NWO/STAR/PWN

20th Winter School on Mathematical Finance

Special topics:

Functional convex ordering of stochastic processes
Optimal transport in finance

January 24–26, 2022
Hotel De Werelt, Lunteren

Sponsored by NWO, STAR, PWN, and FWO
Winter School on Mathematical Finance

In recent years, the mathematical theory associated with financial risk management and the pricing of contingent claims has been a highly active field of research. The area has established itself as one of the most vigorously growing branches of applied mathematics. Model-based analysis of contracts and portfolios has become a standard in the finance industry, and the number of academic institutions offering curricula in financial mathematics has increased rapidly. In this context, the winter school on Mathematical Finance that will take place on January 24–26, 2022 in Lunteren aims at providing a meeting place for participants both from industry and from academia and will be organized in a safe, Coronaproof environment. The program provides ample opportunity for discussion.

The special topics of the 20th winter school are Functional convex ordering of stochastic processes, and Optimal transport in finance. These are the subjects of minicourses that will be taught by two distinguished speakers: Professors Jan Obłój (University of Oxford) and Gilles Pagès (Sorbonne Université). Additionally there will be three one-hour special invited lectures by Professors Christa Cuchiero (University of Vienna), Blanka Horvath (Technische Universität München) and Mitja Stadje (Universität Ulm). Thirty-minute lectures on recent research work in the Netherlands will be presented by Ioannis Anagnostou (ING), Thijs Kamma (Maastricht University), Sven Karbach (University of Amsterdam), Ioana Neamțu (Bank of England) and Anna Sulima (Wrocław University of Economics and Business).

Auspices, sponsoring and grants

The Winter School takes place under the auspices of the mathematics cluster STAR and of PWN. The stochastics groups of the mathematics departments of the universities in the Netherlands cooperate in STAR. PWN (Platform Wiskunde Nederland) is a national organization that aims to strengthen the position of mathematics in the Netherlands in all its aspects. The winter school is supported financially by STAR, PWN, by the Netherlands Organization for Scientific Research (NWO) and by the Research Foundation - Flanders (FWO). Administrative assistance is provided by the Korteweg–De Vries Institute for Mathematics of the University of Amsterdam.

The FWO research network Modelling and Simulation with applications in Finance, Insurance and Economics has made available a limited number of grants for young researchers (PhD students and postdocs) associated to the network to be used as a waiver of the registration fee for the winter school. For those researchers the grants completely cover the registration fee. For other young researchers a limited number of grants of € 250 each is available as a reduction on the registration fee for the winter school. Priority will be given to grant applications from PhD students whose supervisor is a member of the network, but others are invited to apply as well. Applications for the grant can be sent by email to both Michel Vellekoop and Peter Spreij (make sure both are addressed, spreij@uva.nl and m.h.vellekoop@uva.nl). Applications are required to contain a brief motivation why the grant should be beneficial for the research of the applicant, a brief motivation why the applicant has a specific need for the grant, a (link to) a CV of the applicant and the name of her/his principal supervisor. Applications should be submitted before the deadline, November 20, 2021.
Organizers

The winter school is organized by:
Michel Vellekoop (Faculty Economics and Business, University of Amsterdam; e-mail m.h.vellekoop@uva.nl)
Peter Spreij (Korteweg–De Vries Institute for Mathematics, University of Amsterdam and IMAPP, Radboud University; e-mail spreij@uva.nl).

Program outline

The program starts with registration and coffee on Monday, January 24, from 10:30 to 11:30, and ends on Wednesday, January 26, at 16:00. The following events are planned:

Minicourses

Jan Obłój
*Optimal transport techniques in finance*

Gilles Pagès
*Functional convex ordering of stochastic processes: a constructive approach with applications to finance*

Special invited lectures

Christa Cuchiero
*Signature SDEs as affine and polynomial processes*

Blanka Horvath
*Data-driven market simulators and some simple applications of signature kernel methods in mathematical finance*

Mitja Stadje
*Hedging and optimal portfolio choice under endogenous permanent market impacts*

Short contributions

Ioannis Anagnostou
*Financial market community detection and an application to portfolio risk modelling*

Thijs Kamma
*Dual formulation of the optimal consumption problem with ratio habit formation*

Sven Karbach
*An affine stochastic volatility model in Hilbert spaces with state-dependent jumps*

Ioana Neamțu
*Risk-taking and uncertainty: do contingent convertible (CoCo) bonds increase the risk appetite of banks?*

Anna Sulima
*Completeness, arbitrage and optimal portfolio strategy in an Itô-Markov additive market*
Schedule of lectures

<table>
<thead>
<tr>
<th>Time</th>
<th>Monday</th>
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<tr>
<td>09:00 - 10:00</td>
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<td>16:00 - 17:00</td>
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<td>Horvath</td>
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<td>17:30 - 18:00</td>
<td>Kamma</td>
<td>Karbach</td>
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<td>18:00 - 18:30</td>
<td>Neamţu</td>
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<td>18:30 - 19:00</td>
<td>Anagnostou</td>
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Venue

The winter school will take place in Hotel De Werelt, Westhofflaan 2, Lunteren, tel. +31-(0)318-484641, fax +31-(0)318-482924. Located in the heart of the Veluwe forest, De Werelt is one of the top accommodations in the Netherlands in terms of attractiveness of surroundings. Access by car or by public transportation is easy. By train, the village of Lunteren can be reached in twenty minutes from Amersfoort, and in ten minutes from Ede-Wageningen. It takes about fifteen minutes to walk from the railway station in Lunteren to the conference center (see directions below). If you come by car, ANWB signs in Lunteren will guide you to the venue. It is also possible to take a taxi from the taxi stand at railway station Ede-Wageningen. To get a taxi in Lunteren, call +31-(0)318-484555. For further details please see [https://dewerelt.nl/en/](https://dewerelt.nl/en/) under Contact in the dropdown Menu or the Google map on [https://dewerelt.nl/en/contact/](https://dewerelt.nl/en/contact/).

Directions from the railway station: leaving the station, turn right across the pebble-covered parking lot. Turn left into the forest (Boslaan). At the crossroads, turn right into Molenweg. The first turn left is Westhofflaan.

Website

[https://staff.fnwi.uva.nl/p.j.c.spreij/winterschool/winterschool.html](https://staff.fnwi.uva.nl/p.j.c.spreij/winterschool/winterschool.html)
Abstracts

MINICOURSE I

Jan Obłój (University of Oxford)

Optimal transport techniques in finance

The recent decade has seen an explosion of interest in optimal transport (OT) techniques in the context of statistics, stochastic processes, optimization and beyond. These lecture series explore various case studies from these novel crossroads with the unifying theme of applications in quantitative finance. We will start with an introduction to the classical optimal transport with an emphasis on probabilistic methods. We will then consider the martingale version of the problem, explore its structure, and show how it translates into the robust pricing-hedging problem in finance. In this problem, one is given market prices of vanilla European call/put options and asks what are the no-arbitrage bounds on prices of other options. We will discuss many variants - motivated by the type of market information that is available in practice - and also touch on numerical methods. We will restrict ourselves to a discrete time setting but I will link these problems with the so-called Skorokhod embeddings in continuous time. Subsequently, we will turn to small model uncertainty and use OT methods to build small perturbations to a given model. I will show how this is analogous to OT applications in image classification. Fundamentally, we will see that given $n$ data points, instead of perturbing these as points in a finite-dimensional space, it is more elegant to think of them via their empirical measure and do the perturbations in the infinite-dimensional space of measures. This will lead us to distributionally robust optimization and its sensitivity analysis. In particular, we will see how OT yields a non-parametric version of the classical Black-Scholes Vega.

MINICOURSE II

Gilles Pagès (Sorbonne Université)

Functional convex ordering of stochastic processes: a constructive approach with applications to finance

We will start by some background on convex (resp. monotone convex) ordering of $\mathbb{R}^d$-valued random vectors, namely random vectors (or probability distributions) satisfying

$X \leq_{\text{cv}} Y$ if $\mathbb{E} f(X) \leq \mathbb{E} f(Y)$

for every convex (resp. non-decreasing convex) function $f : \mathbb{R} \to \mathbb{R}$

provided both expectations have a sense. In particular we will recall classical results like Strassen and Kellerer’s theorems ([Str65] and [Kel72]) which make the connection between convex ordering, martingales and peacocks (for p.c.o.c. itself acronym for the French “processus croissant pour l’ordre convexe”) extensively investigated by Yor and co-authors.

We will first investigate both convex orderings of the marginal for Brownian diffusions with respect to their diffusion coefficients. We will show that these results also holds in functional sense i.e. for functional of their whole path. Then we will extend this result
to jump stochastic differential equations driven by Lévy processes but also Brownian stochastic integrals, etc., following [Pag16]. Some of these results are classical (see [Haj85]) for monotone convex ordering of diffusion) or more recently known (see Rüschendorf and co-authors [BR06, BR07, BR08], Hobson [Hob98, Hob10] among others).

We apply these results to establish in local volatility models sensitivity results of path-dependent options with respect to their volatility. Doing so, we extend a result by [EKJPS98] and [BGW96] which produce upper- and lower-bounds based on Black-Scholes formula for a vanilla option with convex payoff when the volatility function is itself bounded and bounded away from 0.

As a second step we investigate optimal stopping theory, replacing the path-dependent functional by the Snell envelope of a “vanilla” reward process written on a martingale diffusion, with an obvious connection with American option and again an application to their sensitivity to the volatility process (see [Pag16]).

The specificity of our approach is to be constructive in the sense that we first establish our results in discrete time (which has its own interests), typically for a discretization scheme of the underlying process and then rely on functional limit theorems “à la Jacod-Shiryaev” [JS03] to transfer the property to the continuous time model. When dealing with numerics in Finance, it usually produces arbitrage free approximating numerical methods (as far as volatility modeling is concerned). This can be seen as a paradigm.

In view of the importance taken by McKean-Vlasov equations (for mean-field games but also for Langevin algorithm) we will apply the above paradigm to McKean-Vlasov equations (see [LP20, LP21]) for both regular and monotone convex ordering.

A natural question on our way is to wonder whether it is possible to extend such an approach to non-Markovian dynamics. The answer is positive since our approach successfully applies to Volterra equations [JP21], with applications to rough stochastic volatility models.

References


Special invited lectures

Christa Cuchiero (University of Vienna)

Signature SDEs as affine and polynomial processes

Signature methods represent a non-parametric way for extracting characteristic features from time series data which is essential in machine learning tasks. This explains why these techniques become more and more popular in Econometrics and Mathematical Finance. Indeed, signature based approaches allow for data-driven and thus more robust model selection mechanisms, while first principles like no arbitrage can still be easily guaranteed. In view of option pricing the key quantity that one needs to compute in these models is the expected signature of some underlying process. Surprisingly this can be achieved for generic classes of jump diffusions (with possibly path dependent characteristics) via techniques from affine and polynomial processes. More precisely, we show how the signature process of these jumps diffusions can be embedded in the framework of affine and polynomial processes, which have been – due to their tractability – the dominating process class prior to the new era of highly over-parametrized dynamic models. In other words, this means that the infinite dimensional Feynman Kac PIDE of the signature process can be reduced to (sometimes finite dimensional) ODEs either of linear or Riccati type. We illustrate our findings by means of one dimensional diffusion processes with analytic characteristics.

Blanka Horvath (Technische Universität München)

Data-driven market simulators and some simple applications of signature kernel methods in mathematical finance

Techniques that address sequential data have been a central theme in machine learning research in the past years. More recently, such considerations have entered the field of finance-related ML applications in several areas where we face inherently path dependent problems: from (deep) pricing and hedging (of path-dependent options) to generative modelling of synthetic market data, which we refer to as market generation. We revisit Deep Hedging from the perspective of the role of the data streams used for training and highlight how this perspective motivates the use of highly accurate generative models for synthetic data generation. From this, we draw conclusions regarding the implications for risk management and model governance of these applications, in contrast to risk-management in classical quantitative finance approaches. Indeed, financial ML applications and their risk-management heavily rely on a solid means of measuring and
efficiently computing (similarity-) metrics between datasets consisting of sample paths of stochastic processes. Stochastic processes are at their core random variables with values on path space. However, while the distance between two (finite dimensional) distributions was historically well understood, the extension of this notion to the level of stochastic processes remained a challenge until recently. We discuss the effect of different choices of such metrics while revisiting some topics that are central to ML-augmented quantitative finance applications (such as the synthetic generation and the evaluation of similarity of data streams) from a regulatory (and model governance) perspective. Finally, we discuss the effect of considering refined metrics which respect and preserve the information structure (the filtration) of the market and the implications and relevance of such metrics on financial results.

Mitja Stadje (Universität Ulm)

Hedging and optimal portfolio choice under endogenous permanent market impacts

We consider hedging and expected utility maximization problems of a large investor who is allowed to make transactions on a tradable asset in a financial market with endogenous permanent market impacts. The asset price is assumed to follow a nonlinear price curve quoted in the market as the utility indifference curve of a representative liquidity supplier. Under this market impact model, we introduce a completeness condition under which any derivative can be perfectly replicated by a dynamic trading strategy. For the optimal portfolio choice problem, we show that optimality can be fully characterized via a system of coupled forward-backward stochastic differential equations (FBSDEs) which is equivalent to a highly non-linear backward stochastic partial differential equation (BSPDE). We show existence and uniqueness solutions for FBSDEs in the case where the driver function of the representative market maker grows at least quadratic or the utility function of the large investor falls faster than quadratic or is exponential. Explicit examples are provided when the market is complete or the driver function is positively homogeneous. This talk is based on joint works with Masaaki Fukasawa and Thai Nguyen.

Short contributions

Ioannis Anagnostou (ING)

Financial market community detection and an application to portfolio risk modelling

One of the most challenging aspects in the analysis and modelling of financial markets is the presence of an emergent, intermediate level of structure standing in between the microscopic dynamics of individual financial entities and the macroscopic dynamics of the market as a whole. This mesoscopic level of organisation is often sought for via factor models that ultimately decompose the market according to geographic regions and economic industries. However, at a more general level, the presence of mesoscopic structure might be revealed in an entirely data-driven approach, looking for a modular and possibly hierarchical organisation of the empirical correlation matrix between financial time series. The crucial ingredient in such an approach is the definition of an appropriate null model for the correlation matrix. Recent research showed that community detection techniques developed for networks become intrinsically biased when applied to correlation matrices. For this reason, a method based on Random Matrix Theory has been developed, which identifies the optimal hierarchical decomposition of the system into internally correlated
and mutually anti-correlated communities. Building upon this technique, here we resolve
the mesoscopic structure of the CDS market and identify groups of issuers that cannot be
traced back to standard industry/region taxonomies, thereby being inaccessible to stan-
dard factor models. We use this decomposition to introduce a novel default risk model
that is shown to outperform more traditional alternatives.

**Thijs Kamma** (Maastricht University)

*Dual formulation of the optimal consumption problem with ratio habit formation*

This paper provides a dual formulation of the optimal consumption problem with internal ratio habit formation. In this problem, the agent derives utility from the ratio of consumption to the internal habit component. Due to this multiplicative specification of the habit model, standard Lagrangian techniques fail to supply a candidate for the dual problem. Using a slight modification of the conventional Legendre transform, we manage to identify a candidate formulation and prove that it specifies a well-posed dual problem. This formulation discloses the analytical links between optimal consumption, the habit level and the portfolio process. On the basis of these links, we propose two analytical approximations to optimal consumption. Small duality gaps demonstrate the potential accuracy of these approximations.

**Sven Karbach** (University of Amsterdam)

*An affine stochastic volatility model in Hilbert spaces with state-dependent jumps*

We present a flexible and tractable stochastic volatility model in Hilbert spaces with constant and state-dependent jumps. The model consists of a Hilbert-valued linear SDE joined by an affine Markov process, which we use to model the operator-valued instantaneous variance process of the former. The linear SDE admits for a possibly unbounded drift operator and infinite-dimensional Wiener noise perturbed by the stochastic volatility dynamics. The variance process itself is recruited from a class of pure-jump Markov processes with values in the positive self-adjoint Hilbert-Schmidt operators and such that its Laplace transform exhibits an exponential-affine form in the initial value of the process. We show that this desired affine property inherits to the characteristic function of the joint model and we discuss applications of our infinite-dimensional stochastic volatility model to commodity forward markets, where the dynamics of forward price curves can be specified by a SPDE in the Heath-Jarrow-Morton-Musiela modeling framework, which, formulated as a linear SDE on some Hilbert space containing the forward curves, fits into our model setting.

**Ioana Neamţu** (Bank of England)

*Risk-taking and uncertainty: do contingent convertible (CoCo) bonds increase the risk appetite of banks?*

We assess the impact of contingent convertible (CoCo) bonds and the wealth transfers they imply conditional on conversion on the risk-taking behaviour of the issuing bank. We also test for regulatory arbitrage: do banks try to maintain risk-taking incentives by issuing CoCo bonds, when regulators reduce them through higher capitalization ratios? While we test for, and reject sample selection bias, we show that CoCo bonds issuance has a strong positive effect on risk-taking behaviour, and so do conversion parameters.
that reduce dilution of existing shareholders upon conversion. Higher economic volatility amplifies the impact of CoCo bonds on risk-taking. [Joint work with Mahmoud Fatouh and Sweder van Wijnbergen.]

Anna Sulima (Wroclaw University of Economics and Business)

Completeness, arbitrage and optimal portfolio strategy in an Itô-Markov additive market

We study a market with the prices of financial assets described by Itô-Markov additive processes, which combine Lévy processes and regime switching models. Such a process evolves as an Itô-Lévy process between changes of states of a Markov chain, that is, its parameters depend on the current state of the Markov chain. Thus, the model takes into account two sources of risk: the jump diffusion risk and the regime switching risk. Due to the presence of these sources of risk, our market model is incomplete. We show how to complete the Itô-Markov additive market model by adding Markovian jump securities, Markovian power-jump securities and impulse regime switching securities. Using these securities, all contingent claims can be replicated by a self-financing portfolio. Moreover, we give conditions for the market to be asymptotic-arbitrage-free, namely, we find a martingale measure under which all the discounted price processes are martingales. We also consider the problem of identifying the optimal strategy that maximizes the expected value of the utility function of the wealth process at the end of some fixed period. The analysis is conducted for the logarithmic and power utility functions.

References


Registration procedure

To register for the winter school, please use the electronic registration form that is available at the web page of the winter school, https://staff.fnwi.uva.nl/p.j.c.spreij/winterschool/winterschool.html.

The registration fee includes accommodation (single room) for the nights of January 24 and 25, all meals starting with lunch on Monday up to and including lunch on Wednesday, and tea and coffee during breaks. Payment can be made by transfer to IBAN account number: NL27 INGB 0007388994 of Winter School Amsterdam, Secretariaat Korteweg–De Vries Instituut, Amsterdam and (for international money transfers) BIC: INGBNL2A.

The fee schedule is as follows:

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<th>early registration (before December 1)</th>
<th>late registration (after December 1)</th>
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<tr>
<td>industry professional</td>
<td>€1195</td>
<td>€1350</td>
</tr>
<tr>
<td>full-time academic</td>
<td>€395</td>
<td>€445</td>
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Inquiries concerning fees for partial attendance may be directed to ms. Bijl at the address given below. Registration will be valid after full payment has been received. Refunds can be given only for cancellations received before January 1, 2022. However, should the situation of the spreading of the Corona virus deteriorate in such a way that governmental rules force the organization to cancel the winter school, all fully paid registrations will be completely restituted.

Please note that PhD students and postdocs which receive an FWO grant should also register at the website, to ensure accommodation.

Accommodation at the venue is limited. Therefore, reservations will be treated on a first-come-first-served basis with priority for full arrangements. Participants who cannot be lodged at the venue will be accommodated in a hotel nearby. Transportation from the hotel to the venue and vice versa will be taken care of by the organization.

Further information

For further information regarding the scientific program, please contact one of the members of the organizing committee. For information concerning registration please contact:

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