double-touch options work the same way with two barriers.

In this article we would like to take a detailed look at the pricing of one-touch options, which are often also called *binary* or *digital* options, *hit options* or *rebate options*. They trade as listed[1] and are over-the-counter products.

Touch-time is the first time the underlying trades at or beyond the touch-level. The barrier determination agent, which is specified in the contract, determines the touch-event. The *Foreign Exchange Committee*[2], which includes representatives of major financial institutions engaged in foreign currency trading worldwide, has recommended a set of best practices for the barrier options market to the foreign exchange community and is planning to publish a revision of the International Currency Options Master Agreement (ICOM) reflecting some new recommendations. Following, some of its key regulations

- The barrier needs to have been traded within the foreign exchange markets
- All transactions have to be completed between 5:00 A.M. Sydney time on Monday and 5:00 P.M. New York time on Friday
- All transactions need to have a commercial size (generally at least $3 million in liquid markets)

The barrier or touch-level is usually monitored continuously over time.

Applications of one-touch options

Speculative market participants like to use one-touch options as bets on a rising or falling exchange rate. Clients, who prefer to hedge, trade one-touch options as a rebate in order to secure themselves a compensation in case their strategy doesn't work out. One-touch options are also often integrated into structured products to increase returns on forward and interest rates.

Quotation conventions and bid-ask spreads

If the payoff is at maturity, the undiscounted value of the one-touch is the touch probability under the risk-neutral measure. The market standard is to quote the price of a one-touch in per cent of

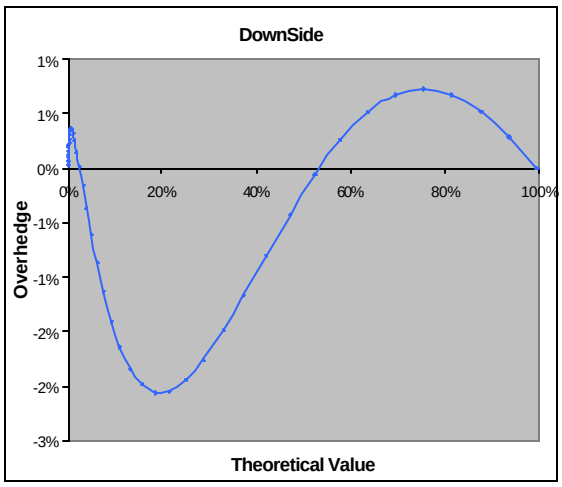
[1] Often named *Hit Warrants*, e.g. on Reuters page COBAWAR600
[2] For details see http://www.ny.frb.org/fxc/fxann000217.html

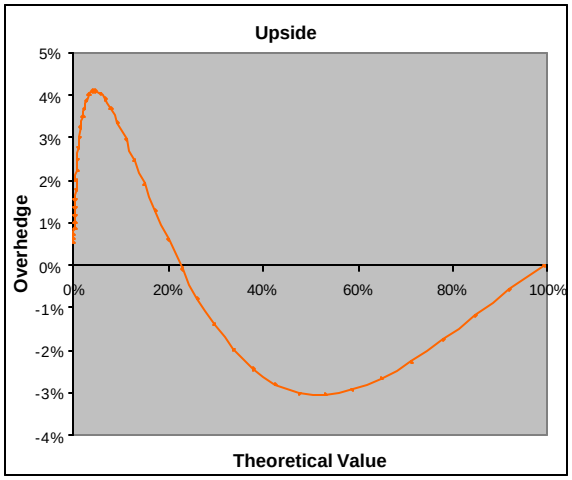
the payoff, which is a number between 0% and 100%. The price of a one-touch depends on the overhedge (explained below) and the bid-ask spread. The spread in turn depends on the currency pair and the client. Interbank trading spreads are usually between 2% and 4% for liquid currency pairs.

Cost of trading and its implication on the market price of one-touch options

The theoretical value (TV) of a one-touch is based on the standard Black-Scholes model for the underlying exchange rate of EUR/USD, a geometric Brownian motion with constant parameters EUR interest rate, USD interest rate and volatility. The TV is defined as the expectation under the risk neutral measure of the discounted payoff of the one-touch.[3] The tradable price is the sum of the TV and the overhedge. Two typical examples, one for an upper touch level in EUR/USD, one for a lower touch level, are shown in the charts below.

Clearly, there is no overhedge for one-touch options with a TV of 0% or 100%, but it is worth noting that low-TV one-touch options can be twice as expensive, and sometimes even more expensive, than their TV. SuperDerivatives.com has become a standard reference for pricing exotic FX options up to the market. The overhedge arises from the cost of risk managing the one-touch. In the Black-Scholes model the only source of risk is the underlying exchange rate, whereas the volatility and interest rates are assumed constant. However, volatility and rates themselves vary in dependence of e.g. spot movements as the options trader's positions are exposed to unstable vega and rho (change of the value with respect to volatility and rates). For short dated options, the interest rate risk is negligible compared to the volatility risk. Hence the overhedge of a one-touch is a reflection of a trader's cost of managing his vega exposure.

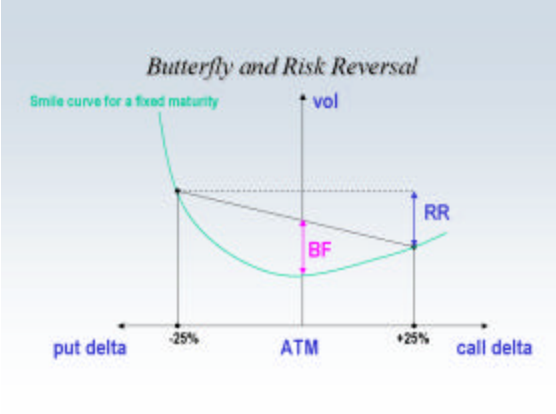




[3] for details see *Foreign Exchange Risk*, ed. by J. Hakala and U. Wystup, Risk Books 2002.

Example

Let's reflect over a one-year one-touch in USD/JPY with payoff in USD. Our assumed market parameters are a spot of 117.00 JPY per USD, JPY interest rate 0.10%, USD interest rate 2.10%, volatility 8.80%, 25delta risk reversal –0.45%[4], 25delta butterfly 0.37%[5]. The notion of risk reversals and butterflies are illustrated in the graph shown below.



The touch level is 125.00, and the TV is at 38.2%. If we only hedge the vega exposure we need to consider two main risk factors:

(1) the change of vega as the spot changes, often called vanna and
(2) the change of vega as the volatility changes, often called volga (or volgamma or vomma).

To hedge this exposure we need to deal with both effects separately.
The vanna of the one-touch is –9.0, the vanna of the risk reversal is 4.5. We therefore need to sell 2 (=-9/4.5) risk reversals, and for each of them we need to pay 0.15% of the USD amount, which causes an overhedge of 0.3%. The volga of the one-touch is –1.0, the volga of the butterfly is 0.035. So we need to sell 28 (=-1/0.035) butterflies, each of which pays us 0.27% of the USD amount, which causes an overhedge of -7.7%. Therefore, the overhedge is –7.4%.
However, we will get to the touch level with a risk-neutral probability of 36%, in which case we would have to unwind the hedge. Therefore the total overhedge is -64%*7.7%=-4.7%. This leads to a mid market price of 33.5%.

There are different beliefs among market participants about the unwinding cost. Other prices for one-touch options can be caused by different vega profiles in the trader's portfolio, a marketing campaign or a hidden additional sales margin.

Comment on the cost of risk reversals and butterflies

The cost of the risk reversal mentioned above is computed as follows: Difference of the call with correct implied volatility and the call with at-the-money volatility minus the difference of the put with correct implied volatility and the put with at-the-money volatility. It is easy to see that this can be well approximated by the vega of the at-the-money vanilla times the risk reversal in terms of volatility. Similarly, the cost of the butterfly can be approximated by the vega of the at-the-money volatility times the butterfly in terms of volatility.

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[4] This means that a 25delta USD call is 0.45% cheaper than a 25delta USD put in terms of implied volatility.
[5] This means that a 25delta USD call and 25delta USD put is on average 0.37% more expensive than an at-the-money option in terms of volatility

Further applications

The method illustrated above shows how important the current smile of the vanilla options-market is for the pricing of simple exotics. Similar types of approaches are commonly used to price other exotic options. For long-dated options the interest rate risk will take over the lead in comparison to short dated options where the volatility risk is dominant. As a matter of consistency the method can also reproduce the smile for the vanilla market.

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