Modified numerals as split disjunctions

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Peter van Ormondt (joint work with Maria Aloni)

P.vanOrmondt@uva.nl

http://www.vanormondt.net/~peter

http://www.illc.uva.nl

Introduction

- Pragmatic reasoning is the process of integrating contextual information in order to interpret what is meant (Grice, 1975, 1989).
- We will look at a class of inferences triggering ignorance effects and the obviation of these effects in the presence of modal operators and quantifiers.
- In particular we will look at constructions called modified numerals, the effects of which are considered to be pragmatic. However, we show they in fact exhibit a more hybrid behaviour.
- We will present a logic in which these (and other) inferences can be formally derived and accounted for.

An inferential puzzle (1)

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(1) a. The band has three players.

b. $\rightsquigarrow$ The band has exactly three players.

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(2) a. The band has more than two players.
  b. $\not\rightsquigarrow$ The band has exactly three players.
  $\not\rightsquigarrow$ The speaker conveys ignorance about the exact number of players.

An inferential puzzle (1)

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b. $\not\rightsquigarrow$ The band has exactly three players.
$\not\rightsquigarrow$ The speaker conveys ignorance about the exact number of players.

(3) a. The band has at least three players.
b. $\not\rightsquigarrow$ The band has exactly three players.
$\rightsquigarrow$ The speaker conveys ignorance about the exact number of players.

Modified numerals

Superlative quantifiers: at least n / at most n

Comparative quantifiers: more than n / fewer than n

Superlative quantifiers are known to trigger ignorance effects, while comparative quantifiers do not (Nouwen, 2010).

Joanna says:

(4) a. ?I have at least three children.
    b. I have more than two children.

The ignorance effect of modified numerals is cancellable only under special circumstances.

(5) I have at least three children. Guess how many?!

Disjunction

Plain disjunctions give rise to ignorance effects (Grice, 1989; Gazdar, 1976)

(6) a. Klaus has three or four children.
$\rightsquigarrow$ The speaker does not know how many.
b. $\varphi \vee \psi \rightsquigarrow \Diamond\varphi \wedge \Diamond\psi$ [Epistemic $\Diamond$]

The ignorance effect is strong.

(7) ?I have two or three children.

These inferences are also only cancellable under special circumstances.

(8) I have two or three children. Guess how many?!

Disjunction (2)

- Disjunctions have similar effects as modified numerals.
- Hypothesis: Modified numerals can be analyzed as disjunctions (cf., e.g., Geurts and Nouwen, 2007)

(9) a. The band has at least three players. [Superlative]
    b. `three` $\vee$ `more`

(10) a. The band has more than two players. [Comparative]
     b. `more-than-two`

An Inferential puzzle (2): Obviation

It has been observed (Nouwen, 2010; Blok, 2019) that the ignorance reading can be obviated once modified numerals appear in the scope of certain operators (quantifiers, modals).

(11) a. Everyone read at least three books.
     b. $\not\rightsquigarrow$ Speaker does convey ignorance.

(12) a. To pass the course, you're required to read at least three books.
     b. $\not\rightsquigarrow$ Speaker does convey ignorance.

Again similar effects obtain with disjunction:

(13) a. Everyone read two or three books.
     b. To pass the course, you're required to read two or three books.
     c. $\not\rightsquigarrow$ Speaker does convey ignorance.

An inferential puzzle (3): Distribution

Sentences with disjunction in the scope of a universal quantifier tend to give rise to distributive inferences that each of the disjuncts hold (Spector, 2006; Fox, 2007; Klinedinst, 2007).

(14) a. Every woman in my family has two or three children.
$\rightsquigarrow$ Some woman has two and some woman has three children.
b. $\forall x(\texttt{two}(x) \vee \texttt{three}(x)) \rightsquigarrow \exists x\, \texttt{two}(x) \wedge \exists x\, \texttt{three}(x)$

This works similarly for superlative modified numerals:

(15) a. Every woman in my family has at least three children.
$\rightsquigarrow$ Some woman has three and some woman has more than three children.
b. $\forall x(\texttt{three}(x) \vee \texttt{more}(x)) \rightsquigarrow \exists x\, \texttt{three}(x) \wedge \exists x\, \texttt{more}(x)$

Summary

Ignorance

(16) a. Klaus has at least three children.
b. $(\texttt{three} \vee \texttt{more}) \rightsquigarrow \Diamond\texttt{three} \wedge \Diamond\texttt{more}$

Obviation

(17) a. Every woman in my family has at least three children.
b. $\forall x(\texttt{three}(x) \vee \texttt{more}(x)) \not\rightsquigarrow \forall x(\Diamond\texttt{three}(x) \wedge \Diamond\texttt{more}(x))$

Distribution

(18) a. Every woman in my family has at least three children.
b. $\forall x(\texttt{three}(x) \vee \texttt{more}(x)) \rightsquigarrow \exists x\,\texttt{three}(x) \wedge \exists x\,\texttt{more}(x)$

A logic based account

We will consider a logic-based account where all these inferences will follow as “reasonable inferences” (cf. Stalnaker, 1975).

The system we propose extends the bilateral framework of Aloni (2018) to the first-order case and is a modal predicate logic with state-based semantics that defines conditions of assertion/rejection rather than conditions of truth.

State-based semantics

- In state-based semantics formulas are interpreted wrt states, rather then possible worlds.
- Classical modal propositional logic: $\mathcal{M}, w \models \varphi,$ where $w \in W$
- State-based modal propositional logic: $\mathcal{M}, s \models \varphi,$ where $s \subseteq W$ (cf. Aloni, 2018)
- In our framework the possibilities are pairs of possible worlds and (partial) assignments.
- We have a *bilateral system* where we define conditions of assertability and rejectability rather than truth:

$\mathcal{M}, s \models \varphi$ "φ is assertable wrt a model $\mathcal{M}$ and a state s"

$\mathcal{M}, s \dashv \varphi$ "φ is rejectable wrt a model $\mathcal{M}$ and a state s"

Language, model and states

Language

$$\varphi := Px_1, \ldots, Px_n \mid \neg\varphi \mid \varphi \vee \varphi \mid \Diamond\varphi \mid \exists x\varphi \mid \forall x\varphi \mid \mathsf{NE},$$

where $\Diamond\varphi$ is an epistemic modal.

Model

A model for our language is a tuple $\mathcal{M} = \langle W, R, D, I, s_{\mathcal{M}} \rangle$, where $s_{\mathcal{M}}$ is the designated state.

State

An index $i = \langle w_i, g_i \rangle$ is a world-assignment pair. A state s is a set of indices. The indices in the designated state $s_{\mathcal{M}}$ have the empty assignment function. Basically, this means that $s_{\mathcal{M}}$ is equivalent to a set of possible worlds.

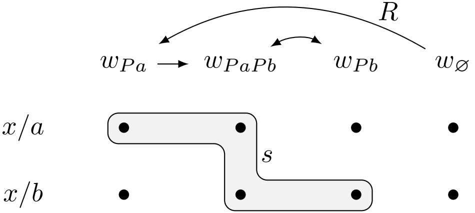
Support

Atomic formula

An atomic formula φ is *supported* by a state s and model $\mathcal{M}$ iff every $i \in s$ makes φ classically true.

An atomic formula φ is *rejected* by a state s and model $\mathcal{M}$ iff every $i \in s$ makes φ classically false.

Example



$$s \models Px$$
$$s \not\models Pa$$
$$s \not\models Pb$$

Logical consequence

Logical consequence as preservation of support w.r.t. to the designated state $s_{\mathcal{M}}$:

$$\varphi \models \psi \text{ iff for all } \mathcal{M} : \mathcal{M}, s_{\mathcal{M}} \models \varphi \implies \mathcal{M}, s_{\mathcal{M}} \models \psi.$$

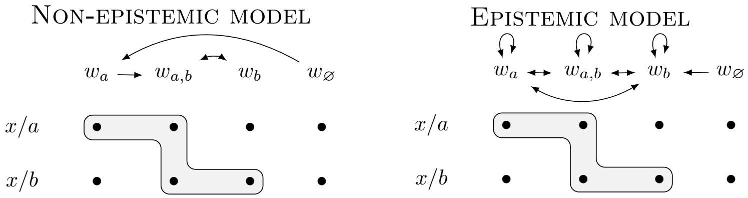
Stalnaker on "reasonable inference":

> *[...] an inference [...] is reasonable just in case, in every context in which the premisses could appropriately be asserted or supposed, it is impossible for anyone to accept the premisses without committing himself to the conclusion (Stalnaker, 1975, p. 271)*

Epistemic models

In order to capture the epistemic modals we put constraints on the accessibility relation R. We will consider only models such that within the state $s_{\mathcal{M}}$ the accessibility relation R is universal: all and only worlds in $s_{\mathcal{M}}$ are accessible within $s_{\mathcal{M}}$. We will call such models *epistemic models*.

Example

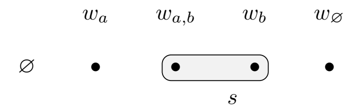


In what follows we will only consider *epistemic models* and omit the arrows for convenience.

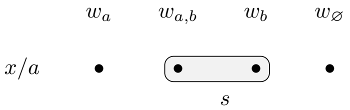
Some operations on states (cf. Dekker, 1993, Chapter 5)

$D = \{a, b\}$

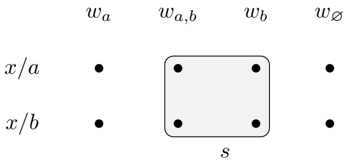
A state can have an empty assignment.



Individual x-extension of s, $s[x/a]$. We define $s[x/a]$ to be the state which results from s by replacing the assignment g_i in each index $i \in s$ by $g_i[x/a]$.



Universal x-extension of s, $\bigcup_{d \in D} s[x/d]$.



Quantifiers

$s \models \forall x\varphi$ iff the universal x-extension of s supports φ

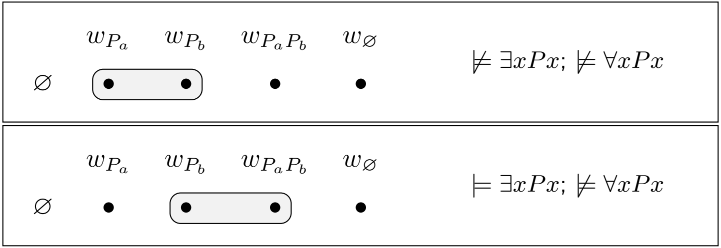
$s \Dashv \forall x\varphi$ iff there is an individual x-extension which supports φ

$s \models \exists x\varphi$ iff there is an individual x-extension which supports φ

$s \Dashv \exists x\varphi$ iff the universal x-extension of s rejects φ

Examples (1)

$D = \{a, b\}$



Quantifiers

$s \models \forall x\varphi$ iff the universal x-extension of s supports φ

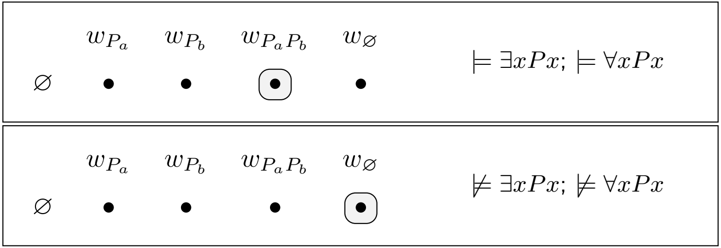
$s \dashv \forall x\varphi$ iff there is an individual x-extension which supports φ

$s \models \exists x\varphi$ iff there is an individual x-extension which supports φ

$s \dashv \exists x\varphi$ iff the universal x-extension of s rejects φ

Examples (2)

$D = \{a, b\}$



Modals

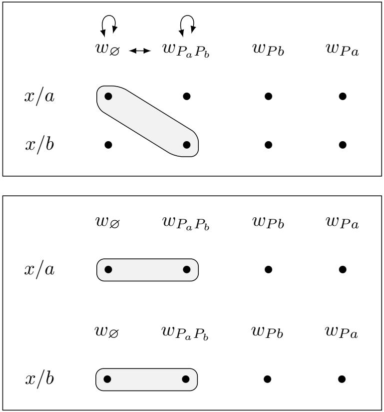
We interpret modal formulas $\Diamond\varphi$ by evaluating φ wrt to a state contructed by combining the worlds accessible from w_i with g_i.

The following is the case:

$$s \not\models Px$$
$$s \models \Diamond Px$$

In order to evaluate $\Diamond Px$ in state s.

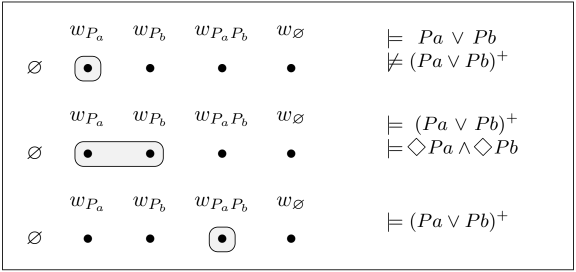
Px needs to be supported at least in a non-empty substate of the first state and in a non-empty substate of the second state.



Split disjunction and pragmatic enrichment

- We adopt a split notion of disjunction from team logic (Väänänen, 2007; Hawke and Steinert-Threlkeld, 2018).
  - A state s supports $\varphi \vee \psi$ iff s can be split into two substates, each supporting one of the disjuncts
  - A state s rejects $\varphi \vee \psi$ iff s rejects φ and rejects ψ.
- A *pragmatic enrichment function* is a mapping from formula's to formula's adding the NE operator recursively (Aloni, 2018).
  - After pragmatic enrichment: $(\varphi \vee \psi)^+ := (\varphi^+ \wedge \mathsf{NE}) \vee (\psi^+ \wedge \mathsf{NE})$
  - A state s supports $(\varphi \vee \psi)^+$ iff s can be split into two non-empty substates, each supporting one of the disjuncts.

Example

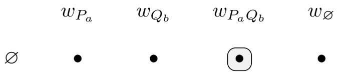


Obviation

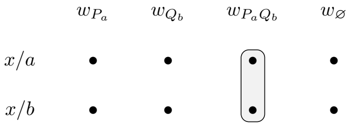
$$\forall x(\varphi \vee \psi)^+ \not\models \forall x(\Diamond\varphi \wedge \Diamond\psi) \qquad D = \{a, b\}$$

Counterexample

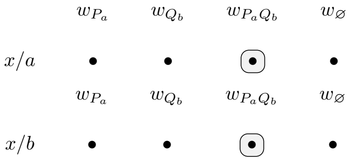
This state supports $\forall x(Px \vee Qx)^+$



Because it's universal extension supports $(Px \vee Qx)^+$.



But it does not support $\forall x(\Diamond Px \wedge \Diamond Qx)^+$, because the universal extension does not support $\Diamond Px \wedge \Diamond Qx$. E.g. The first state does not support $\Diamond Qx$ and the second state does not support $\Diamond Px$.



Results

Before pragmatic enrichment

Classical logic can be recovered (as NE-free fragment)

After pragmatic enrichment

The following facts obtain

$$(\varphi \vee \psi)^+ \models \Diamond\varphi \wedge \Diamond\psi \quad \text{(Ignorance)}$$

$$\forall x(\varphi(x) \vee \psi(x))^+ \not\models \forall x(\Diamond\varphi(x) \wedge \Diamond\psi(x)) \quad \text{(Obviation)}$$

$$\forall x(\varphi(x) \vee \psi(x))^+ \models \exists x(\varphi(x) \wedge \psi(x)) \quad \text{(Distribution)}$$

Conclusion & further work

- We have shown a class of pragmatic inferences that do not exhibit all typical pragmatic characteristics, in particular modified numerals, giving rise to an intricate pattern of inferences.
- We have presented a logic that is able to model the inference patterns by adopting a split notion of disjunction taken from team logic and by using a formally defined pragmatic enrichment function.
- We have not yet included implication in our language. This makes comparisons with axiom systems for standard modal predicate logics difficult.

One final remark: my specific motivation for developing this account of indicative conditionals is of course to solve a puzzle, and to defend a particular semantic analysis of conditionals. ***But I have a broader motivation which is perhaps more important. That is to defend, by example, the claim that the concepts of pragmatics (the study of linguistic contexts) can be made as mathematically precise as any of the concepts of syntax and formal semantics****; to show that one can recognize and incorporate into abstract theory the extreme context dependence which is obviously present in natural language without any sacrifice to standards of rigor. (Stalnaker, 1975, p. 281–282)*

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