Exhaustive interpretation of complex sentences

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Abstract. In terms of Groenendijk & Stokhof's (1984) formalization of exhaustive interpretation, many conversational implicatures can be accounted for. In this paper we justify and generalize this approach. Our justification proceeds by relating their account via Halpern & Moses' (1984) non-monotonic theory of 'only knowing' to the Gricean maxims of Quality and the first sub-maxim of Quantity. The approach of Groenendijk & Stokhof (1984) is generalized such that it can also account for implicatures that are triggered in subclauses not entailed by the whole complex sentence.

1. Introduction

One of the most influential pragmatic theories of this century is the theory of conversational implicatures proposed by Grice (1967). It has not only been applied to various semantical problems, but also received considerable attention in philosophy and the social sciences. The main purpose of this theory was to defend a simple, truth-conditional approach to semantics, particularly to the meaning of sentential operators and quantificational phrases. Traditionally, the semantic meaning of natural language expressions like 'and', 'or', 'every', 'some', 'believe', and 'possibly' has been analyzed in terms of their intuitive analogs in classical logic: '$\wedge$', '$\vee$', '$\forall$', '$\exists$', '$\Box$', and '$\Diamond$', respectively. However, in many contexts these expressions receive interpretations that are different from what is predicted by this approach to their semantics. It turned out to be extremely difficult to come up with an alternative semantic theory that can account for the observed interpretations. This led some ordinary language philosophers such as Ryle and Strawson even to question the logical approach to natural language semantics in general.

According to Grice (1967), the mistake in this line of reasoning is the assumption that the problematic interpretations have to be explained by semantics only. He proposes to single out within the 'total significance' of a linguistic utterance the class of *conversational implicatures.* Grice takes conversational implicatures (from now on: implicatures) to be not part of the semantic meaning of an utterance, but to be due to principles of pragmatics. More particularly, they are inferences an interpreter can draw from taking the speaker to behave rationally in a cooperative conversational situation. According to Grice, this means that the speaker is assumed to obey certain rules that govern such behavior: the *maxims of conversation.* The idea, then, is to account for the interpretation of sentential operators and quantificational expressions in terms of both their semantic meaning as described in classical logic and a set of conversational implicatures.

While Grice's notion of conversational implicature is generally accepted, his proposal concerning the way these inferences are determined is still under debate. One central issue is the question whether (i) the conversational implicatures of an utterance are generated globally, after the grammar assigned a meaning to it, or, whether (ii) the generation refers to intermediate states of the grammar-driven semantic derivation. Following Grice's theory one should adopt the first position. However, it has been argued that a global derivation is not able to account for the implicatures actually observed (Landman (2000), Chierchia (ms)). The central argument brought forward by defenders of a local derivation is the behavior of implicatures in complex sentences, where the expression whose interpretation is to be explained is embedded under other sentential operators or quantificational expressions. For instance, the semantic meaning of numerals such as '100' is often analyzed as 'at least 100' and then conversational implicatures are taken to be responsible for the 'exactly'-reading these expressions often receive. Chierchia now claims that globalists cannot explain why sentence (1) is normally interpreted as implying that John believes that his colleague makes not more than \$100 an hour, hence, why the numeral in scope of the belief-operator receives an 'exactly'-interpretation.

(1) John believes that his colleague makes \$100 an hour.

A closer investigation of the argumentation of the localists Landman (2000) and Chierchia (ms) reveals that they discuss only one particular approach to a global description of certain conversational implicatures: the simple scalar approach. Theories that fit into this scheme assume that sentences can be associated with *expression scales* (ordered sets of expressions). They derive the conversational implicatures of an utterance of sentence s as follows. If s contains an item i from an expression scale that s can be associated with, let s' be a sentence one obtains by replacing i in s by another element of this scale that is ranked higher than i. Then s conversationally implies *not* s'. Such a kind of derivation is, for instance, proposed in Horn (1972). The conversational implicatures these theories aim to describe are now generally called – after this approach – *scalar implicatures.* To give a concrete example of a derivation, the simple scalar approach can, for instance, account for the exactly-readings of numerals occurring in simple sentences such as 'John's colleague makes \$100 an hour'. Assume that the sentence is associated with the scale containing the numerals and ordered by increasing height. Then its utterance is predicted to conversationally imply that John's colleague does not earn more than \$100 an hour. Together with the asserted meaning that John's colleague earns *at least* \$100 an hour we derive the *exactly* interpretation. The simple scalar approach easily gets into trouble with examples such as (1). The only implicature derivable this way is that John did not believe that his colleague makes more than \$100 an hour. This does not give us the exactly-reading of the embedded numeral that we intuitively perceive.

However, the argumentation of localists such as Landman and Chierchia would only be conclusive if they could show that *all* global accounts get into this kind of trouble. But the simple scalar approach that they criticize is not the only possible theory of this kind. A quite different global account has been introduced by Groenendijk & Stokhof (1984). Even though they address the exhaustive interpretation of answers, and not directly conversational implicatures, their description of exhaustivity is able to account for many phenomena analyzed under the latter heading, in particular for scalar implicatures. Except for its appealing predictions, this proposal also overcomes other shortcomings of previous approaches to conversational implicatures, such as the neglect of contextual interactions and dependence on the conceptually difficult notion of expression scales/alternatives. Recently, it has been shown (van Rooij & Schulz, submitted) how some well-known problems faced by Groenenendijk & Stokhof's (1984) account can be overcome by using results from decision theory and dynamic semantics.

In this article we will study whether this approach can deal with conversational implicatures of complex sentences. We will see that while it easily accounts for some of the counterexamples to the simple scalar approach brought forward by localists, other predictions it makes are not satisfying. We will then develop a generalization of the approach that can deal with the problematic cases.

At the same time, the generalization will address another open question. While the work of Groenendijk & Stokhof (1984) and van Rooij & Schulz (submitted) provide us with a powerful formal description of exhaustive interpretation and many conversational implicatures, neither of these works gives us a satisfying theory of the conceptual status of the inferences that they describe. Are they part of the semantic meaning? Are they products of pragmatic rules? Can they be explained by Grice's theory, hence, as due to taking the speaker to obey the maxims of conversation? As we will see, the generalization of Groenendijk & Stokhof's (1984) approach we are going to develop can be interpreted as formalizing some of the maxims of conversation.[1] Thereby it links

[1] Part of this observation can also be found in Spector (2003).

Groenendijk & Stokhof's (1984) exhaustivity operator to Grice's theory of conversational implicatures.

The rest of the paper is organized as follows. The next section will be dedicated to a discussion of the subtle data of implicatures in complex sentences. We will then introduce Groenendijk & Stokhof's (1984) approach to exhaustive interpretation in section 3 and discuss the predictions it makes concerning implicatures of complex sentences. Afterwards, a new pragmatic interpretation function is defined that tries to capture parts of Grice's theory of conversational implicatures. We will show that it contains Groenendijk & Stokhof's account as a special case. The fifth part is devoted to the application of the introduced framework to various problems involving conversational implicatures of complex sentences. We conclude with a discussion of the results.

2. Conversational implicatures of complex sentences: The data

A general problem one always has to face when discussing conversational implicatures is that the observations on which the whole subject is based are rather subtle and controversial. As the reader will agree with us very soon, this gets even worse if it comes to implicatures of complex sentences.[2] It is widely accepted that contextual features – in particular in what kind of exchange we are involved and what is relevant at the present state of conversation – have a great impact on the issue which implicatures are generated. For instance, Hirschberg (1985) argues convincingly that question-answer sequences are important for the analysis of scalar implicatures, and, just like Groenendijk & Stokhof, gives some examples where an implicature does not arise when the scalar term used is part of the answer's background (see example (11) in the sequel). We will take this observation seriously by restricting our discussion to implicatures that arise in a particular type of conversation: cooperative exchange of information. Furthermore we will make the information structure of the context explicit by taking all examples to be answers to overt questions. We will choose the questions such that the expressions whose interpretation is to be explained by implicatures will always occur in that part of the sentence that could have been used as term-answer. In this way we make sure that it contributes to the new, relevant information of the sentence. For instance, we are only interested in the implicatures induced by (1) when uttered in the

[2] Though we take it to be one of the advantages of Grice's pragmatic theory that it can explain this diversity of intuitions.

context of a question like 'How much does John believe his colleague makes?'.

As already mentioned in the introduction, the specific implicatures that have been used by localists to support their point belong to a particular rather well-studied group of conversational implicatures: the scalar ones. These inferences are traditionally associated with Grice's first sub-maxim of Quantity (we will call implicatures due to this maxim Quantity1-implicatures). Example (1) falls in this group. Quantity1-implicatures of a sentence s are, roughly speaking, sentences of the form $\neg s'$ where s' is an alternative to s that is in some sense stronger than s itself. Controversial in the literature is the issue with which epistemic force these sentences $\neg s'$ should actually be generated. Roughly speaking, the issue is whether Quantity1-implicatures should receive a strong or a weak reading. Proponents of the existence of a strong reading argue either with Horn (1972) that it is indeed $\neg s'$ that is conversationally implied (we will call this the *factive strong* reading), or with Gazdar (1979) (for scalar implicatures) that it is implicated that the speaker knows or believes $\neg s'$ (what we will call the *epistemic strong* reading). In the latter case, the derivation of $\neg s'$ is taken to be due to other rules such as veridicality of knowledge. Proponents of the existence of a weak reading have either argued that sometimes no Quantity1-implicature is generated at all (the *factive weak* reading, see Gazdar (1979) for scalar items under negation) or that one only infers that the knowledge of the speaker is limited with respect to $\neg s'$. Here a distinction should be made between the inference that the speaker thinks it is possible that $\neg s'$, and, hence, does not know/believe *that* s' (the *epistemic weak* reading, see Soames (1982) for scalar implicatures) and the inference that the speaker takes both $\neg s'$ and s' to be possible, and, hence, does not know or does not believe *whether* $\neg s'$ *or* s' (what we will call the *ignorance* reading, see Gazdar (1979) on clausal implicatures[3]). The different readings of the implicatures $\neg s'$ are summarized in figure 1 with some associated names.

strong readings		weak readings		
fact. strong	epist. strong	fact. weak	epist. weak	ignorance
Horn '72	Gazdar scalar	Gazdar neg.	Soames scalar	Gazdar claus.
$\neg s'$	$\Box\neg s'$	no implicature	$\Diamond\neg s'$	$\Diamond\neg s' \wedge \Diamond s'$

Figure 1.

[3] Clausal implicatures are another class of inferences Gazdar takes to be due to the first sub-maxim of Quantity. We will come back to them in section 4.

Let us now discuss some reported observations concerning Quantity1 implicatures of complex sentences using this vocabulary. The critical data we will discuss here fall roughly in three groups: (i) the scalar item occurs in the scope of a negation; (ii) the scalar item occurs in the scope of an existential quantifier; and (iii) the scalar item occurs in the scope of an all-quantifier (such as a belief operator). This is not a complete classification of the examples that have been brought forward against global theories of implicatures. But the classification does capture a wide range of these examples[4] and we cannot discuss all of them in one paper. The reader is invited to try the account we will propose to the other cases by herself.

The first context we are going to discuss is one of *negation*. Look at the following examples.

(2) (A: What did John eat?)
B: John didn't eat the apples or the pears.

(3) (A: How many apples did John eat?)
B: John didn't eat three apples.

In the literature, mainly two readings are reported for such examples[5]: (a) a factive weak reading, according to which no Quantity1-implicature is present if the scalar item occurs under negation (see e.g. Gazdar (1979), Hirschberg (1985), Landman (2000)); and (b) a reading where the sentence raises factive strong implicatures, for (3), for instance, that John did not eat less than two apples (e.g. Atlas & Levinson (1981), Levinson (2000), Chierchia (ms)). According to our informants the answers in (2) and (3) normally imply that the speaker cannot provide a complete answer and the given response is the best she can do. Hence, they report epistemic weak or ignorance implicatures. Some informants also can get the strong factive inferences but others rigorously exclude them.

A second group of examples brought forward by localists can be characterized as existence-quantifying contexts.[6] We start with the simple case of multiple disjunction.

(4) (A: Who knows the answer?)
B: Peter, Mary, or Sue.

[4] One may even argue, the most frequent ones.
[5] Though they are not always discussed in the context of such a question.
[6] We understand here under existence-quantifiers also '*or*', which quantifies over propositions, and modal existential quantifiers such as '*possibly*'.

Intuitively, the answer given in (4) has an interpretation according to which only one of the three persons knows the answer. Notice that the sentence B uses in her response only counts as being complex if one assumes that multiple disjunction constructions are based on the iterative application of a binary disjunction operator. But given that the opinions on the question how to analyze such constructions still diverge and that many global theories do have a problem with this example no matter whether they assume an analysis with one n-ary or two occurrences of a binary 'or', we thought it being a good idea to discuss this example here.[7] The next example is discussed in Landman (2000).

(5) (A: Who invited whom?)
B: Three boys invited four girls.

Landman analyzes the semantic meaning of the cumulative reading of (5) as follows: $\exists e \in {}^*INVITE : \exists x \in {}^*BOY : card(x) = 3 \wedge {}^*Agent(e) = x \wedge \exists y \in {}^*GIRL : card(y) = 4 \wedge {}^*Theme(e) = y$. Hence, the groups of boys and girls are introduced in the scope of an existential quantifier over events. According to Landman the factive strong implicature that should be described is that no more than three boys invited a girl and not more that four girls were invited by a boy. He takes this to be a problem for global accounts given that the noun phrases are interpreted under the scope of an existential quantifier.

Another example, adapted from Chierchia (ms), is the following:

(6) (A: What did John eat?)
B: John ate the apples or some of the pears.

Here, the scalar item 'some' occurs under 'or'. According to Chierchia the answer should get a reading according to which John either ate the apples, or some, but not all of the pears. This is again a factive strong inference. He claims that a global account cannot make this prediction.

Finally, we discuss some examples where classical scalar items occur in the scope of an all-quantification.

[7] Though Merin (1994) already observed that by a slight (though disputable) adaption of Gazdar's (1979) analysis, examples like (4) could be accounted for. As it turns out, a slight modification of Horn's (1972) analysis of scalar implicatures would do the trick as well. These modifications will not be of great help, however, for most other complex sentences discussed in this paper. More recently, Sauerland (2004) proposed yet another modification of traditional analyses of scalar implicatures to account for (4) and (6). But also this analysis will not be able to account for the set of data discussed in this paper.

(7) (A: Who kissed whom?)
B: Every boy kissed three girls.

According to Landman the answer of B in (7) has the factive strong implicature that every boy kissed no more than three girls. A similar intuition he reports for (8).

(8) (A: Who does Bill believe were at the party?)
B: Bill believes that there were four boys at the party.

Again, we should obtain the implicature that Bill believes that there were no more than four boys at the party. Chierchia makes similar observations. However, opinions diverge whether these implicatures are indeed generally observed in all-quantifying contexts. An anonymous referee questioned whether a sentence like 'Every admirer of Dickens read *Bleak House* or *Great Expectations*.' comes with the implicature that no admirer read both of the books. While we admit that these implicatures do not have to occur, we think nevertheless that they represent reasonable readings – particularly in the kinds of context we use. For instance, (9) seems to us to come with a reading implying that every student took not all three courses, Semantics 1 and Phonology 1 and 2.

(9) A: Which courses did your students take?
B: Every student took Semantics 1 or Phonology 1 and 2.

3. Implicatures in non-monotonic logic

The examples we have discussed in the last section have been used by Landman (2000) and Chierchia (ms) to argue against a global approach to conversational implicatures. However, as we have pointed out in the introduction, their argumentation is not conclusive because they only showed for one particular global theory that it fails to make the correct predictions. In this section we will introduce a promising alternative global approach, the description Groenendijk & Stokhof (1984) propose for exhaustive interpretation, and discuss the predictions this account makes for the implicatures of complex sentences.

3.1. Circumscription

According to Grice (1989), one of the defining features of conversational implicatures is that they may be cancelled. This is still by far the most commonly used property to identify these inferences. But calling implicatures cancelable is nothing but calling them non-monotonic inferences. This suggests that techniques and results from non-monotonic logic are useful for the analysis of implicatures. There is one approach that successfully uses such techniques to account for a particular class of conversational implicatures – though without noticing the connection to non-monotonic logic: the proposal of Groenendijk & Stokhof (1984) (from now on abbreviated as G&S).[8] Actually, they did not intend to describe implicatures, their aim was to account for the particular way we often interpret answers: we take an answer such as 'Peter' to question 'Who called yesterday?' not only as conveying that Peter was among the callers, but additionally that he is the *only* person who called yesterday. This reading is known as the *exhaustive interpretation* of the answer. It is a well-know fact – also illustrated by the paraphrase of the exhaustive interpretation just given – that this mode of interpretation is closely connected with the way we understand sentences containing 'only'. However, 'only'-paraphrases are also often given to reinforce implicatures. This holds in particular for inferences that are analyzed as scalar implicatures. To give an example, in a context where it is relevant how many cookies Paula ate, (10a) is quite generally reported to come with the cancelable inference that Paula did not eat all of the cookies. This meaning can also be expressed by (10b) – but now it is no longer cancelable.

(10) (a) Paula ate some of the cookies.
(b) Paula ate only $[\text{some}]_F$ of the cookies.[9]

Given this connection between exhaustive interpretation, the meaning of 'only', and scalar implicatures, it should not come as a surprise that as far as G&S are successful in accounting for the exhaustive interpretation of answers, they can also describe many classical scalar implicatures (and the meaning of 'only') – but of course, now dependent on the particular question the sentence is meant to answer.[10] Before we illustrate the descriptive power of the approach with some examples, let us first quickly review their proposal. G&S describe the exhaustive interpretation as the following interpretation function, taking as argu-

[8] But see also Wainer (1991) for a more explicit use of non-monotonic reasoning techniques.

[9] The notation $[\cdot]_F$ means that the relevant item is focussed, i.e. intonationally marked.

[10] Notice, by the way, that what we called an exhaustive interpretation in this paper is explicitly treated by Harnish (1976) as a Quantity1-implicature.

ments (i) the predicate B of the question, and (ii) the meaning of the term-answer, or focus, F to the question.[11]

$$exh(F, B) =_{def} F(B) \wedge \neg\exists B' \subseteq D : F(B') \wedge B' \subset B$$

Van Benthem (1989) first observed that this function can be seen as instantiating one of the first and best-known mechanisms to describe non-monotonic inferences: *predicate circumscription*, introduced by McCarthy (1980). Predicate circumscription is an operation that maps theories A and predicates P on the following theory that is then called the circumscription of P with respect to A.[12]

$$CIRC(A, P) \equiv_{def} A \wedge \neg\exists P' \subseteq D : A[P'/P] \wedge P' \subset P$$

It is obvious that $exh(F, B)$ can be obtained from $circ(A, P)$ by taking B for P and A to be $F(B)$, hence instead of the term-answer, the sentential answer.[13] For our purposes it is important to notice the following model-theoretic analog of circumscription: interpretation in *minimal models*. In order to make the connection, we have to enrich the model theory for classical predicate logic by defining an order on the class W of possible models of our predicate logical language in the following way: a model v is said to be more minimal than model w with respect to some predicate P, $v <_P w$, in case they agree on everything except the interpretation they assign to P and here it holds that $P(v) \subset P(w)$. In this setting fact 1 is a well-known result.

FACT 1.
$\forall w \in W : w \models CIRC(A, P) \Leftrightarrow w \models A \wedge \neg\exists v \in W : v \models A \wedge v <_P w.$

This fact shows that we can equivalently describe predicate circumscription by the interpretation function $circ^W(A, P)$ defined as follows.

[11] D stands for the domain of individuals. Even though the operation is described for n-ary predicates, we simplify and assume B to be of type $\langle e, t\rangle$.

[12] $A[P'/P]$ is the theory that is obtained by replacing all occurrences of P in A with P'.

[13] However, the two approaches are not equivalent. One thing to notice is that G&S took *exh* to be a description of an operation on semantic representations while $CIRC(A, P)$ is an expression in the object language. Second, one anonymous referee called our attention to the fact that both operations make indeed different predictions in case there are occurrences of the question-predicate in the focus- or term-answer-part F. The circumscription of A w.r.t. P minimizes P in all occurrences of $A = F(P)$. The operation of G&S does so for the background-occurrence only. To see the difference, take the answer 'Men that wear a hat' to a question 'Who wears a hat?'. Circumscribing this answer has the result that the extension of 'wears a hat' is empty – which is not the intuitive reading. *exh* correctly predicts that exactly those people wear a hat that are men that wear a hat. In this paper we will assume that the question-predicate will not occur in the focus or term-answer-part.

$$circ^W(A,P) =_{def} \{w \in W | w \models A \wedge \neg\exists v \in W : v \models A \wedge v <_P w\}$$

3.2. Prospects and problems of the circumscription account

In terms of G&S's exhaustivity operator, or of circumscription, quite a number of conversational implicatures (including *scalar* ones) can be accounted for straightforwardly. Except for the obvious result that from the answer '$P(a)$' to a question with question-predicate P we derive that a is the only object that has property P, we also derive (i) for 'John ate three apples' that John ate *exactly* three apples[14]; for P as question-predicate (ii) the *exclusive* reading of a disjunctive sentence like '$P(a) \vee P(b)$'; (iii) the implicature that not everybody has property P from the assertion that *most* have; (iv) the so-called *conversion*-inference that every P-thing is a Q-thing, if the answer is 'Every Q is a P'; and (v) the biconditional reading of 'John will come if Mary will go', if this sentence is given as answer to the polar question 'Will John come?'. Another pleasing property of an exhaustivity analysis of implicatures is that it predicts that it depends on the context, or question-predicate, whether we observe these inferences. If, for instance, the scalar item occurs in the question-predicate P instead of in the focus F of the answer, as for instance in example (11), no implicatures are predicted.[15]

(11) A: Do you have some apples?
B: Yes, I have some apples.

This may account (at least partly) for the often cited context- and relevance-dependence of implicatures and the observed factive weak readings of sentences containing scalar items. All these predictions are appealing and they show that this approach outperforms many other accounts of Quantity1-implicatures. Note, furthermore, that G&S's description of exhaustive interpretation (and this is even more true for $circ(A,P)$) is a *global* account of implicatures, because it can be assumed to work on the output of the grammar, or on some kind of discourse representation.[16]

[14] Given an *at least* -semantics for numerals and in the context of a question 'How many apples did John eat?'.

[15] At least, in case the scalar item is not focussed in the question itself.

[16] There is a strong fraction of semanticists that have argued that the output of the grammar are meanings structured in focus and background. For *circ* we would only need the question-predicate accessible in the context.

Still, there are some serious limitations of an analysis of implicatures in terms of circumscription or exhaustive interpretation.[17] First, it is quite obvious that such an analysis cannot account for the context-dependency of exhaustive interpretation on other factors than the predicate of the question, which plays, for instance, a role for phenomena such as domain restriction, or answers that receive a mention-some, instead of a mention-all reading. This is inevitable given the functionality of *exh* as defined by G&S. The restricted functionality of the operation causes also other problems: because circumscription (or exhaustification) works immediately on the *semantic meaning* of an expression, it is predicted that if two sentences have the same semantic meaning, they will give rise to the same implicatures as well. This, however, does not seem to be the case. It is, for instance, standardly assumed in generalized quantifier theory (adopted by G&S) that 'three men' has the same semantic meaning as '*at least* three men'. But sentences in which the former occurs seem to give rise to an 'at most' implicature, while the latter do not. In Van Rooij & Schulz (submitted) solutions to these and some other problems are proposed by bringing the approach of G&S (particular in the form of $circ(\cdot)$) together with some independent developments in natural language semantics and pragmatics. For instance, by adopting *dynamic semantics* (e.g. Kamp, 1981; Heim, 1982), which allows more fine-grained distinctions than static semantics does, one can account for at least part of the functionality problem. As for the context dependence of exhaustive answers, it is proposed in van Rooij & Schulz (submitted) that the ordering between worlds should not be defined in terms of the extensions the question-predicate has in different worlds, but rather in terms of the *utility* or *relevance* of the propositions that express what those extensions are in those worlds. In this way we make the exhaustification operator more sensible to the beliefs and preferences of the agents involved, and can account for, among others, both mention-all and mention-some readings of answers. Furthermore, this may also help us to get an even better grasp of the context (and relevance) dependence of implicatures.

In this paper we are interested in the issue to what extent this global account of implicatures can deal with implicatures of complex sentences. So, let us start to check the examples discussed in section 2. It proves to be the case that some of the observations that are claimed by Landman (2000) and Chierchia (ms) to be out of reach of global accounts are correctly predicted immediately.

[17] For an elaborate discussion see, among others, Groenendijk & Stokhof (1984), Wainer (1991), and van Rooij & Schulz (submitted).

First, even though this is not an example from localists, notice that by exhaustive interpretation or circumscription we straightforwardly get the correct prediction for (4) that only one of the three disjuncts is true. This is even independent of the issue which position is taken with respect to the functionality of 'or'. In the same way, *circ* predicts for (5) and (6) the reported factive strong implicatures. The approach can also deal with the all-quantification examples (7) and (9). We discuss shortly the second one. Consider the sentence $\forall x(P(x,a) \vee P(x,b))$ in the context of a question $?x, y[P(x,y)]$ (or $?y[\forall x P(x,y)]$).[18] Applying *circ* will minimize the number of tuples $\langle x, y\rangle$ for which $P(x,y)$ holds. In particular it will minimize for every x the number of y such that $P(x,y)$. For example (9) this number is smallest if every student either took semantics 1 or Phonology 1 and 2, but not both. Hence, it is implied that no student took all three courses. However, G&S's approach is not able to deal with the other examples we have discussed in section 2, and also the improvements on this account proposed in van Rooij & Schulz (submitted) are of no help here. These were the cases where the relevant expression occurs, for instance, under negation or a verb of belief. G&S were already well-aware of the problem concerning negation: by taking the exhaustive interpretation of $\neg P(a)$ with respect to predicate P, we end up with the wrong result that in the actual world P has an empty extension. Some improvement can be made by proposing that if a negation occurs in the answer then it is not P that is circumscribed, but the complement $\bar{P}$ (as proposed by von Stechow & Zimmermann (1984)). In this way we can account for the readings of (2) and (3) with factive strong implicatures. However, as we have already discussed above, while some speakers of English can get these inferences, many others claim that they do not. They understand an answer like 'Not Peter' to the question 'Who called yesterday?' as stating that Peter did not call yesterday and that as far as the speaker knows other individuals might have called, and, hence, only get a reading with epistemic weak implicatures. Furthermore, sentences concerning the belief-state of the speaker or of other agents such as (1) are problematic for the reason that an account of exhaustive interpretation in terms of circumscription is purely extensional. *circ* totally ignores information about P in other (epistemic) possibilities than the actual world. To give another example where this causes a problem, take (12).

(12) A: Who knows the answer?
B: Peter and possibly Mary.

Following our informants, B's response can have two different readings. According to the first, it gives rise to the factive strong inference that

[18] Where '$?x[P(x)]$' represents the question 'Who has property P?'.

except for Peter and perhaps Mary, nobody knows the answer. According to the second, the answer exhausts the knowledge of the speaker in the sense that she knows that Peter knows the answer, she has some evidence that also Mary knows the answer, but for all other people they may as well not know the answer. Hence, in this case the interpreter only derives epistemic weak implicatures. For the reason given above, *circ* can account for neither of these readings.

4. All that the speaker knows. Setting the stage

In this section, a generalization of the circumscription account to exhaustive interpretation is introduced that will help us to account for the problematic implicatures of complex sentences. But before we start we should decide first which of the reported readings we actually want to describe. At the beginning of section 2 we distinguished 5 readings of Quantity1-implicatures that can be found in the literature: the factive strong, the epistemic strong, the factive weak, the epistemic weak, and the ignorance reading. However, they may not all exist independently of each other. For instance, some of the readings entail others. The latter may, therefore, be distinguished as an independent reading only because some additional inferences were ignored. There is, for instance, some evidence that at least in some cases when a factive weak reading is diagnosed what was actually observed was an epistemic weak interpretation.[19] Furthermore, we have seen in section 3 that *circ* can account for weak factive readings if the 'trigger' of the implicature occurs in the question-predicate or background of the answer. We will propose that in those cases discussed here, where the expression that triggers an implicature appears in the focus- or term-answer-part of an answer to an explicitly asked question we do not have to distinguish a weak factive reading. Likewise, there is some evidence that also the ignorance reading is due to a misinterpretation of the data. For instance, according to Gazdar's (1979) formal account of clausal implicatures at page 59, the sentence 'My sister is either in the bathroom or in the kitchen' has the implicatures *the speaker does not know that her sister is in the bathroom, the speaker does not know that her sister is not in the bathroom, the speaker does not know that her sister is in the kitchen, the speaker does not know that her sister is not in the kitchen.* Earlier

[19] For example, in Zimmermann (2000) the author distinguishes an open list reading for disjunctions/conjunctions 'expressing undecidedness or uncertainty whether the list is exhaustive' (p. 261), but he models this interpretation as a factive weak reading.

on at page 50, however, he describes the implicatures of the sentence as being *I don't know that my sister is in the bathroom and I don't know that my sister is in the kitchen* and thus only reports epistemic weak implicatures. Based on such observations we decided to assume that for the cases discussed here, we can also dial out the ignorance reading. Finally, we would like to claim that there is no factive strong implicature without the inference that the speaker knows the implicature to be true. Then, under the additional assumption that the factive strong implicatures come about via the veridicality of knowledge, we can restrict our considerations to one strong reading: the epistemic strong reading of implicatures.

Given this analysis, what we have to model is a function that makes the interpretation of a sentence dependent on the question-predicate and allows both epistemic strong and weak implicatures. Furthermore, we have seen that we already have with *circ* a promising description of the factive consequences of the epistemic strong inferences. Hence, any approach to the epistemic strong reading should turn out to predict factive inferences that are strongly related to those of *circ*.

The proposed reduction to two context-dependent readings of the implicatures of an answer nicely reflects a central tenor in the literature on implicatures. Many authors acknowledge the existence of both, a weak and a strong reading of Quantity1-implicatures – even though they may disagree on the exact form of these readings. There are also some divergences with respect to the question what determines which reading should be predicted in a certain context. Recall from section 2 that the Quantity1-implicatures of a sentence s are based on sentences s' that are, in some sense, stronger alternatives to s. According to Gazdar (1979) it is the form of s' that decides the epistemic force with which a Quantity1-implicature is generated. He distinguishes between two classes: *scalar* and *clausal* Quantity1-implicatures. Scalar implicatures are based on sentences s' that imply s and that are obtained from s by replacing an item in s by an alternative from a certain expression scale. The actual implicature is that the speaker knows $\neg s'$ (epistemic strong reading). A *clausal implicature* is based on a sub-sentence s' of s that is not decided by s. Here, the actual implicature is that the speaker does not know $\neg s'$ and does not know s' (ignorance reading). According to such an approach, an utterance may raise at the same time both weak and strong Quantity1 implicatures.

Gazdar has often been criticized for the prediction that scalar implicatures always have to have strong epistemic force. Instead, it has been argued by many students of conversational implicatures that scalar implicatures should be generated primarily with weak epistemic force.

Only in contexts where the speaker is assumed/believed to know that $\neg s'$ if $\neg s'$ is true, or, in other words, if the speaker is taken to be *competent* on s', the epistemic strong reading is derived (see, among others, Soames (1982), Leech (1983), Horn (1989), Matsumoto (1995), and Green (1995)). Apart from intuitions, one other argument given for this analysis is that in the Gricean derivation often used to explain scalar implicatures one explicitly has to make this additional assumption on the competence of the speaker; the maxims alone are not strong enough to derive the strong reading.[20,21] We agree with these authors that, indeed, scalar implicatures can, depending on the context, be generated with either strong or with weak epistemic force and that Gazdar's predictions are not adequate here. An additional pleasing property of such an analysis is that Gazdar's unmotivated difference between the epistemic force of scalar and clausal implicatures is weakened. Now both, scalar and clausal implicatures are primary generated with a weak epistemic reading. However, the distinction between the two kinds of implicatures does thereby not disappear. For one thing, clausal implicatures are still claimed to have the stronger ignorance reading. For another, both classes of implicatures are described by two different generation processes for s'. This is not very convincing given that Gazdar ascribes both types of inferences to the same maxim: the first sub-maxim of Quantity.

Schulz (2003) shows how some developments in non-monotonic logic, namely the work of Halpern & Moses (1984) on the concept of 'only knowing', recently generalized by van der Hoek et al. (1999, 2000), can be used to improve on Gazdar's account of clausal implicatures. One of the advantages of the approach is that it is much closer to Grice's formulation of the first sub-maxim of Quantity than Gazdar's account. This raises the following question: given that we have an approach that nicely describes clausal implicatures, can we extend this approach so

[20] Consider, for instance, the derivation given by Levinson: 'The speaker S has said s; if S was in a disposition to [...] assert s' then he would be in breach of the first maxim of Quantity if he asserted s. Since I the addressee assume that S is cooperating, and therefore will not violate the maxim of Quantity without warning, I take it that S wishes to convey that he is not in a position to state that the stronger s' holds *and indeed knows that it does not hold.*' (Levinson, 1983, pp. 134-5, slightly modified, italics added by the authors).

[21] An interesting additional argument is due to Soames (1982). To avoid making some false predictions, Gazdar (1979) has to assume that clausal implicatures are added to the beliefs of the interpreter before scalar implicatures. But he does not give any independent motivation for why the generation of implicatures has to be ranked this way. As Soames points out, if one assumes that scalar implicatures have epistemic weak force and that the strong readings are due to additional beliefs about the competence of the speaker, one can do without this additional assumption.

that it can also deal with scalar implicatures and the different epistemic readings observed? This is the topic of the present section.

We will start by applying this new approach to clausal implicatures to the situation at hand. In order to do so, we have to introduce some technical machinery. The relevant aspects of natural language interpretation will be modeled using a formal language $\mathcal{L}$ of modal predicate logic that is generated from predicate and function symbols of various types, variables, the logical connectors $\neg, \wedge$, and $\forall$ (we will use $\phi \vee \psi$ and $\exists x.\phi$ to abbreviate $\neg(\neg\phi \wedge \neg\psi)$ and $\neg\forall x.\neg\phi$, respectively), and for the moment we are satisfied with having one modal operator $\Box$ in our language that refers to the knowledge-state of the speaker. $\Box\phi$ should be read as *the speaker knows that* ϕ. $\Diamond\phi \equiv_{def} \neg\Box\neg\phi$ expresses in turn that *the speaker takes it as possible that* ϕ. $\mathcal{L}$ is the set of all sentences (hence, formulas containing no free variables) that can be constructed from these primitives in the standard way. We will occasionally refer to the *basic language* $\mathcal{L}^0 \subseteq \mathcal{L}$, which is the set of sentences containing no modal operators.

$\mathcal{L}$-formulas are interpreted with respect to states s and assignments g. A state is a tuple $\langle M, w\rangle$ consisting of a model $M = \langle W, R, D, V\rangle$ (where W is a set of points (possible worlds), R a binary relation on W, D a set of individuals, and V an interpretation function for our non-logical vocabulary) and a point $w \in W$. $R[w]$ denotes the set $\{v \in W | R(w, v)\}$. We will use a strong version of the unique domain assumption: not only do we take the domain D to be the same for all worlds in a state, but we assume in addition that all states have the same domain. Truth of a formula $\phi \in \mathcal{L}$ with respect to a state $s = \langle M, w\rangle$ and an assignment g is defined in standard ways. We will give here only the definition of truth for a modal formula and assume the reader's familiarity with the standard definitions: $M, w, g \models \Box\psi$ iff$_{def}$ for all worlds v such that $v \in R[w]$ it holds that $M, v, g \models \psi$. When we talk about the truth of a sentence, the assignment will be dropped. We will work with a restricted class of states: those states where the relation R is an equivalence relation. This is a standard way to turn R into a relation that can represent the knowledge-state of the speaker: she is assumed to be fully introspective, and her beliefs are taken to be true. Let $\mathcal{S}$ be the class of states that fulfill this restriction.

We will now directly start from Grice's theory of conversational implicatures and try to capture the central ideas of his maxims Quantity1 and Quality in terms of a pragmatic interpretation function for answers: this function will map sentences on the set of states where the speaker *knows* what she claims (she believes her utterance and has evidence for its truth) and provides all the relevant knowledge she has – she gives the best (i.e. most informative) answer she can, given her knowledge.[22] To account for the 'maximally informative' part, we enrich the class of states $\mathcal{S}$ for our language with an order that compares how much relevant information the speaker has in different states. For the application at hand the order is intended to compare how much the speaker knows about the extension of predicate P in different states. Our pragmatic interpretation function selects among those states where the speaker knows that her utterance is true only the minimal elements with respect to this order, hence, those states where the speaker knows least about the extension of P.[23]

DEFINITION 1. *(The Epistemic Weak Reading)*
Given a sentence A of $\mathcal{L}$ and a predicate P, we define the pragmatic meaning $eps_1^S(A,P)$ of A with respect to P and a set of states S as follows:

$$eps_1^S(A,P) =_{def} \{s \in S | s \models \Box A \wedge [\forall s' \in S : s' \models \Box A \rightarrow s \preceq_\Box s']\}$$

But how to define the order $\preceq_\Box$? The concept is very simple. When does a speaker know more about the extension of the question-predicate P? If she takes less possible extensions of P to be compatible with her beliefs. And because for the epistemic weak reading only positive information about P counts (hence, knowledge that $P(a)$ but not knowledge that $\neg P(a)$), it is sufficient to call $s_1 = \langle M_1, w_1\rangle$ as least as small as $s_2 = \langle M_2, w_2\rangle$ if for every epistemic possibility in s_2 the speaker distinguishes an epistemic possibility in s_1 where the extension of P is smaller than or equal to the extension of P in s_2. Hence, we define:

DEFINITION 2.
For all $s_1 = \langle M_1, w_1\rangle, s_2 = \langle M_2, w_2\rangle \in \mathcal{S}$:

$$\begin{array}{lll} s_1 \preceq_\Box s_2 & \textit{iff}_{def} & \forall v_2 \in R_2[w_2] \exists v_1 \in R_1[w_1] \\ & & V_1(P)(v_1) \subseteq V_2(P)(v_2) \\ s_1 \cong_\Box s_2 & \textit{iff}_{def} & s_1 \preceq_\Box s_2 \textit{ and } s_2 \preceq_\Box s_1 \end{array}$$

To illustrate the workings of the order and also of $eps_1^S(\cdot)$, assume that D has only two elements, a and b. Then, the extension of P in each point v of every state $s = \langle M, w\rangle \in \mathcal{S}$ can only have four different

[22] Hence, 'relevance' is here interpreted as relevant to the question to which the interpreted sentence is intended as answer.
[23] Obviously, this interpretation function is an instance of interpretation in preferential structures and, hence, the basis of a non-monotonic notion of entailment.

values. We use this observation to define the following function f on $R[w]$:

$$f(v) = 0 \quad \text{iff}_{def} \quad V(P)(v) = \emptyset, \qquad f(v) = b \quad \text{iff}_{def} \quad V(P)(v) = \{b\},$$
$$f(v) = a \quad \text{iff}_{def} \quad V(P)(v) = \{a\}, \quad f(v) = ab \quad \text{iff}_{def} \quad V(P)(v) = \{a, b\}.$$

We can then classify states according to which of those four cases the speaker considers possible. For $X \subseteq \{0, a, b, ab\}$ define $[X] = \{\langle M, w\rangle | x \in X \Leftrightarrow \exists v \in R[w] : f(v) = x\}$. For instance, $[\{a, ab\}]$ stands for those states where the speaker distinguishes at least one epistemic possibility where a but not b is in the extension of P and one epistemic possibility where both a and b are in P. With these definitions at hand we can now represent the structure $\preceq_\Box$ imposes on $\mathcal{S}$ – see figure 2.

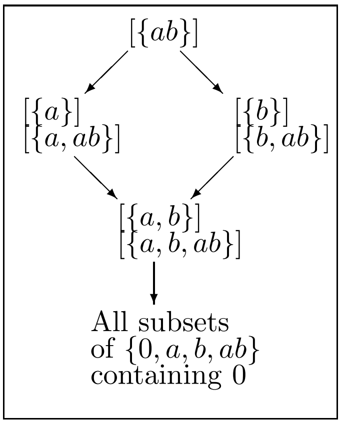

Figure 2.

In this picture an arrow from some set of states y to some set of states x represents that for all $s_1 \in x, s_2 \in y$: $s_1 \preceq_P s_2$. Not very surprising: for states $s_1, s_2 \in [X]$, $X \subseteq \{0, a, b, ab\}$: $s_1 \cong_\Box s_2$. This picture shows that the $\preceq_\Box$-smallest states in S are those where the speaker takes it to be possible that neither a nor b has property P. The maximum is constituted by those states where the speaker knows that both a and b have property P.

We can now use figure 2 to calculate the pragmatic interpretation of some examples. Let us for the moment make the simplifying assumption that the speaker knows which individual bears which name and that there is only one name for each individual. Hence, we can identify the individuals with their names. To calculate $eps_1^{\mathcal{S}}(P(b), P)$ we first select the states where the speaker knows $P(b)$: $[\{b\}] \cup [\{ab\}] \cup [\{b, ab\}]$. Then, figure 2 helps us to select the minimas among these states. We end up with $eps_1^{\mathcal{S}}(P(b), P) = [\{b\}] \cup [\{b, ab\}]$. Hence, according to the interpretation function eps_1 the speaker considers it (at least) possible that a does not have property P. In general, under the assumption made about the relation between names and individuals, $eps_1^{\mathcal{S}}(P(b), P)$ implies that for all $x \in D$, $x \neq b$ the speaker takes $\neg P(x)$ to be possible.

Applied to the sentence $P(a) \vee P(b)$ we obtain as the pragmatic meaning the states that are of type $[\{a, b\}]$ or type $[\{a, b, ab\}]$. Hence, applying the pragmatic interpretation function eps_1 allows the interpreter to conclude from an utterance of the form $P(a) \vee P(b)$ that the speaker neither knows $P(a)$ nor $P(b)$. This captures exactly Gazdar's (1979, p. 50) reported intuition concerning clausal implicatures of a disjunction. Furthermore, we obtain the inference $\neg\Box(P(a) \wedge P(b))$ which according to Soames (1982), among others, is the (epistemic weak) scalar implicature of such a sentence. In sum, at least for these two examples, it looks as if eps_1 really allows us to describe the epistemic weak inferences we wanted to account for. And, as the second example shows, we can describe with one and the same operation both scalar and clausal implicatures.

However, so far we have done nothing to account for the strong reading that Quantity1-implicatures sometimes receive. To describe this occasional strengthening of eps_1, we want to use the intuition expressed so often in the literature that the epistemic strong reading is obtained in case the speaker is taken to be competent, or be an authority. For our setting this means to take the speaker to know the answer to the question asked. In Zimmermann (2000), Groenendijk & Stokhof's (1984) analysis of '*knowing whether*' is used to define what it means for a speaker to be competent with respect to a predicate P.

DEFINITION 3. *(Competence)*
A speaker is competent in state $\langle M, w\rangle \in \mathcal{S}$ (where $M = \langle W, R, D, V\rangle$) with respect to a predicate P iff$_{def}$ $\forall v \in R[w].\forall x \in D : w \in P(x) \Leftrightarrow v \in P(x)$.

Given the intuition that competence should play a role in the derivation of the strong reading of Quantity1-implicatures, a first idea that comes to mind is that we simply have to apply eps_1 to the set COMP of states where the speaker is competent with respect to P to obtain the epistemic strong reading. Unfortunately, this will not work. According to this approach the epistemic strong reading can occur only in situations where the speaker is taken to be competent, i.e., if her utterance is interpreted with respect to COMP (or a subset of COMP). However, there are sentences that have epistemic strong implicatures but cannot stem from a speaker that is (i) competent in the sense just defined, and (ii) obeying the maxims Quantity1 and Quality as interpreted by eps_1. A good example for such a sentence is a disjunction like $P(a) \vee P(b)$. Let us calculate $eps_1^{COMP}(P(a) \vee P(b), P)$. A speaker competent on P knows whether $P(a)$ holds and whether $P(b)$ holds. Hence, if she believes $P(a) \vee P(b)$ she can only be in one of the following three types of states: $[\{a\}]$, $[\{b\}]$ or $[\{ab\}]$. Now, take a look at figure 2 again. Applying eps_1 means that we have to select the $\preceq_\Box$-minimas among these states. In this case the speaker can be in different types of minimal knowledge-states: $[\{a\}]$ and $[\{b\}]$. But then the speaker was withholding the relevant information which of $P(a)$ and $P(b)$ does in fact hold. Hence, it is obvious for the interpreter that the speaker must be breaking the maxim Quantity1. eps_1 reflects this by assigning to $P(a) \vee P(b)$ on COMP the empty interpretation: the sentence is predicted to be pragmatically not well-formed. This certainly does not match our intuitions. Such a sentence can be read as implying that the speaker *knows* that not both $P(a)$ and $P(b)$ hold, which is an epistemic strong implicature (while raising at the same time the epistemic weak implicature that the speaker does not know which of the disjuncts is true). Given this result we have to conclude that eps_1^{COMP} does not provide an adequate description of the epistemic strong reading of implicatures.

Where should we locate the mistake? Did we choose the wrong formalization of the maxims? That does not seem to be the case, because we do get the right results for the epistemic weak inferences. Therefore, it might be a better idea to keep this part and rethink the role of competence. Consider our example again. The inference we were after was $\Box\neg(P(a) \wedge P(b))$. But to derive this, we do not have to assume that the speaker is fully competent on the extension of P – the assumption that caused the counterintuitive interpretation above –, it is enough to assume that she knows whether or not the conjunction $P(a) \wedge P(b)$ is true. This suggests modeling the strong reading by adding competence only as far as this is consistent with the assumption that the speaker obeys the maxims Quantity1 and Quality, an idea that can also be found in Spector (2003).[24] Thus, (i) also competence should be treated as something that is maximized, and (ii) competence should be maximized *after* the application of eps_1.

To model maximizing competence we will use the same strategy as for minimizing knowledge. We take $\preceq_\Diamond$ to denote the order defined as follows:

DEFINITION 4.
For all $s_1 = \langle M_1, w_1 \rangle, s_2 = \langle M_2, w_2 \rangle \in \mathcal{S}$*:*

$$\begin{array}{lll} s_1 \preceq_\Diamond s_2 & \textit{iff}_{def} & \forall v_1 \in R_1[w_1] \exists v_2 \in R_2[w_2] \\ & & V_1(P)(v_1) \subseteq V_2(P)(v_2) \\ s_1 \cong_\Diamond s_2 & \textit{iff}_{def} & s_1 \preceq_\Diamond s_2 \textit{ and } s_2 \preceq_\Diamond s_1 \end{array}$$

[24] Gazdar's (1979) idea, recently modified by Sauerland (2004), to generate strong $\Box\neg s'$ implicatures (his scalar ones) only in as far as they are compatible with (the set of) weak implicatures of the form $\neg\Box s''$ (his clausal ones) is, of course, closely related as well. However, Gazdar does not motivate this strengthening by appealing to the informedness of the speaker.

We can use the same notation introduced earlier to visualize the structure that $\preceq_\Diamond$ imposes on $\mathcal{S}$ in case there are only two individuals a and b – see figure 3. One can see that the lower a state is in the hierarchy imposed by $\preceq_\Diamond$, for the more individuals the speaker knows in s that they do not have property P. To model maximizing competence we will thus select minima with respect to this order.

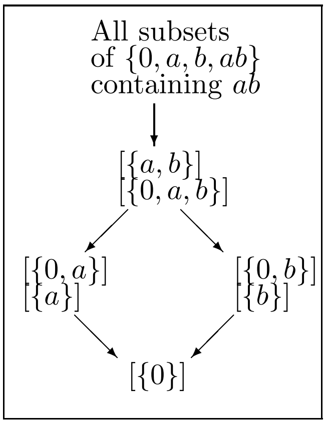

Figure 3.

$\preceq_\Diamond$ works quite opposite to $\preceq_\Box$. The $\preceq_\Diamond$-biggest states are those where the speaker takes it to be possible that both a and b have property P. The minimum is constituted by those states where the speaker knows that both, a and b, do not have property P.

We define a new pragmatic interpretation function, $eps_2^S(A, P)$, that strengthens $eps_1^S(A, P)$ by selecting among the states in $eps_1^S(A, P)$ those that are minimal with respect to $\preceq_\Diamond$.

DEFINITION 5. *(The Epistemic Strong Reading)*
Given a sentence A of $\mathcal{L}$ and a predicate P, we define the pragmatic meaning $eps_2^S(A, P)$ of A with respect to P and a set of states S as follows:

$$\begin{aligned} eps_2^S(A, P) \;&=_{def}\; \{s \in S | s \models \Box A \wedge [\forall s' \in S : s' \models \Box A \rightarrow [s \preceq_\Box s' \wedge \\ &\qquad\qquad (s \cong_\Box s' \rightarrow s' \not\prec_\Diamond s)]]\} \\ &= \quad \{s \in eps_1^S(A, P) | \forall s' \in eps_1^S(A, P) : s' \not\prec_\Diamond s\} \end{aligned}$$

In this interpretation function the application of the order $\preceq_\Box$ has priority over $\preceq_\Diamond$. The latter only comes to work if the former does not see any difference between two states.[25] This captures our earlier conclusion that maximizing competence should only be executed as far as it does not conflict with the maxims Quantity1 and Quality as formalized in eps_1.

To illustrate the working of $eps_2^S(\cdot)$, let us calculate the pragmatic interpretation assigned by eps_2 to the sentence $P(b)$ with respect to P

[25] Thus, $eps_2^S(\cdot)$ falls under the heading of *Prioritized Circumscription.*

and $\mathcal{S}$. Again, we assume for simplicity that the speaker knows which individual bears which name and that there is only one name for each individual. We already know that the $\preceq_\Box$-minimal states where $\Box P(b)$ holds are $eps_1^{\mathcal{S}}(A, P) = [\{b\}] \cup [\{b, ab\}]$. Using figure 3 we can now select the $\preceq_\Diamond$-minimal states among the elements of $eps_1^{\mathcal{S}}(A, P)$ and end up with $eps_2^{\mathcal{S}}(P(b), P) = [\{b\}]$. Hence, according to this pragmatic interpretation function the speaker knows that $P(b)$ and $\neg P(a)$ – the second part is the epistemic strong implicature we wanted to derive. With the veridicality of knowledge we obtain $eps_2^{\mathcal{S}}(P(b), P) \models P(b) \wedge \neg P(a)$ – exactly the predictions of $circ^{\mathcal{S}}(P(b), P)$! [26]

Let us discuss one more example. $eps_2^{\mathcal{S}}(P(a) \vee P(b), P)$ is the set of $\preceq_\Diamond$-minima among $eps_1^{\mathcal{S}}(A, P) = [\{a, b\}] \cup [\{a, b, ab\}]$, which is the set $[\{a, b\}]$. In these states the speaker takes both $P(a)$ and $P(b)$ as possible. Hence, $eps_2^{\mathcal{S}}(P(a) \vee P(b), P)$ entails the epistemic weak clausal implicatures $\Diamond P(a)$ and $\Diamond P(b)$ (as did eps_1). We also derive that the speaker knows that $P(a)$ and $P(b)$ are not both true at the same time. Thus, together with the veridicality of knowledge this interpretation function can account for the exclusive interpretation of 'or': $eps_2^{\mathcal{S}}(P(a) \vee P(b), P) \models \neg(P(a) \wedge P(b))$, as was also predicted by $circ^{\mathcal{S}}(P(a) \vee P(b), P)$.

In the two cases discussed above the pragmatic interpretation function eps_2 makes very promising predictions: we obtain the intended epistemic strong readings of certain implicatures which, together with the veridicality of knowledge, imply the inferences of *circ*. Of course, we would rather like to establish the adequacy of eps_2 in some generality. Let us see what we can achieve here. The following fact is quite easy to prove.

FACT 2. *For $A \in \mathcal{L}^0$ and P a predicate of $\mathcal{L}$, $eps_2^{\mathcal{S}}(A, P)$ implies circumscription of the predicate P with respect to A and $\mathcal{S}$:*

$$eps_2^{\mathcal{S}}(A, P) \subseteq circ^{\mathcal{S}}(A, P).$$

PROOF : Assume that for $s = \langle M, w\rangle \in \mathcal{S}$, $s \in eps_2^{\mathcal{S}}(A, P)$. By reflexivity of R it follows that $s \models A$. Assume additionally that $s \notin$

[26] At this point we straightforwardly extend the definition of $circ^W$ to the operation $circ^S$ on states of our language $\mathcal{L}$ of modal predicate logic. This comes down to the following adapted definition of $\leq_P$. We say for two states $s_1 = \langle M_1, w_1\rangle, s_2 = \langle M_2, w_2\rangle$ that $s_1 \leq_P s_2$ if the respective interpretation functions V_1 and V_2 agree in w_1 and w_2 on everything except possibly the interpretation of P and $V_1(P)(w_1) \subseteq V_2(P)(w_2)$. We implicitly used $circ^S$ already in section 3 when we discussed the predictions of $circ^W$ for sentences referring to the epistemic state of some agent as in example (1).

$circ^{\mathcal{S}}(A, P)$. In consequence $\exists s' = \langle M', w' \rangle \in \mathcal{S} : s' \models A$ and V' of s' is defined in w' as is V of s in w except that $V'(P)(w') \subset V(P)(w)$.

We choose an $s^* = \langle M^*, w \rangle \in \mathcal{S}$ where M^* is like M except that the valuation function V^* deviates from V of M as follows: if for $v \in R[w]$: $V(P)(w) \subseteq V(P)(v)$, then V^* evaluates the non-logical vocabulary in v as does V' of M' in w'; in all other points of $R[w]$, V and V^* assign the same interpretation to the non-logical vocabulary.[27] We will show that (i) $s^* \models \Box A$, (ii) $s^* \preceq_{\Box} s$ and (iii) $s^* \prec_{\Diamond} s$. Thus, s^* falsifies that s is in $eps_2^{\mathcal{S}}(A, P)$. This would prove the claim.

Ad i). Let us introduce the notation $\langle N, u \rangle \equiv^0 \langle N', u' \rangle$ iff$_{def}$ the valuation functions of the models N and N' agree on the interpretation of the non-logical vocabulary in u and u'. For every $v \in R[w]$ it is either the case that $\langle M^*, v \rangle \equiv^0 \langle M, v \rangle$ or (in case $V(P)(w) \subseteq V(P)(v)$) $\langle M^*, v \rangle \equiv^0 \langle M', w' \rangle$. It is easy to see that if for two states $\langle N, u \rangle \equiv^0 \langle N', u' \rangle$ then they make the same sentences $\phi \in \mathcal{L}^0$ true. Because $A \in \mathcal{L}^0$, $s \models \Box A$ and $s' \models A$ we can conclude $s^* \models \Box A$. *Ad* ii). We have to show that $\forall v \in R[w] \exists v^* \in R[w] : V^*(P)(v^*) \subseteq V(P)(v)$. Take $v^* = v$. By construction either $V^*(P)(v) = V(P)(v)$ or (in case $V(P)(w) \subseteq V(P)(v)$) $V^*(P)(v) = V'(P)(w') \subseteq V(P)(v)$. *Ad* iii). To show that $s^* \preceq_{\Diamond} s$ and, hence, $\forall v^* \in R[w] \exists v \in R[w] : V^*(P)(v^*) \subseteq V(P)(v)$ take again $v = v^*$. Finally, if $s \preceq_{\Diamond} s^*$ then we would have $\forall v \in R[w] \exists v^* \in R[w] : V(P)(v) \subseteq V^*(P)(v^*)$. But take $v = w$. By construction there will be no v^* in $R^*[w^*]$ where P is as least as big as in w. Hence, $s \not\preceq_{\Diamond} s^*$. q.e.d.

Fact 2 shows that if applied to an answer that contains no modal operators, the new pragmatic interpretation function eps_2 will give us all the inferences we also obtain by using predicate circumscription – a quite nice result. It does not extend to arbitrary $A \in \mathcal{L}$. Consider, for instance, $A \equiv \Diamond P(a)$. By circumscription we obtain: $circ^{\mathcal{S}}(A, P) \models \neg P(a)$. It is easy to see that this inference is not supported by $eps_2^{\mathcal{S}}(A, P)$.[28] But, as emphasized earlier, the predictions of *circ* for sentences containing modal operators are not in accordance with our intuitions. Therefore, this restrict! ion in fact 2 to modal-free formulas is a blessing rather than a curse.

However, with the results of fact 2 we are only half of our way. Of course, we do not want $eps_2^{\mathcal{S}}$ to allow additional inferences that

[27] Hence, s^* is 'constructed' from s by substituting for the valuation V in all worlds of $R[w]$ where the extension of P is at least as large as in w the valuation of V' in w'.

[28] Let $s = \langle M, w \rangle \in circ^{\mathcal{S}}(\Diamond P(a), P)$. It follows that $\exists v \in R[w] : M, v \models P(a)$. Take $s' = \langle M, v \rangle$. Obviously $s \cong_{\Box} s'$ and $s \cong_{\Diamond} s'$. Hence, if $s \in eps_2^{\mathcal{S}}(A, P)$ then $s' \in eps_2^{\mathcal{S}}(A, P)$ as well. However, $s' \not\models \neg P(a)$.

destroy the nice predictions made by circumscription for the factive strong reading. Hence, we would like to establish something like the following: if $A, \phi \in \mathcal{L}^0$ then $eps_2^{\mathcal{S}}(A,P) \models \phi \Rightarrow circ^{\mathcal{S}}(A,P) \models \phi$. This, however, does not hold. Recall that the order $\leq_P$ on which *circ* relies relates two states $s_1 = \langle M_1, w_1\rangle, s_2 = \langle M_2, w_2\rangle \in \mathcal{S}$ if (i) $V_1(P)(w_1) \subseteq V_2(P)(w_2)$ and (ii) V_1 and V_2 assign in w_1 and w_2 respectively the same interpretation to all other elements of the non-logical vocabulary. The conditions imposed by $\preceq_\Box$ and $\preceq_\Diamond$ are in some sense weaker: here we have to find pairs of epistemic possibilities that only have to fulfill the first, (i), of these two conditions. This results in stronger predictions for pragmatic inferences. In particular, in case A contains other items of the non-logical vocabulary besides P one may now obtain pragmatic information about the denotation of these items. The additional condition (ii) has been added to the definition of circumscription to explicitly prevent that circumscribing P also imposes additional restrictions on the interpretation of the non-logical vocabulary apart from P. But how far is this relevant to model the exhaustive interpretation of answers? In a context where it is known that *if the weather was fine Peter was there*, the exhaustive interpretation of the answer 'Mary (was there)' to a question 'Who was there?' intuitively allows the inference that the weather was not fine. Exactly for the reason described above, standard circumscription cannot account for this inference. This can be taken as evidence that to model the factive strong reading we do not want to exclude that minimizing P influences the interpretation of other items. Hence, we should rather adopt the following order for $s_1 = \langle M_1, w_1\rangle, s_2 = \langle M_2, w_2\rangle \in \mathcal{S}$: $s_1 \leq_P^2 s_2$ iff$_{def}$ $V_1(P)(w_1) \subseteq V_2(P)(w_2)$, and use $circ_2^{S}(A,P)$ to model strong factive readings.[29]

$$circ_2^S(A,P) =_{def} \{s \in S | s \models A \wedge \neg\exists s' \in S : s' \models A \wedge s' \leq_P^2 s\}$$

For this version of circumscription we can indeed establish not only an analogue claim to fact 2 but additionally the following result:

FACT 3. *Let A be an element of $\mathcal{L}^0$ and P a predicate of $\mathcal{L}$. Assume that for all $s \in \mathcal{S}$ such that $s \models A$ there is some $s' \in circ_2^{\mathcal{S}}(A,P)$ such that $s' \leq_P^2 s$. Then we have:*

$$\forall \phi \in \mathcal{L}^0 : eps_2^{\mathcal{S}}(A,P) \models \phi \Rightarrow circ_2^{\mathcal{S}}(A,P) \models \phi.$$

[29] Notice that $circ_2$ does not make the right predictions for answers like 'If the weather was fine Peter was there' to the question 'Who was there?'. But according to many of our informants these answers are not interpreted exhaustively anyway and, thus, $circ_2$ should not be applied. Hence, they do not necessarily constitute counterexamples for this approach.

Proof : We will show that for every $s \in circ_2^{\mathcal{S}}(A,P)$ there is some $s' \in eps_2^{\mathcal{S}}(A,P)$ such that $s \equiv^0 s'$. This proves the claim. Take an arbitrary $s = \langle M, w\rangle \in circ_2^{\mathcal{S}}(A,P)$. We choose $s' = \langle M', w'\rangle$ as follows: (1) $\forall t \in circ_2^{\mathcal{S}}(A,P)\exists v' \in R'[w'] : t \equiv^0 \langle M', v'\rangle$, (2) $\forall v' \in R'[w']\exists t \in circ_2^{\mathcal{S}}(A,P) : t \equiv^0 \langle M', v'\rangle$, and (3) $s \equiv^0 s'$.[30] It follows immediately from (3) that $s \equiv^0 s'$. Hence, we only have to show that $s' \in eps_2^{\mathcal{S}}(A,P)$, thus (i) $s' \models \Box A$ and (ii) $\forall s'' \in \mathcal{S} : s'' \models \Box A \Rightarrow [s' \preceq_\Box s'' \wedge [s'' \cong_\Box s' \Rightarrow s'' \not\prec_\Diamond s']]$ to conclude the proof of the claim.

Ad i). This is an immediate consequence of the definition of s' and the fact that $A \in \mathcal{L}^0$. *Ad* ii). Take an arbitrary $s'' \in \mathcal{S}$ such that $s'' \models \Box A$. We show first that it cannot be the case that $s' \not\preceq_\Box s''$. If it were, we would have $\exists v'' \in R''[w'']\forall v' \in R'[w'] : V'(P)(v') \not\subseteq V''(P)(v'')$. By (1) it follows $\exists v'' \in R''[w'']\forall\langle M, w\rangle \in circ_2^{\mathcal{S}}(A,P) : V(P)(w) \not\subseteq V''(P)(v'')$. This contradicts the assumption that $\forall \in \mathcal{S} : s \models A \Rightarrow [\exists s' \in \mathcal{S} : s' \in circ_2^{\mathcal{S}}(A,P) \wedge s' \leq_P^2 s]$. Hence, $s' \preceq_\Box s''$. Now we have to show that also the following cannot hold: $s' \cong_\Box s'' \wedge s'' \prec_\Diamond s'$. From $s' \cong_\Box s''$ it follows that $\forall v' \in R'[w']\exists v'' \in R''[w''] : V''(P)(v'') \subseteq V'(P)(v')$. Furthermore, if $s' \not\prec_\Diamond s''$ then $\exists v' \in R'[w']\forall v'' \in R''[w''] : V'(P)(v') \not\subseteq V''(P)(v'')$. Together, this gives $\exists v' \in R'[w']\exists v'' \in R''[w''] : V''(P)(v'') \subset V'(P)(v')$. By (2) it follows that $\exists\langle M, w\rangle \in circ_2^{\mathcal{S}}(A,P)\exists v'' \in R''[w''] : V''(P)(v'') \subset V(P)(v)$, what contradicts the definition of $circ_2^{\mathcal{S}}(A,P)$. Hence, $s' \cong_\Box s'' \rightarrow s'' \not\prec_\Diamond s'$. This concludes the proof of (ii). q.e.d.

So far, we have introduced two pragmatic interpretation functions that seem to model adequately the epistemic weak (eps_1) and the epistemic strong reading (eps_2) of Quantity1-implicatures. The first function eps_1 was motivated directly by Grice's theory of conversational implicatures and intended to describe conversational implicatures due to the maxims of Quality and the first sub-maxim on Quantity. eps_2 was obtained by strengthening eps_1 with an additional principle to maximize the competence of the speaker. We have seen that the factive inferences of eps_2 are closely related to the inferences of predicate circumscription, which provided a promising description of the factive strong reading of Quantity1-implicatures. In sum, our approach meets all the general requirements any analysis of these implicatures has to fulfill that we formulated at the beginning of this section.

Before we start to discuss the adequacy of this formalization for the implicatures of complex sentences not treated correctly by circumscription, some general remarks are in order.

[30] The first two conditions are totally independent of the choice of s: for all s we chose the same knowledge-state of the speaker for s', the state consisting exactly of the elements in $circ_2^{\mathcal{S}}(A,P)$ (modulo $\equiv^0$).

First, concerning the epistemic force with which an implicature is generated, a central question in the debate is what determines which reading should be predicted in a certain context. According to the approach developed here, the difference is based on whether or not an additional assumption of competence can be made. For the sentences discussed until now we think that this assumption is made as long as it is consistent with what the interpreter already knows and the assumption that the speaker is obeying the maxims of Quality and Quantity1. Hence, we predict the strong reading to occur in those contexts where such a competence assumption can be made. Otherwise only the weak reading is obtained.

Secondly, one of the central advantages of any rigorous formalization is that it clarifies the consequences of one's ideas. This is also very true for the work presented here. When we were discussing some examples for the predictions of eps_1 and eps_2, we made the simplifying assumption that we can identify constants and individuals. Let us now drop this assumption. If we then, for instance, calculate $eps_1^S(P(e), P)$ in a context S with two individuals a and b where the speaker may but need not know which individual is denoted by e, the states selected are those in $[\{a, b\}]$ and $[\{a, b, ab\}]$ where the speaker does not know whether e denotes a or b. This may at first sight not be a very intuitive result, but it is a consequence that is to be expected given the assumptions underlying our formalization. If the interpreter is interested in the true individuals that have property P, then a speaker who said $P(e)$ and knows who is denoted by e is withholding this relevant information from the hearer. Therefore eps_1 concludes that the speaker cannot have this knowledge. This shows that our formalization may still miss relevant variables for the calculation of implicatures – something to study in further work.

5. Complex sentences

Fact 2 of the previous section shows that implicatures triggered by sentences of $\mathcal{L}^0$ that we could account for by means of circumscription can be described as well in terms of our epistemic notion of pragmatic interpretation. But our richer machinery allows us, additionally, to predict exhaustivity effects – particularly connected with the beliefs of the speaker – that could not be accounted for in terms of circumscription, or of G&S's operator *exh*. This will be illustrated in the present section.

5.1. Possibility statements

Consider again example (12), here repeated as (13), a statement that explicitly refers to the belief state of the speaker.

(13) A: Who knows the answer?
B: Peter and possibly Mary.

In section 3 we reported two readings for this sentence: one reading according to which the speaker knows that Peter knows the answer, but does not know of any other individual that he or she knows the answer, and a second one that says that the speaker knows that Peter knows the answer, she does not know that Mary knows the answer, but she does know that all other individuals besides Mary and Peter do not know the answer. The application of *circ* to a sentence like $P(p) \wedge \Diamond P(m)$ will predict the inference that Mary did not know the answer – which is obviously inadequate. $eps_1^S(P(p) \wedge \Diamond P(m), P)$ correctly describes the first reading, including the 'scalar' implicature from $\Diamond P(m)$ to $\neg\Box P(m)$. Furthermore, by applying eps_2^S we are also able to account for the second reading of the example.

What about examples involving a 'scalar' expression under the scope of a possibility statement, like $\Diamond(P(p) \vee P(m))$? By minimization with respect to $\preceq_\Box$ we conclude that for every individual c the speaker does not know that c has property P. We also infer that $\neg\Box(P(p) \vee P(m))$. When additionally minimizing with respect to $\preceq_\Diamond$, one obtains that the speaker even knows for each c that it does not have property P, and that $\neg\Diamond(P(p) \wedge P(m))$ holds. Thus, we obtain the scalar implicature from 'or' to 'not and', but now under the scope of the possibility operator. When comparing these results with the observations of the localists Landman and Chierchia on examples where the 'implicature-triggering' expression stands in the scope of an existential quantifier, it turns out that we predict exactly the reading they claimed a global approach cannot account for.

5.2. Negation

As has been discussed already in section 3, negation is a problem for G&S's approach, whether or not we use a 'scalar' expression in its scope: both $\neg P(a)$ and $\neg(P(a) \vee P(b))$ receive by *exh* or *circ* the interpretation that no individual actually has property P. From fact 2 stated above it follows that negation is a problem for eps_2 as well.

To solve this problem, we propose elsewhere (van Rooij & Schulz, submitted) to follow the suggestion of von Stechow & Zimmermann (1984) that for the exhaustive interpretation of negative sentences we should not minimize the extension of P, but rather that of the complement of P, i.e. $\bar{P}$, hence, to calculate $circ^{S}(\cdot,\bar{P})$ instead. As we suggested already in our discussion of section 3, however, we believe that negative sentences (just like positive ones) might have two pragmatic readings. The first reading – which only some people seem to get – is correctly described by $circ^{\mathcal{S}}(\cdot,\bar{P})$ and, hence, also by $eps_2^{\mathcal{S}}(\cdot,\bar{P})$. In terms of *circ*, however, we were not able to account for the epistemic weak reading, according to which a sentence like $\neg P(a)$ gives rise to the inference that for every other individual the speaker considers it possible that it has property P. But this reading can now be described correctly by $eps_1^{\mathcal{S}}(\cdot,\bar{P})$. To 'explain' the fact that for 'negative' sentences the strong reading is more exceptional than for their positive counterparts, we suggest that negation functions as a trigger signaling not only that it is the extension of $\bar{P}$ that is at issue, but also that the interpreter should (normally) not try to maximize the speaker's competence.

5.3. Belief

To account for implicatures of belief attributions we have to extend our formal framework so that it can also express facts about the epistemic states of other agents. In general this can be easily done: we add to our language $\mathcal{L}$ modal operators $\Box_i/\Diamond_i$ and extend the states with respect to which this new language is interpreted with accessibility relations R_i for every $\Box_i$. A sentence $\Box_i\phi$ should then be read as *agent i believes* ϕ. The question how to extend our pragmatic interpretation functions is somewhat more tricky. We will simplify things a bit here. Let us assume that we have in D only finitely many individuals and a finite set N of names for these individuals such that every name denotes exactly one individual. In this case there is another language-oriented way to define our orderings $\preceq_\Box$ and $\preceq_\Diamond$. Let $\mathcal{L}(P)\subseteq\mathcal{L}$ be the sub-language defined by the BNF $\varphi ::= P(a)(a\in N)|\varphi\vee\varphi|\varphi\wedge\varphi$. Furthermore, $\Box\mathcal{L}(P)$ is defined as the set $\{\Box\phi|\phi\in\mathcal{L}(P)\}$ and $\Diamond\mathcal{L}(P)$ as $\{\Diamond\phi|\phi\in\mathcal{L}(P)\}$. In this simplified setting, the following connection can be proven.

FACT 4. $\forall s_1,s_2\in\mathcal{S}$:

$$s_1\preceq_\Box s_2 \Leftrightarrow \forall\phi\in\mathcal{L}(P): s_1\models\Box\phi\rightarrow s_2\models\Box\phi$$
$$s_1\preceq_\Diamond s_2 \Leftrightarrow \forall\phi\in\mathcal{L}(P): s_1\models\Diamond\phi\rightarrow s_2\models\Diamond\phi$$

In order to account for implicatures triggered by expressions occurring in belief attributions, we have to consider another language in terms of which we define the ordering relations. The idea is now not to take $\mathcal{L}(P)$ as the basic language, but rather for each agent j

the language $\mathcal{L}^j(P) =_{def} \{\Box_j\phi | \phi \in \mathcal{L}(P)\} \cup \{\Diamond_j\phi | \phi \in \mathcal{L}(P)\}$. We define two new orders $\preceq_{\Box,j}$ and $\preceq_{\Diamond,j}$ as follows: $s_1 \preceq_{\Box,j} s_2$ iff$_{def}$ $\forall\phi \in \mathcal{L}^j(P) : s_1 \models \Box\phi \rightarrow s_2 \models \Box\phi$ and $s_1 \preceq_{\Diamond,j} s_2$ iff$_{def}$ $\forall\phi \in \mathcal{L}^j(P) : s_1 \models \Diamond\phi \rightarrow s_2 \models \Diamond\phi$. Now take a belief attribution like 'John believes that P(a)' that we represent by $\Box_j P(a)$. One reading we obtain is by looking at the minimal state which verifies $\Box_j P(a)$ with respect to the order $\preceq_{\Box,j}$. On this reading we cannot infer much more from the sentence than its semantic meaning. Another reading we obtain by interpreting the belief attribution as the minimal state which verifies $\Box_j P(a)$ with respect to the prioritized order defined in terms of $\preceq_{\Box,j}$ and $\preceq_{\Diamond,j}$. According to the resulting interpretation, the speaker knows that John believes that only a has property P. It is this latter reading that accounts for the inferences in (1) and (8) as reported in sections 1 and 2 of this paper. In analogy with 'scalar' terms occurring under possibility statements, we obtain the standard exhaustive interpretation, but now under the scope of a belief operator. Hence, again we correctly predict those readings that Landman (2000) and Chierchia (ms) take to be problematic for global approaches.

6. Conclusion

In this paper we introduced two related pragmatic interpretation functions for answers to overt questions. The first function, eps_1, was motivated directly by Grice's theory of conversational implicature and intends to describe conversational implicatures due to the maxims of Quality and the first sub-maxim of Quantity. The second, eps_2, was obtained by strengthening eps_1 with an additional principle to maximize the competence of the speaker. In both cases the definition makes crucial use of the work of Halpern & Moses (1984) on 'only knowing' and its recent generalization by van der Hoek et al. (1999, 2000).

These two functions predict two different pragmatic readings for answers to overt questions. eps_1 describes the conversational implicatures that are always obtained if the speaker is taken to obey the maxim of Quality and the first sub-maxim of Quantity. The predicted inferences have weak epistemic force and say, roughly, that for certain stronger statements the speaker does not know whether they are true. The interpretation function eps_2 strengthens eps_1 in the contexts where the interpreter can make additional assumptions about the competence of the speaker. This strengthening has the result that some of the inferences of eps_1 are now generated with a strong epistemic force, claiming that the speaker knows that certain claims that are stronger than the statement made by the speaker are not true. As these paraphrases show, the implicatures predicted by our approach are closely related to many other descriptions of these inferences given in the literature. The specific advantages of our proposal, we claim, are, on the one hand, its rigorous formal outset that allows for clear, testable predictions, and, on the other, that it provides a unified account of implicatures due to the first sub-maxim of Quantity.

Apart from these points, this paper contributes in two other respects to the research on conversational implicatures. First, in section 4 we have seen that eps_2 generalizes Groenendijk & Stokhof's (1984) description of exhaustive interpretation, which was already a very promising approach to a wide class of implicatures. Thereby it links their proposal to Grice's theory of conversational implicatures and gives this more descriptive approach a conceptual, explanatory foundation. Second, the proposed formalization represents a strong argument against defenders of a local approach to conversational implicatures as Landman (2000) and Chierchia (ms). For a wide range of examples it falsifies their claim that global approaches cannot account for the implicatures of complex sentences.

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References

Atlas, J.D. & S.C. Levinson, 1981, '*It*-clefts, informativeness and logical form', In: P. Cole (ed.), *Radical Pragmatics*, Academic Press, New York.

Benthem, J. van, 1989, 'Semantic parallels in natural language and computation', In. H. D. Ebbinghaus et al. (eds.), *Logic Colloquium '87*, Elsevier Science Publishers, Amsterdam, pp. 331-375.

Chierchia, G., 'Scalar Implicatures, Polarity Phenomena, and the Syntax/Pragmatics Interface', Manuscript, University of Milan.

Gazdar, G., 1979, *Pragmatics*, Academic Press, London.
Green, M., 1995, 'Quantity, volubility, and some varieties of discourse', *Linguistics and Philosophy*, **18**: 83-112.
Grice, H. P., 1967, 'Logic and Conversation', typescript from the William James Lectures, Harvard University. Published in P. Grice (1989), *Studies in the Way of Words*, Harvard University Press, Cambridge, Massachusetts, 22-40.
Groenendijk, J. and M. Stokhof, 1984, *Studies in the Semantics of Questions and the Pragmatics ofAnswers*, Ph.D. thesis, University of Amsterdam.
Halpern, J.Y. and Y. Moses, 1984, 'Towards a theory of knowledge and ignorance', *Proceedings 1984 Non-monotonic reasoning workshop*, American Association for Artificial Intelligence, New Paltz, NY, pp.165-193.
Harnish, R. M., 1976, 'Logical form and implicature', In: T. G. Bever et al. (eds.) *An integrated theory of linguistic ability*, Crowell, New york, pp. 313-391.
Heim, I., 1982, *The Semantics of Definite and Indefinite Noun Phrases*, Ph.D. dissertation, University of Massachusetts, Amherst.
Hirschberg, J., 1985, *A theory of scalar implicature*, Ph.D. thesis, UPenn.
Hoek, W. van der, J. Jaspers and E. Thijsse, 1999, 'Persistence and minimality in epistemic logic', *Annals of Mathematics and Artificial Intelligence*, **27**: 25-47.
Hoek, W. van der, J. Jaspers and E. Thijsse, 2000, 'A general approach to multi-agent minimal knowledge', In: M. Ojeda-Aciegoet. al. (eds.) *Proceedings JELIA 2000*, LNAI 1919, pp. 254-268, Springer-Verlag, Heidelberg.
Horn, L., 1972, *The semantics of logical operators in English*, Ph.D. thesis, UCLA.
Horn, L., 1989, *A Natural History of Negation*, University of Chicago Press, Chicago.
Kamp, H., 1981, 'A theory of truth and semantic representation', In: Groenendijk et al. (eds.), *Formal Methods in the Study of Language*, Amsterdam, 277-322.
Landman, F., 2000, *Events and Plurality: The Jeruzalem lectures*, Kluwer, Dordrecht.
Levinson, S.C., 1983, *Pragmatics*, Cambridge University Press, Cambridge.
Levinson, S.C., 2000, *Presumptive Meanings. The Theory of Generalized Conversational Implicature*, MIT Press, Cambridge, Massachusetts.
Matsumoto, Y., 1995, 'The conversational condition on Horn scales', *Linguistics and Philosophy*, **18**: 21-60.
McCarthy, J., 1980, 'Circumscription - a form of non-monotonic reasoning, *Artificial Intelligence*, **13**: 27 - 39.
Merin, A., 1994, *Decision-Theoretic Pragmatics*, Lecture Notes read in Course BA 5, Copenhagen Business School, Denmark.
Leech, G.N., 1983, *Principles of Pragmatics*, Longman, London.
Rooij, R. van and K. Schulz, 'Pragmatic meaning and non-monotonic reasoning: The case of exhaustive interpretation', submitted, University of Amsterdam.
Sauerland, U., 2004, 'Scalar implicatures of complex sentences', *Linguistics and Philosophy*, **27**, 367-391.
Schulz, K., 2003, 'You may read it now or later', Master thesis, University of Amsterdam.
Soames, S., 1982, 'How presuppositions are inherited: A solution to the projection problem', *Linguistic Inquiry*, **13**: 483-545.
Spector, B., 2003, 'Scalar implicatures: Exhaustivity and Gricean reasoning?', In: B. ten Cate (ed.), *Proceedings of the ESSLLI 2003 Student session*, Vienna.
Stechow, A. von and T.E. Zimmermann, 1984, 'Term answers and contextual change', *Linguistics*, **22**: 3-40.
Wainer, J., 1991, *Uses of nonmonotonic logic in natural language understanding: generalized implicatures*, Ph.D.thesis, Pennsylvania State University.

Zimmermann, T.E., 2000, 'Free choice disjunction and epistemic possibility', *Natural Language Semantics*, **8**: 255-290.

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