An Introduction to Financial Networks and Systemic Risk

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Some References

1. Andrew G Haldane's 2009 talk "Rethinking the Financial Network";
2. Tobias Adrian and Hyun Song Shin, "Liquidity and Leverage", 2009.
3. Darrell Duffie, "How Big Banks Fail ", PUP 2011.
4. C. Upper, "Simulation methods to assess the danger of contagion in interbank markets", Journal of Financial Stability, forthcoming, 2011.
5. L. Eisenberg and T. H. Noe,"Systemic risk in nancial systems". Management Science, 47(2):236 249, 2001.
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4. Duncan Watts, "A simple model of global cascades on random networks", Proc. Nat. Academy Sciences, **99**, 2002.
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2. T. Hurd, J. Gleeson, “A framework for analyzing contagion in banking networks”, submitted paper, 2011.
3. P. Gai, A. Haldane, S. Kapadia, “Complexity, Concentration and Contagion”, Journal of Monetary Economics, **58**, 2011.

Systemic Risk: various definitions

1. “the likelihood of a sudden, usually unexpected, event that disrupts information in financial markets, making them unable to effectively channel funds to those parties with the most productive investment opportunities” (Mishkin 1995).
2. “probability that cumulative losses will accrue from an event that sets in motion a series of successive losses along a chain of institutions or markets comprising a system. . . . That is, systemic risk is the risk of a chain reaction of falling interconnected dominos” (Kaufman 1995).
3. “the risk that the failure of a participant to meet its contractual obligations may in turn cause other participants to default with a chain reaction leading to broader financial difficulties” (Bank for International Settlements 1994).

1. “In the payments system, systemic risk may occur if an institution participating on a private large- dollar payments network were unable or unwilling to settle its net debt position. If such a settlement failure occurred, the institutions creditors on the network might also be unable to settle their commitments. Serious repercussions could, as a result, spread to other participants in the private network, to other depository institutions not participating in the network, and to the nonfinancial economy generally.” (Federal Reserve System 2001, 2)
2. U.S. Commodity Futures Trading Commission: “[t]he risk that a default by one market participant will have repercussions on other participants due to the interlocking nature of financial markets. For example, Customer A’s default in X market may affect Intermediary B’s ability to fulfill its obligations in Markets X, Y, and Z.”

Systemic Risk: S. L Schwarcz’ definition

“ The risk that (i) an economic shock such as market or institutional failure triggers (through a panic or otherwise) either (X) the failure of a chain of markets or institutions or (Y) a chain of significant losses to financial institutions, (ii) resulting in increases in the cost of capital or decreases in its availability, often evidenced by substantial financial-market price volatility.”

Cascades of shocks to banks plus general drop in liquidity

The 2009 Perspective

Andrew G Haldane's 2009 talk "Rethinking the Financial Network" is a brilliant summary of the nature of networks. He compares the 2002 SARS epidemic to the 2008 collapse of Lehman Bros. In both cases:

- an external event strikes;
- panic ensues and system seizes up;
- "collateral damage" is wide and deep;
- in hindsight, trigger event was modest;
- dynamics was chaotic.

Manifestation of a complex adaptive system

Haldane: Rethinking the Financial Network

> *[Haldane 2009, p. 3] Both events [the failure of Lehman Brothers and the unfolding of the SARS epidemic] were manifestations of the behavior under stress of a complex, adaptive network. Complex because these networks were a cats-cradle of interconnections, financial and non-financial. Adaptive because behavior in these networks was driven by interactions between optimizing, but confused, agents. Seizures in the electricity grid, degradation of ecosystems, the spread of epidemics and the disintegration of the financial system: each is essentially a different branch of the same network family tree.*

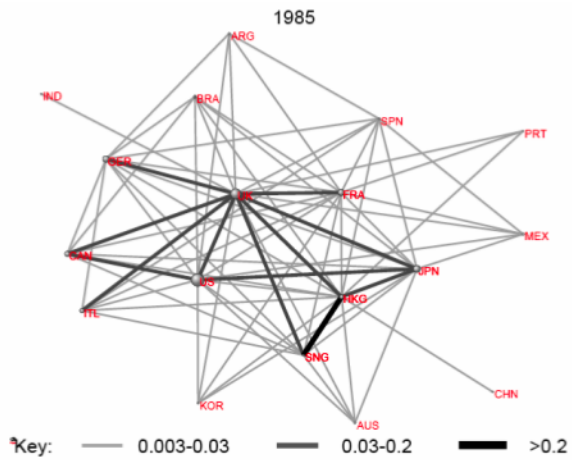
Complexity and Stability

What went wrong with the financial network?

- increasing complexity;
- decreasing diversity.

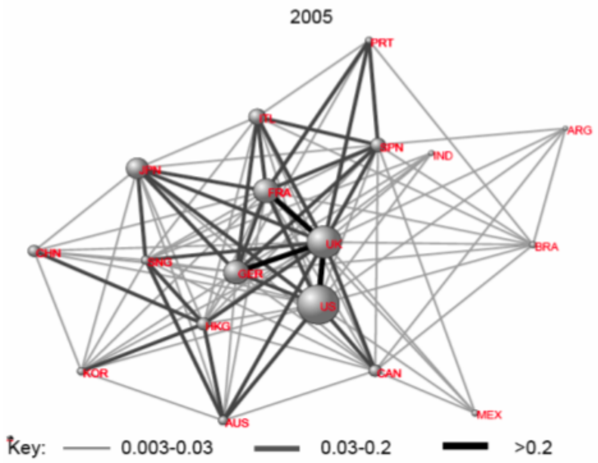
These two facts imply fragility and ring alarm bells for ecologists, engineers, geologists.

Global Financial Network 1985



(line denotes link strength as fraction of total GDP)

Global Financial Network 2005



Connectivity and Stability

Highly connected networks may be "robust yet fragile":

- In a network, connections may be either shock absorbers or shock amplifiers;
- There may be a "tipping point" that separates these two regimes.
- A fat-tailed "degree distribution" (the number of links per node) implies robustness to random shocks but vulnerability to shocks that target highly connected nodes.

Feedback and Stability

How do agents respond to a crisis?

- Epidemics: "hide" vs "flight";
- Finance: "hoard liquidity" vs "sell assets".

In finance, both responses are rational, but make the systemic problem worse. Government intervention is important to provide liquidity when it is most needed!

Uncertainty and Stability

Networks generate chains of claims. At times of stress, these chains can amplify uncertainties about true counterparty exposures.

- In good times, counterparty risk is small, and thus "Knightian" uncertainty is small: stability improves with connectivity;
- In bad times, counterparty risk can be large and uncertain, due to the complicated web: stability declines with connectivity.

Innovation and Stability

Financial innovation, particularly "securitization", created instability.

- CDOs, MBSs, RMBSs and similar high dimensional products became pervasive internationally;
- The structure of these contracts was opaque, not transparent;
- They dramatically expanded the size and scope of the precrisis bubble (see Shin 2009, "Securitisation and Financial Stability");
- They dramatically increased the connectedness and complexity of the network;
- "Adverse selection" made them hard to evaluate.
- "With no time to read the small-print, the instruments were instead devoured whole. Food poisoning and a lengthy loss of appetite have been the predictable consequences. "

Diversity and Stability

- In ecosystems, biodiversity is known to improve stability;
- In "Great Moderation" period, financial diversity has been reduced;
- Pursuit of returns lead to many agents following similar strategies: portfolio correlations grew to $> 90\%$.
- Risk management regulation (a la Basel II) lead to similar risk management strategies for banks;
- As a result, bank balance sheet became increasingly homogeneous;

Finance became almost a "monoculture", and vulnerable to "viral infection".

Haldane: Summary

- Networks arising in ecology, engineering, the internet, finance, etc are complex and adaptive;
- They typically are "robust yet fragile";
- There is a role for intervention to create more stable networks;
- Key determinants for financial stability may be deduced by studying other types of networks.

What properties of the financial network most influence stability?

Nature of Banking Balance Sheets

From "Liquidity and Leverage" by Tobias Adrian and Hyun Song Shin 2009.

> *[Adrian and Shin] In a financial system in which balance sheets are continuously marked to market, asset price changes appear immediately as changes in net worth, eliciting responses from financial intermediaries who adjust the size of their balance sheets. We document evidence that marked-to-market leverage is strongly procyclical.*

Balance Sheet Arithmetic: a Household

- Suppose household is worth $A = 100$ (asset)...
- and mortgage value is $D = 90$ (debt):
- then net worth $E = A - D = 10$ (equity)
- and leverage $L = A/E = 10$.

Assets	Liabilities
100	10
	90

What happens to leverage as total assets A fluctuate?

Leverage for a Passive Investor

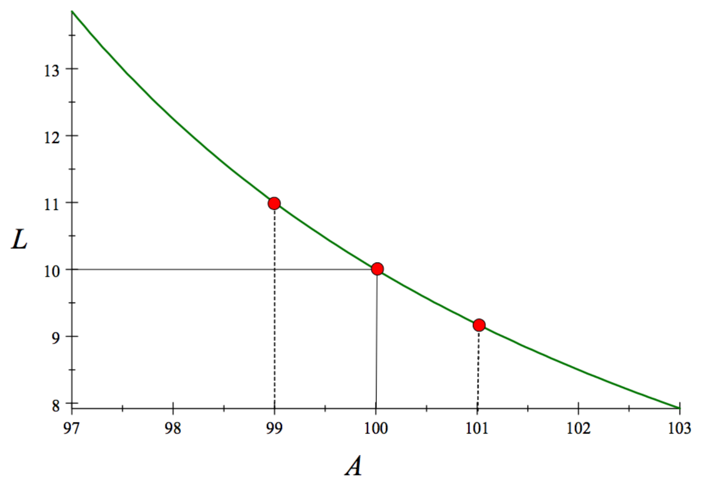


Figure: Leverage for a Passive Investor

Quarterly percentage changes in household leverage and asset value for period 1963-2006

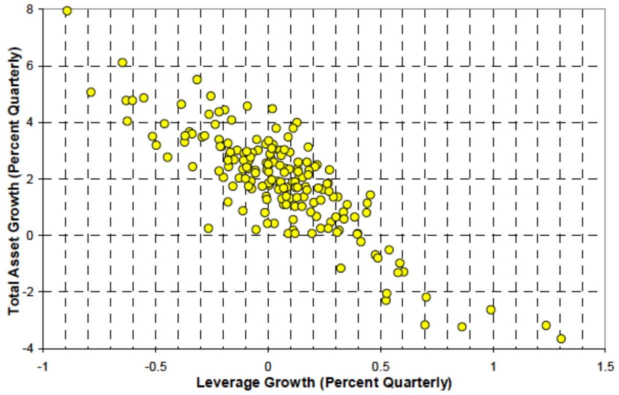


Figure 2.2: Total Assets and Leverage of Household

Non-Financial/Non-Farm Corporates

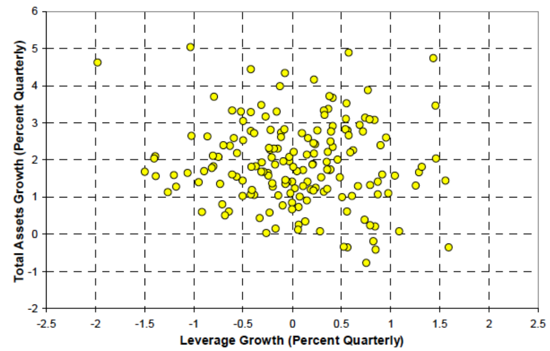


Figure 2.3: Total Assets and Leverage of Non-financial, Non-farm C

Commercial Banks

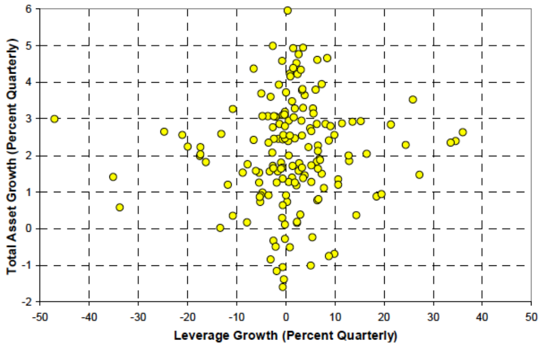


Figure 2.4: Total Assets and Leverage of Commercial Banks

Investment Banks

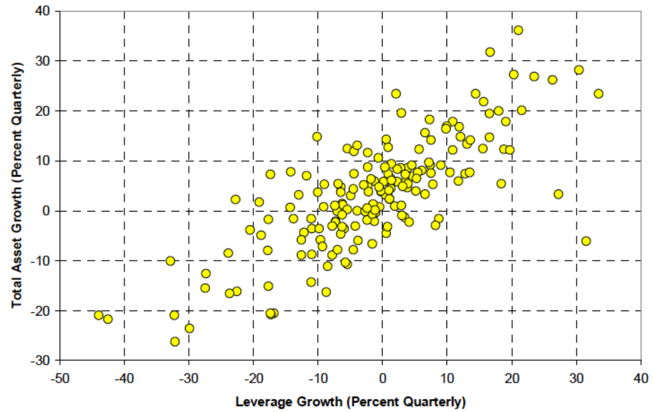


Figure 2.5: Total Assets and Leverage of Security Brokers and Deal

Active Balance Sheets: Constant Leverage

Commercial bank that maintains $L = 10$:

Assets	Liabilities
securities 100	equity 10 debt 90

- Suppose asset value rises: $A \to 101...$
- new leverage: $L = 101/11 = 9.18...$
- raise debt by 9: $D \to 99...$
- buy 9 units of new assets: $A \to 110...$
- new leverage $L = 110/11 = 10.$

1% rise in security values leads to increase of 10% in assets:

demand curve is upward sloping!

Imperfectly liquid markets

If increase in demand leads to increase in security price:

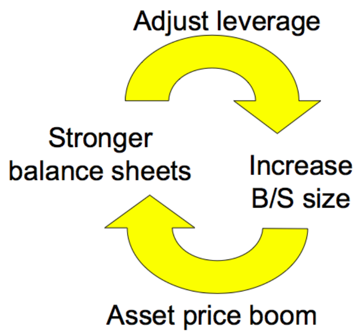


Figure: Leverage Spiral in an Upturn

Imperfectly liquid markets: (ctd)

If decrease in demand leads to decrease in security price:

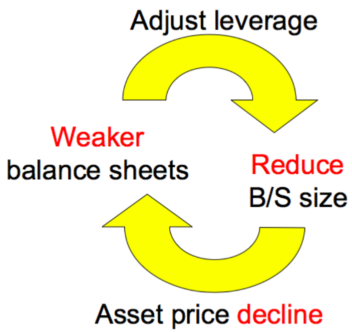


Figure: Leverage Spiral in a Downturn

Investment Banks 1997 Q1-2007 Q1

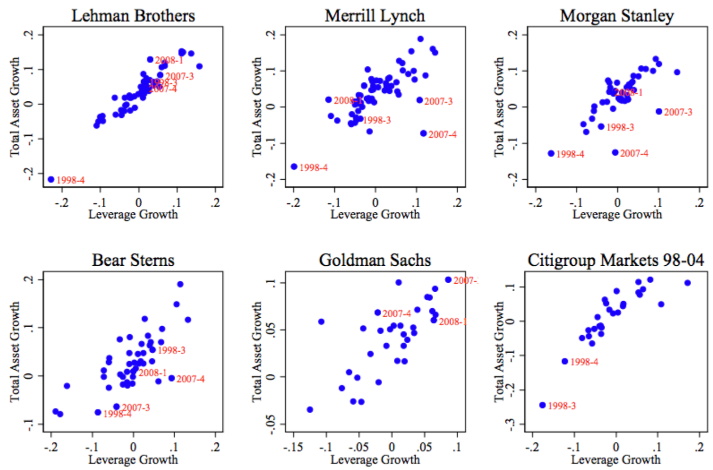


Figure 3.2: Total Assets and Leverage

Growth of the Investment Bank and Hedge Fund Sectors

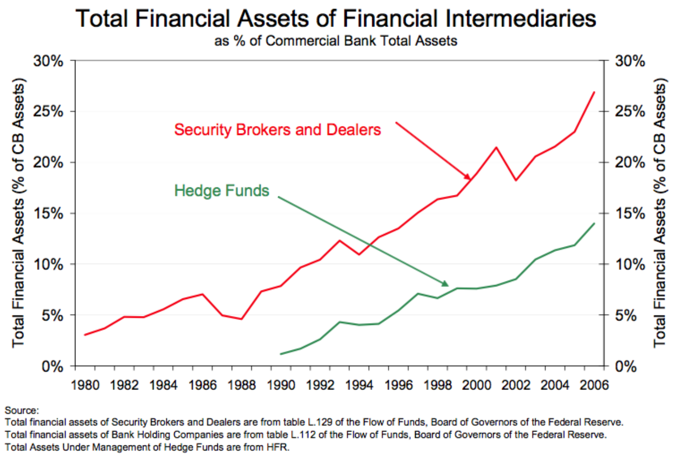


Figure 3.1: Total Financial Intermediary Assets

Dealer Bank

Following D. Duffie, "How Big Banks Fail ", PUP 2011, Dealer Banks are

- Financial institutions that intermediate the backbone markets for securities and over-the-counter (OTC) derivatives.
- They also act as "Prime Brokers" for hedge funds;
- and underwrite securities issuances;
- and trade speculatively on their own behalf ("proprietary trading", like a hedge fund).

Their failure (eg Lehman Bros. 2008) is a major component of "systemic risk".

Dealer Bank: A Stylized Balance Sheet

Assets	Liabilities
Trading assets	Short positions
Reverse repos	Repos
Other assets	Long term debt
	Shareholder equity

Long term debt is a small fraction of the balance sheet. Apparently, traditional unsecured overnight loans are a small part of a dealer bank's balance sheet.

Repos

Overnight collateralized interbank loans:

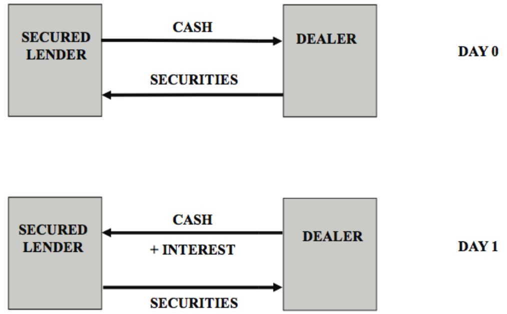
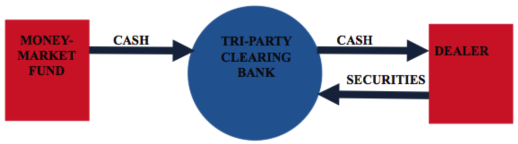


Figure 2.1: A repurchase agreement, or "repo."

A "haircut" of 1-5% is usual (i.e $105 in security to raise $100). Repos often used for levered financing: securities are used as collateral to purchase further securities, hence $L = \frac{1+h}{h}$.

Triparty Repos

Often an intermediary "clearing bank" stands in the repo contract:



Interbank Exposures to OTC Derivatives: June 2009

Asset class	Exposure ($ billions)
Credit default swap	2,987
Interest Rate Swap	15,478
Equity Linked	879
Foreign Exchange	2,470
Commodity	689
Unallocated	2,868
Total	25,372
Total after netting	3,744

CDS Contracts

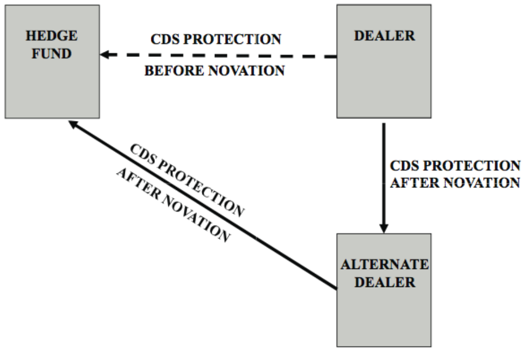


Figure: Novation of a CDS

Off-Balance-Sheet Financing

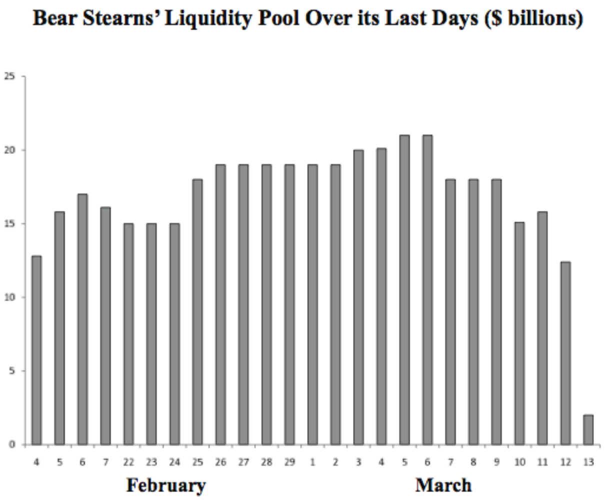
- Banks can purchase residential mortgages and other loans...
- Financing them by selling them to a "special purpose entity (SPE)".
- The SPE pays for the assets with the proceeds of debt that it issues to third-party investors;
- Principal and interest payments on the SPEs debt are paid from the cash flows received from the assets that it has purchased from the sponsoring bank.
- Under some conditions, the SPEs assets and debts are treated as "remote" from the bank.
- For example, at June 2008, Citigroup, Inc. reported over $800 billion in off-balance-sheet assets held in such qualified special purpose entities.

Example: Structured Investment Vehicle

An SIV finances residential mortgages and other loans with short-term debt sold to investors such as money-market funds.

- In 2007 and 2008, when home prices fell dramatically in the United States and sub-prime residential mortgage defaults rose, the solvency of many SIVs was threatened.
- Some short-term creditors recognized the solvency concerns and failed to renew their loans to SIVs...
- Forcing some large dealer banks to bail out investors...
- Some of these banks might in hindsight have preferred to allow these investors to fend for themselves!
- Shin 2009: "... far from passing the hot potato down the chain to the greater fool next in the chain, the large financial intermediaries ended up keeping the hot potato."

Bank Failure Mechanisms



Reactions by OTC Derivatives Counterparties

A derivative counterparty may try to reduce exposure to a failing dealer by:

- taking a loan from the dealer; or
- Restriking in-the-money options at-the-money; or
- “novating” derivative to a third party;
- attempting to enter a new trade that takes cash out of the bank.

In addition to being direct stresses, such requests on Bear-Stearns were a strong signal of that bank’s distress. The third party may even decide to refuse the novation.
Another factor is that credit downgrades of a bank typically force higher collateral amounts or early termination on derivative positions, thus leaking further cash.

The Flight of Short-Term Creditors

Repos are typically over-night loans, and are used by dealer banks for a large fraction of their borrowing. Repo creditors will likely make huge trouble for a distressed bank. Creditors may:

- Raise haircuts (in 2008, average haircuts on US treasuries rose from $< 2\%$ to $> 20\%$);
- Dispute collateral valuations (since often market valuations are falling quickly);
- Most severely, refuse to renew the repo.
- Also, a clearing bank might withdraw tri-party repo and other clearing services.

The Flight of Creditors Continues...

In response, the bank needs to look elsewhere for cash:

- make use of existing lines of credit;
- sell assets (perhaps enflaming a “fire sale”);
- look to a “lender of last resort”, such as ECB or US Fed.

Disappearance of Prime Brokerage Clients

Prime broker is a large fee generator for some dealer banks. But banks also finance themselves using their clients cash and securities accounts as collateral.

- Such accounts may be “unsegregated” or partially segregated, allowing the bank to raise cash (through repo type contracts);
- Ex: such “pledgable” securities in Morgan Stanley dropped from \$B 877 to \$B 294 from Aug 08 to Nov 08, forcing MS to raise approximately \$B 80 in cash financing.

Loss of Clearing and Settlement Privileges

- Clearing banks typically offer "daylight exposure" to client banks (ie intraday overdrafts without interest). What matters is the client's FedFunds balance at 18:30 each day;
- However, if the client is failing, the clearing bank may apply "right to offset", which means denial of this overdraft.
- For example, on Sept 11, 2008, JP Morgan demanded an additional $5 billion in cash collateral to cover its daylight exposure to Lehman; on Sept 15, Lehman went bankrupt.

Studies of Specific Financial Systems

"Simulation methods to assess the danger of contagion in interbank markets" by Christian Upper (2011) reviews 15 recent studies of specific financial systems.

Channels for Contagion: Liability Side

Possible channels of contagion in the banking system.

Channel	References
Liability side	
Bank runs	
Multiple equilibria/fear of other withdrawals	Diamond and Dybvig (1983), Temzelides (1997), Goldstein and Pauzner (2004)
Common pool of liquidity	Aghion et al. (2000), Acharya and Yorulmazer (2008b), Diamond and Rajan (2005), Brunnermeier and Pedersen (2009)
Information about asset quality	Chen (1999), Acharya and Yorulmazer (2008a)
Portfolio rebalancing	Kodres and Pritsker (2002)
Fear of direct effects	Dasgupta (2004), Iyer and Peydró-Alcalde (2005), Lagunoff and Shreft (2001), Freixas et al. (2000)
Strategic behaviour by potential lenders	Acharya et al. (2008)

Channels for Contagion: Asset Side

Asset side	
Direct effects	
Interbank lending	Rochet and Tirole (1996), *studies reviewed in this paper*
Payment system	Humphrey (1986), Angelini et al. (1996), Bech and Garratt (2006)
Security settlement	Northcott (2002)
FX settlement	Blavarg and Nimander (2002)
Derivative exposures	Blavarg and Nimander (2002)
Equity cross-holdings	
Indirect effects	
Asset prices	Cifuentes et al. (2005), Fecht (2004)

Assumptions These Studies Make

Upper 2011 identifies the type of assumptions implicit in such studies.

1. Banks have limited liability.
   Virtually all banking systems feature institutions whose liabilities are either explicitly or implicitly guaranteed by the government or by other players.
2. Nonbank liabilities are senior to interbank liabilities.
   This is an open issue. Falsely assuming that all interbank claims are junior to claims by non-banks will overstate both the possibility and the severity of contagion.
3. Losses on interbank assets are shared equally across lenders.
   In fact, biases can go into either direction.
4. Nonbank assets can be sold at their book value.
   Failing banks liquidate their assets, which would tend to depress prices and thus increase the severity of contagion.

Balance Sheets

Eisenberg-Noe 2001 identifies the stylized elements of a financial system consisting of N "banks":

- The assets A_i of bank i
  1. *external assets* $\bar{\mathsf{Y}}_i$
  2. *internal assets* $\bar{\mathsf{Z}}_i$
- The liabilities of the bank i
  1. *external debts* $\bar{\mathsf{D}}_i$
  2. *internal debt* $\bar{\mathsf{X}}_i$
  3. *equity*, defined by $e_i = \bar{\mathsf{Y}}_i + \bar{\mathsf{Z}}_i - \bar{\mathsf{D}}_i - \bar{\mathsf{X}}_i \geq 0$
- $\bar{L}_{ij}$, the amount i owes j. Note the constraints

$$\bar{\mathsf{Z}}_i = \sum_j \bar{L}_{ji}, \quad \bar{\mathsf{X}}_i = \sum_j \bar{L}_{ij}, \quad \sum_i \bar{\mathsf{Z}}_i = \sum_i \bar{\mathsf{X}}_i$$

These represent "notional amounts".

Default cascades

- Healthy banks maintain e_i/A_i above a fixed threshold Λ_i.
- Following a bank specific catastrophic event, assets of a bank may suddenly contract by more than the equity cushion and bank becomes insolvent.
- The assets of an insolvent bank must be quickly liquidated;
- Any proceeds go to pay off that bank's creditors, in order of seniority.
- Resultant shortfalls can weaken creditors, and some further banks may default, creating a default cascade.

A Simple Liquidation Mechanism

Version A: external debt is senior to internal debt.

- Define $\pi_{ij} = \bar{L}_{ij}/\bar{\mathsf{X}}_i$;
- Let p_i be amount available to pay i's internal debt
- p_i is split amongst creditor banks in proportion to π_{ij}: bank j receives $\pi_{ij} p_i$.
- Given $\mathbf{p} = [p_1, \ldots, p_N]$, the clearing conditions are

$$p_i = \begin{cases} 0 & \text{if } \mathsf{Y}_i + \sum_j \pi_{ji} p_j - \bar{\mathsf{D}}_i < 0 \\ \min(\mathsf{Y}_i + \sum_j \pi_{ji} p_j - \bar{\mathsf{D}}_i, \bar{\mathsf{X}}_i) & \text{if } \mathsf{Y}_i + \sum_j \pi_{ji} p_j - \bar{\mathsf{D}}_i \geq 0 \end{cases}$$

-

$$p_i = F_i^{(A)}(\mathbf{p}) := \min(\bar{\mathsf{X}}_i, \max(\mathsf{Y}_i + \sum_j \pi_{ji} p_j - \bar{\mathsf{D}}_i, 0))$$

Another Simple Liquidation Mechanism

Version B: external and internal debt have equal seniority.

- Define $\tilde{\pi}_{ij} = \bar{L}_{ij}/(\bar{\mathsf{D}}_i + \bar{\mathsf{X}}_i)$.
- Let $\tilde{p}_i$ be amount available to pay i's total debt:
- Then bank j receives $\tilde{\pi}_{ji}\tilde{p}_i$ and clearing conditions are:

$$\tilde{p}_i = F_i^{(B)}(\tilde{\mathbf{p}}) := \min(\bar{\mathsf{D}}_i + \bar{\mathsf{X}}_i, \mathsf{Y}_i + \sum_j \tilde{\pi}_{ji}\tilde{p}_j), \quad i = 1, \dots, N.$$

A Third Simple Liquidation Mechanism

Most simply, Version C supposes that the recovery from any insolvent bank is zero.

- Let p_i be amount available to pay i's internal debt;
- The clearing conditions are:

$$p_i = F_i^{(C)}(\mathbf{p}) := \bar{\mathsf{X}}_i \Theta(\mathsf{Y}_i - \bar{\mathsf{D}}_i + \sum_j \pi_{ji} p_j).$$

Under each of these settlement mechanisms, any solution $\mathbf{p} = (p_1, \dots, p_N) \in \mathbb{R}_+^N$ of the clearing conditions is called a "clearing vector".

Fixed Point Theorem

Proposition

Consider a financial system with $\mathsf{Y} = [\mathsf{Y}_1, \dots, \mathsf{Y}_N], \bar{\mathsf{D}} = [\bar{\mathsf{D}}_1, \dots, \bar{\mathsf{D}}_N]$ *and matrix* $\bar{L} = (\bar{L}_{ij})_{i,j=1\dots,N}$. *Then the mappings* $F^{(A)}, F^{(B)}, F^{(C)} : \mathbb{R}_+^N \to \mathbb{R}_+^N$ *have at least one clearing vector or fixed point* $\mathbf{p}^*$. *If in addition the system is "regular" (a natural economic constraint on the system), the clearing vector is unique.*

Proof: Existence is a straightforward application of the Tarski Fixed Point Theorem.

Clearing Algorithm: Example

Suppose

$$\bar{\mathsf{Y}} - \bar{\mathsf{D}} = [1, 1, 1], \quad \bar{L} = \begin{pmatrix} 0 & 0 & 2 \\ 3 & 0 & 1 \\ 3 & 1 & 0 \end{pmatrix}$$

Further Assumptions Made

5 Banks spread their lending as evenly as possible given the assets and liabilities reported in the balance sheets of all other banks.
This is far from true.

6 Contagion is only driven by domestic exposures.
Assuming away contagion from abroad will lead to an underestimation of both the possibility and the severity of contagion.

Summary of Upper 2011

He identifies two major shortcomings:

- An exaggerated focus on scenarios involving idiosyncratic failure of a single bank, rather than a market shock;
- More important is the absence of "behavioural" foundations that preclude different channels for contagion. These studies assume "Banks sit tight as problems of their counterparties mount". We have seen that "asset hoarding" and "selling assets" are both rational responses that make systemic risk higher.

Elsinger-Lehar-Summers 2006 Overview

Several papers by these authors study financial systems of the UK and Austria in period 2003-2005. These provided perhaps the most complete systemic models prior to the crisis, and give a good case study.

- Adopt Version A of the EN2001 accounting framework ;
- $N = 12$ nodes are 10 large UK banks, aggregated small UK banks, aggregated foreign banks;
- From Bank of England data they infer current state of network $(L, Y - D)$;
- They postulate a stochastic model for Y_t, D_t based on credit risk methods and equity data.
- They assume L is constant over one year.
- They run 10^5 simulations of $(L, Y_T - D_T)$ for $T = 1$, and in each case compute the EN2001 cascade.

Summary of ELS 2006 Results

For 100000 one year simulations:

- In their baseline model, they find default cascades of size $0, 1, 2, 3...$ occur with frequencies $95.3\%.4.0\%, 0.4\%, 0.1\%...$;
- One scenario has nine defaults in total.
- These qualitative results are robust to small changes in the method.
- Turning off firm correlation eliminates most of the cascades.
- Turning off interbank links eliminates few cascades.

ELS 2006: General Conclusions

- UK banking system in December 2003 appears to be extremely stable.
- The probability that one or more defaults occur in the entire system over a one-year horizon given the December 2003 starting position is 4.7 percent.
- The probability of observing a domino effect is practically zero.
- Correlations dominate over interbank linkages in causing systemic events.

Details of the ELS 2006 method

- Available Bank of England balance sheet data only includes row sums $\{b_i\}$ and column sums $\{a_j\}$ of matrix L: they use entropy maximization to determine the matrix $L = L(a, b)$:

$$L(a,b) = \mathsf{argmax}_{\substack{L: L_{ii}=0, \sum_i L_{ij}=a_j \\ \sum_j L_{ij}=b_i}} \sum_{i \neq j} \log L_{ij}$$

- Assume L is constant.
- Assume all bank debt is insured, hence $D_t + X_t = (D_0 + X_0)e^{rt}$;
- Assume $V_i = Y_i + \sum_j L_{ji}$ follows N dimensional correlated Geometric Brownian Motion with parameters μ_i, Σ_{ij};
- They use a one-year time series of bank stock prices to infer μ_i, Σ_{ij}: this is standard Merton/KMV structural credit risk modelling.

Details of the ELS 2006 method

Given these assumptions/building blocks, it is straightforward to simulate V at time $T = 1$, and hence $Y - D$. The default cascade is easy to compute.

Criticisms

In light of recent events, and further research, it is easy to criticize these papers, and their conclusions.

- Assuming L is constant: clearly, from Duffie, we have learned that L is highly stochastic on a daily time-scale. Moreover, during a crisis, they will react even more actively;
- Assuming bank debt is insured: clearly far from the truth, given the nature of interbank liabilities;
- Assuming V is GBM: for risk management over one year, this is never a good assumption;
- Correlation: even if $\rho_{ij}s$ are close to 1, tail events will be negligible, since multivariate normals have tail independence;
- Entropy maximization leads to precisely the wrong statistics for interbank links.
- Although correlations were included, liquidity effects were not included: by current thinking liquidity risk is perhaps the dominant systemic factor.

ELS 2006 Method

Criticisms aside, these authors have been influential in creating a "benchmark" model against which new models are compared.

Boss, Elsinger, Summers, Thurner 2004

This early network study of the Austrian banking system, estimating the matrix L for about 900 Austrian banks. They were able to use a rather complete dataset on interbank links. They found:

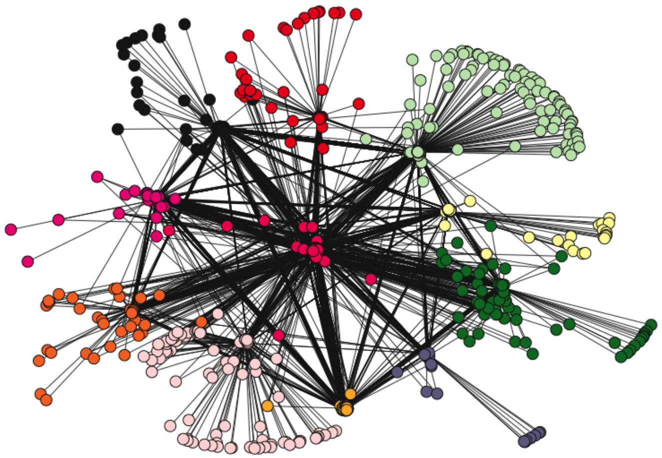
- Out-degree has Pareto tail with exponent ~ 3.1;
- In-degree has Pareto tail with exponent ~ 1.7;
- Contract size distribution degree has Pareto tail with exponent ~ 1.87;
- Relatively small Clustering coefficient

$$C = 3 \times \frac{\text{number of triangles}}{\text{number of connected triples}} \sim 0.12$$

- Average Shortest Path Length ~ 2.59

They concluded that it has a hierarchical structure, with some of the stylized properties of a "small-world" random network.

Austrian Network September 2002



Cont-Moussa-Bastos 2011

This paper includes a detailed study of the Brazilian Interbank network. Their dataset included over 2400 financial institutions. It contained full interbank exposures, reported on six dates (June 2007, December 2007, March 2008, June 2008, September 2008 and November 2008) as follows:

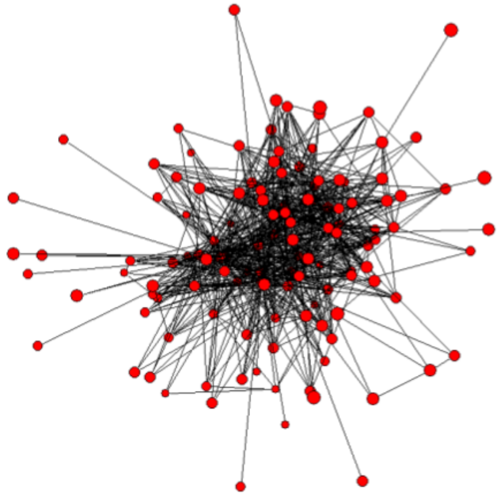
- fixed-income instruments (certicate of deposits and debentures);
- borrowing and lending (credit risk);
- derivatives (including OTC instruments such as swaps);
- foreign exchange and,
- instruments linked to exchange-traded equity risk.

Brazilian Financial Institutions

Type	Jun-07	Dec-07	Mar-08	Jun-08	Sep-08	Nov-08	Dec-0
Multiple Bank	135	135	135	136	139	139	14
Commercial Bank	20	20	21	20	20	18	1
Development Bank	4	4	4	4	4	4	[illegible]
Savings Bank	1	1	1	1	1	1	
Investment Bank	17	17	17	18	18	18	1
Consumer Finance Company	51	52	51	56	55	55	5
Security Brokerage Company	113	107	114	107	107	107	10
Exchange Brokerage Company	48	46	48	46	46	45	4
Security Distribution Company	132	135	133	133	136	136	13
Leasing Company	40	38	41	37	36	36	3
Real Estate Credit Company and Savings and Loan Association	18	18	18	18	18	17	1
Mortgage Company	6	6	6	6	6	6	[illegible]
Development Agency	12	12	12	12	12	12	1
Total Banking Institutions of Type I and II	**597**	**591**	**601**	**594**	**598**	**594**	**59**
Credit Union	1.461	1.465	1.460	1.466	1.460	1.457	1.45
Micro-financing Institution	54	52	54	48	46	45	4
Total Banking Institutions Type III	**2.112**	**2.108**	**2.115**	**2.108**	**2.104**	**2.096**	**2.09**
Non-Banking Institutions	**332**	**329**	**333**	**324**	**317**	**318**	**31**
Total Banking and Non-Banking Institutions	**2444**	**2.437**	**2.448**	**2.432**	**2.421**	**2.414**	**2.40**

Table 1: Number of financial institutions by type of operation for the Brazilian financial system. Source: Sisbacen.

Brazil Network Dec 2007



More Specifics about the Brazilian Network

- By Basel II, "Banks" must maintain

$$\text{Tier 1 capital} := e_i \geq 0.11 \times \text{RWA}$$

- Tier 1 capital is essentially the market value of common shares.
- "Risk-weighted Assets" (RWA) is the total market value of assets, weighted by their credit risk weight (0 for government securities, 0.5 for mortgages, 1 for ordinary loans etc.)
- We see that this provides banks with a buffer to protect against shocks to their balance sheet.

Summary Statistics of Brazilian Network

Their findings about the network on December 2007:

- In and Out-degree distributions have Pareto tails;
- Contract size distribution has Pareto tail;
- Clustering distributions are too weak to be consistent with "small world" assumption.
- Average in-link exposure is dependent on in-degree;
- Average out-link exposure is dependent on out-degree;

Their conclusion: it is superficially like a small world random network. However, observed clustering effects are weaker than expected from a small world random network. Also, exposures (link strengths) are correlated with degree.

Brazil Network: Out-Degree Distributions

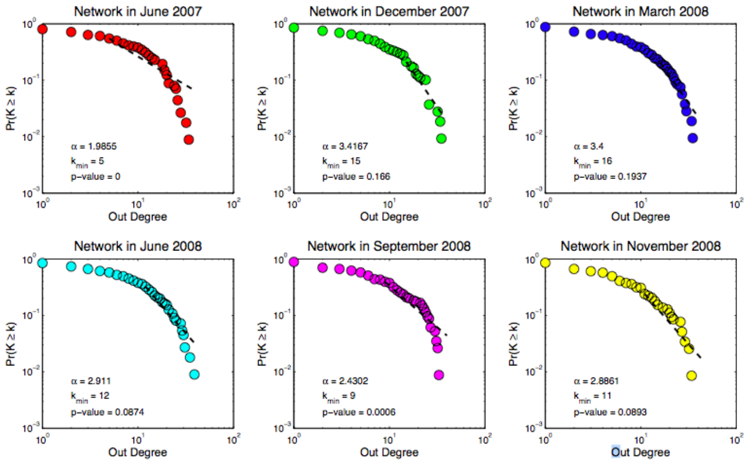


Figure 3: Brazilian interbank network: distribution of out-degree.

Brazil Network Statistics

In-Degree	Jun-07	Dec-07	Mar-08	Jun-08	Sep-08	Nov-08	Mean
$\hat{\alpha}$	2.19	2.70	2.20	3.36	2.16	2.13	2.46
$\hat{\sigma}(\hat{\alpha})$	0.48	0.46	0.47	0.53	0.47	0.44	0.48
$\hat{k}_{in,min}$	6	13	7	21	6	5	9.7
Out-Degree	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**	**Mean**
$\hat{\alpha}$	1.98	3.41	3.40	2.91	2.43	2.88	2.83
$\hat{\sigma}(\hat{\alpha})$	0.63	0.59	0.48	0.43	0.41	0.49	0.51
$\hat{k}_{out,min}$	5	15	16	12	9	11	11.3
Degree	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**	**Mean**
$\hat{\alpha}$	2.61	3.37	2.29	2.48	2.27	2.23	2.54
$\hat{\sigma}(\hat{\alpha})$	0.52	0.47	0.48	0.41	0.43	0.35	0.44
$\hat{k}_{min}$	17	34	12	15	12	10	16.7
Exposures*	**Jun-07**	**Dec-07**	**Mar-08**	**Jun-08**	**Sep-08**	**Nov-08**	**Mean**
$\hat{\alpha}$	1.97	2.22	2.23	2.37	2.27	2.52	2.27
$\hat{\sigma}(\hat{\alpha})$	0.02	0.60	0.21	0.69	0.38	0.98	0.48
$\hat{E}_{min}$	39.5	74.0	80.0	101.7	93.4	336.7	120.9

*values in millions of BRL (Brazilian Reals)

Network Analysis

CMS 2011 make the following assumptions:

- Initial shocks are "systemic": they are drawn from a multivariate distribution with heavy-tailed marginals and a dependence structure described by a Cauchy copula.
- Default of bank happens when $e_i \leq 0$ ("insolvency");
- Any out-link from an insolvent bank is valued at 0 (that is: immediately following insolvency of bank i, counterparties j must "write down" their exposure L_{ij} to 0.
- Then remaining banks recompute their balance sheet;
- Iterate until the cascade is resolved.

Summary: the CMS 2011 framework is precisely EN2001 Version C.

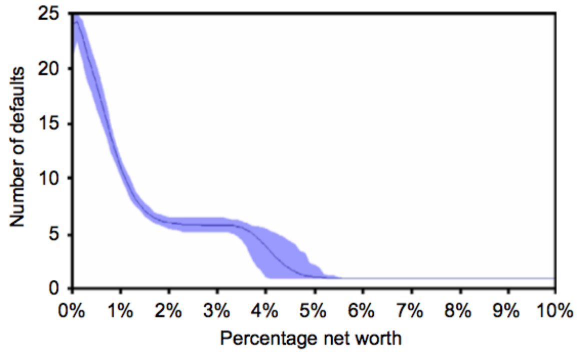
NYYA 2007: General Setup

The 2007 paper "Network models and financial stability" was influential in beginning "network" theoretical studies of systemic risk.

- Random network of $N = 25$ banks with Poisson degree distribution, parameter $P = 0.2$;
- Balance sheets on each bank:
  - Assets a_v;
  - Constant capital buffers $\gamma = 0.05a$;
  - Interbank assets $\theta = 0.20a$
  - Constant interbank link weights w;
- Assume partial recovery after default (almost like Eisenberg-Noe A);

They run Monte Carlo simulations of the resulting cascade, varying one parameter at a time away from their benchmark values.

NYYA Results



NYYA Results

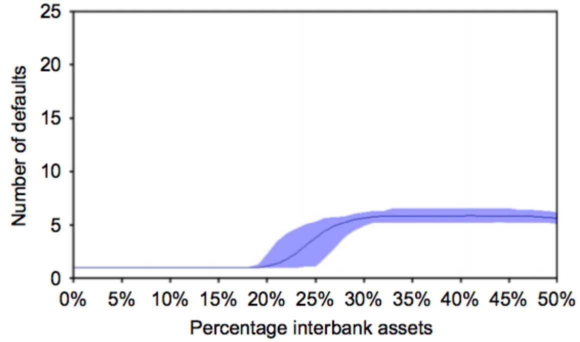


Fig. 2. Number of defaults as a function of the percentage of interbank assets in total assets (θ). Based o 100 runs for each parameter constellation (γ, θ, p, N, E). Parameter values as in Table 1 (except for θ).

NYYA Results

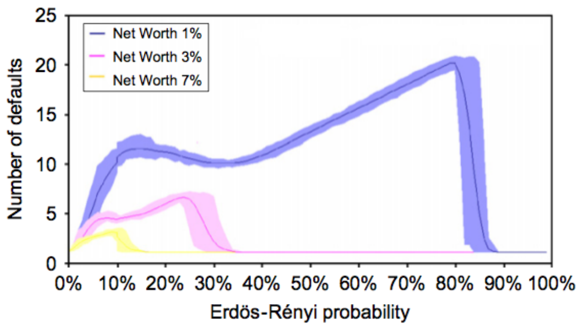


Fig. 3. Number of defaults as a function of the probability of connectedness (p) for different values o percent of net worth (γ). Based on 100 runs for each parameter constellation (γ, θ, p, N, E). Paramete values as in Table 1 (except for p and γ).

NYYA Results

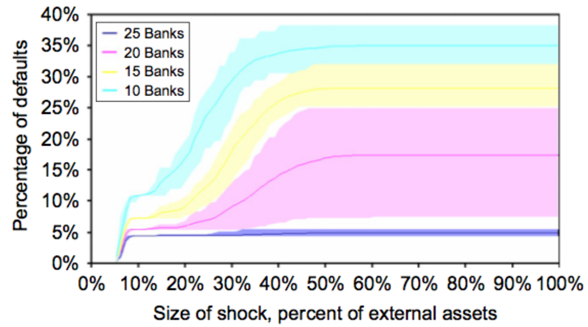


Fig. 4. Percentage of defaults as a function of the shock size as a percentage of external assets for differen N. Based on 100 runs for each shock size. Parameter values (γ, θ, p, E) are as in the benchmark experiment Table 1.

NYYA 2007: Main Conclusions

- First large scale systemic simulation study.
- Contagion decreases in net worth. This effect is non-linear.
- Contagion increases in the size of interbank liabilities. This is the case even if banks hold capital against interbank assets;
- Contagion is a non-monotonic function of the number of interbank connections, all else equal.

NYYA 2007: Main Conclusions

- Important large scale systemic simulation study.
- They implement the Version A EN 2001 Cascade;
- Contagion decreases in net worth. This effect is non-linear.
- Contagion increases in the size of interbank liabilities. This is the case even if banks hold capital against interbank assets;
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