

The propositional and relational syllogistic*

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Abstract

In this paper it is shown how syllogistic reasoning can be extended to account for propositional logic and relations.

Keywords: Syllogistic, Propositional logic, Relations

1 Introduction

Syllogistic logic is a loose term for the logical tradition that originated with Aristotle and survived until the advent of modern predicate logic in the late nineteenth century. The innovation of predicate logic led to an almost complete abandonment of the traditional system. In almost all standard modern textbook introductions one starts with propositional logic, extends this to predicate logic, and then points out that the scope of syllogistic logic is just a very small part of this. It is customary to revile or disparage syllogistic logic in these textbooks. Modern logicians used quite a number of arguments as to why syllogistic logic should be abandoned. The most important complaint is that syllogistic logic is *not rich enough* to account for mathematical reasoning, or to give a serious semantics of natural language. It is only a small fragment of predicate logic, which doesn't say anything about propositional logic, or multiple quantification. Russell (1900) blamed the traditional logical idea that every sentence is of subject-predicate form for giving sentences misleading logical forms.

For several reasons, traditional logic recently regained new popularity. A main motivation for Sommers (1982) — a prominent proponent of traditional logic — to study syllogistic logic is its close relation to natural language. Indeed, due to the development of Montague grammar and especially Generalized Quantifier Theory in the 1960s-1980s the misleading form thesis of early proponents of modern logic is not a mainstream position anymore, and analyzing sentences in subject-predicate form is completely accepted again (cf. Westerståhl, 1989). Computer scientists (e.g. Purdy, Pratt-Hartmann, and Moss), on the other hand, have shown that syllogistic logic and some natural extensions of it behave much better in terms of decidability and complexity than standard predicate logic.

In this paper I will first quickly discuss traditional Aristotelian syllogistic, and how to extend it (also semantically) with negative and singular terms. Afterwards I will discuss how full propositional logic can be seen as an extension of Aristotelian syllogistic. Thus, in distinction with polish logicians like Łukasiewicz (1957) and others, I won't assume that to understand traditional logic we have to presuppose propositional logic, but instead formulate propositional logic by presupposing syllogistic reasoning. It is well-known that once we have full monadic predicate logic, we have full propositional logic as well. Indeed, syllogistic can be viewed as a fragment of propositional logic. In this paper I show how to complete this fragment. There are at least two possible ways to do so. In section 3.1 I sketch a first way by

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introducing *conjunctive* and other complex *terms* to the language. Section 3.2, instead, shows how to account for all of propositional logic in a more direct way. Afterwards I will follow (the main ideas, though not the details of) Sommers (1982) and his followers in showing how traditional logic can be extended so as to even account for inferences involving multiple quantification that almost all modern textbooks claim is beyond the reach of traditional logic: A woman is loved by every man, thus Every man loves a woman. Although the aim of this paper is not to study historical systems by formalizing them, it is sometimes motivated by such historical ideas.

2 Traditional syllogistic reasoning

Syllogisms are arguments in which a categorical sentence is derived as conclusion from two categorical sentences as premisses. As is well-known, a categorical sentence is always of one of four kinds: *a*-type ('All men are mortal'), *i*-type ('Some men are philosophers'), *e*-type ('No philosophers are rich'), or *o*-type ('Some men are not philosophers'). In *a*- and *e*-sentences the predicate is affirmed or denied of 'the whole' of the subject, while in *i* and *o*-sentences, the predicate is affirmed or denied of only 'part of' the subject. The *syntax* of categorical sentences can be formulated as follows: If *S* and *P* are primitive terms, *SaP*, *SiP*, *SeP*, and *SoP* are categorical sentences. Because a syllogism has two categorical sentences as premisses and one as the conclusion, every syllogism involves only three terms, each of which appears in two of the statements.

Aristotle's way to determine which syllogisms are valid was 'proof-theoretic' in nature. He showed that if one takes the four 'perfect' syllogisms of the first figure (Barbara, Celarent, Darii and Ferio) to be axioms, and use any combination of subalternation ($SaP \vdash SiP$ and $SeP \vdash SoP$), conversion ($SeP \vdash PeS$ and $SiP \vdash PiS$), and his so-called *reductio per impossibile* (if contradictory sentences can be derived from a set of premisses, at least one of them is false),¹ one can derive all other valid syllogisms.

We will not make use of subalternation, but instead implement existential import by assuming *TiT* to be an axiom.² We are also not going to make use of conversion, but instead adopt the Law of Identity (*TaT*) and make heavy use of Aristotle's *Reductio per impossibile*. The rule 'reductio per impossibile' can best be formulated in terms of a notion of sentence-negation '¬' which doesn't really exist in traditional logic. However, it can be defined as the contradictory negation featuring in the *square of opposition*: $\neg(SaP) \stackrel{def}{=} SoP$, $\neg(SiP) \stackrel{def}{=} SeP$, $\neg(SeP) \stackrel{def}{=} SiP$, and $\neg(SoP) \stackrel{def}{=} SaP$. Now we can state a proof theory for syllogistic reasoning without negative terms as follows:

1. The first 4 valid Syllogisms of the first figure:
 - (a) Barbara: $MaP, SaM \vdash SaP$.
 - (b) Celarent: $MeP, SaM \vdash SeP$.
 - (c) Darii: $MaP, SiM \vdash SiP$.
 - (d) Ferio: $MeP, SiM \vdash SoP$.
2. Law of Identity: $\vdash TaT$, for all terms *T*.
3. Existential import: $\vdash \neg(TeT)$, or $\vdash TiT$, for all terms *T*.

¹Larry Moss pointed out to me that this rule is not required to axiomatize syllogistic reasoning.

²Thus, in contrast to some traditional logicians I don't require that categorical sentences contain two distinct terms.

4. *Reductio per impossibile*: if contradictory sentences can be derived from a set of premisses, at least one of them is false. If $\Gamma, \neg\phi \vdash \psi$ and $\Gamma, \neg\phi \vdash \neg\psi$, then $\Gamma \vdash \phi$.³

It is easy to see that the validity of *TiT* accounts for subalternation. Following medieval and Leibnizian practice (e.g. Leibniz, 1966d), we can derive all the valid syllogisms of the second and third figure by means of *Reductio per impossibile* from Barbara, Celarent, Darii, and Ferio. Next, it can be shown that the conversion rules follow from some valid syllogisms together with the law of identity. These rules of conversion can then be used, finally, to derive the valid syllogisms of the fourth figure.

It is well-known how to give a satisfactory *semantics* for syllogistic logic. Almost all modern textbooks use Venn diagrams, and thus a set-theoretic *extensional semantics*, to represent the meaning of sentences and to decide whether a syllogistic inference is valid.⁴ A sentence like *SaP*, for instance, is counted as true if the individuals in the extension of *S* are also in the extension of *P*. According to such a semantics, $M = \langle D, E \rangle$ is a model, with *D* a domain of objects and *E* an interpretation function which assigns to each primitive term *T* a non-empty proper subset of *D*: $\emptyset \neq E_M(T) \subset D$. For later generality, we will use a more general interpretation function, V_M , which for primitive terms *T* is identical to *E*: $V_M(T) = E_M(T)$. Categorical sentences are counted as true in the expected way: $V_M(SaP) = 1$ iff $V_M(S) \cap V_M(P) = V_M(S)$ iff $V_M(S) \subseteq V_M(P)$ and $V_M(SiP) = 1$ iff $V_M(S) \cap V_M(P) \neq \emptyset$. *SoP* and *SeP* are interpreted as the negations of *SaP* and *SiP*, respectively. We say that $\phi_1, \dots, \phi_n \models \psi$ iff $\forall M : V_M(\phi_1) = 1$ and ... and $V_M(\phi_n) = 1$, then $V_M(\psi) = 1$. It is well-known that this semantics validates all and only all arguments in classical syllogistic style if and only if they are traditionally counted as valid.⁵

Negative terms didn't play an important role in Aristotle's theory of syllogisms.⁶ This is perhaps due to the *intensional* interpretation of terms preferred by Aristotle.⁷ According to it, *SaP* is true if the *intension*, of *P* is contained in the *intension* of *S*.⁸ When one adopts an extensional semantics, on the other hand (as was popular among scholastic logicians), negative terms can easily be interpreted. We add negative terms to the language as follows: first we say that all primitive terms are categorical terms and second that if *S* is a categorical term, \bar{S} is a categorical term as well. Once negative terms are added to the language, we have to interpret them, and the proof-system has to change. As for (extensional) interpretation, \bar{S} will just denote the complement of what is denoted by *S*. Notice that this means that no term denotes either the empty set or the universal set. As for the proof-theory, we will add two things that account for negative terms: a new double negation axiom, and the rule of contraposition.⁹ However, we will simplify the rule system as well by dropping Celarent and

³I will always assume that the order of premisses is unimportant. It is sometimes argued that because of this assumption together with the *Reductio* rule, syllogistic depends propositional logic. I disagree, because I just see two extra assumptions.

⁴Of course, set theory comes with all the axioms of Boolean algebra, and this much structure is not at all required to model syllogistic reasoning. For the traditional fragment without negative terms, partial, or even pre-orders with a distinguished bottom is already enough.

⁵It is well known how to extend the extensional semantics with negative (and conjunctive) terms (cf. Shepherdson, 1956).

⁶In the history of logic, negative terms are also known as *indefinite* or *infinite* terms.

⁷Leibniz (1966a) claims that Aristotle indeed preferred such an intensional interpretation.

⁸For the simplest fragment, an intensional semantics is easy to give. Let $M = \langle A, I, \perp \rangle$ be a model, with *A* a set of attributes, *I* an interpretation function which assigns to each primitive term *T* a subset of *A*, $I_M(T) \subseteq A$, and \perp a relation between elements of *A*. If for two elements $x, y \in A$ it holds that $x \perp y$, we say that the attributes *x* and *y* are incompatible. We will denote by Δ the set of subsets of *A* which contain such mutually incompatible elements: $\Delta = \{S \subseteq A : \exists x, y \in S : x \perp y\}$. We assume that for each primitive term *T*, $I_M(T) \notin \Delta$, and that the set of supersets of $I_M(T)$ does not equal the set of all maximally consistent sets of *A*. Then we say that $V_M(SaP) = 1$ iff $V_M(S) \cup V_M(P) = V_M(S)$ iff $V_M(S) \supseteq V_M(P)$ and $V_M(SiP) = 1$ iff $V_M(S) \cup V_M(P) \notin \Delta$. Thus, *SiP* is true iff *S* and *P* do not contain mutually incompatible attributes. *SoP* and *SeP* are interpreted as the negations of *SaP* and *SiP*, respectively. It is not trivial to extend this semantics to a language which allows for negative and conjunctive terms.

⁹Though contraposition need not be assumed, and can be derived.

Ferio to be axioms. The categorical sentences SeP and SoP can now be written, or defined, as $Sa\bar{P}$ and $Si\bar{P}$, respectively, where \bar{P} is the negative counterpart of P .¹⁰ Instead of taking Barbara and Darii to be axioms, I prefer to assume with scholastic logicians the *Dictum de Omni* as a rule. This rule is a kind of *substitution rule* and makes crucial use of the *distribution* values of terms. Whether a term is distributed or not is traditionally thought of as a *semantic* question: a term is said to be distributed when it is actually applied to *all* the objects it can refer, and undistributed when it is explicitly applied to only part of the objects to which it can refer. This formulation has been criticized by Geach (1962) and other modern logicians, but as noted by van Benthem (1986), it can be redefined in terms of *monotonicity*: a term occurs distributively when it occurs monotone decreasingly/negatively within a sentence, and undistributively when it occurs monotone increasingly/positively. Denoting a distributed term by $-$ and an undistributed term by $+$, the following follows at once: S^-aP^+ , S^+iP^+ , S^-eP^- , and S^+oP^- , and \bar{P} occurs positively in Γ iff P occurs negatively in Γ , which we might think of now as a *syntactic* characterization. Now we can formulate the *Dictum de Omni* and the related *Dictum de Nullo* as follows:¹¹

- **Dictum de Omni** (DDO): $SaP, \Gamma(S)^+ \vdash \Gamma(P)$, where $\Gamma(S)^+$ is a sentence where term S occurs positively.¹²
- **Dictum de Nullo** (DDN): $SaP, \Gamma(P)^- \vdash \Gamma(S)$, where $\Gamma(P)^-$ is a sentence where term P occurs negatively (or distributively).

Modern logicians are not very familiar with the *Dictum de Omni*. In contrast, they all know Leibniz's famous substitution principle which allows terms S and T to be substituted for one another in any sentence if they are 'the same'. In fact, Leibniz's substitution principle *follows* syllogistically from the much more general *Dictum de Omni*. On the basis of the following passage, Sommers (1982) argues that Leibniz was fully aware of this (Leibniz writes ' AaB ' as 'a is b'):

If a is b and b is a , then a and b are said to be 'the same'. From this it can be easily proved that one can everywhere be substituted in place of the other without loss of truth; for if a is b and b is a , and b is c or d is a , then a is c or d is b . (Leibniz, 1966b, p. 43)

In fact, Leibniz proves here something more general than just the substitution principle for which he is so famous. He proves that if AaB , then (i) if BaC then AaC (by DDO), and (ii) if DaA then DaB (by DDO). Thus, (i) all things (C) predicated of B can also be predicated of A , and (ii) all things (D) A can be predicated of, B can also be predicated of.

The proof system **SYL** of syllogistic reasoning consists of the following set of axioms and rules (for all terms T , S and P):

- | | |
|---|-------------------------------|
| (1) $SaP, \Gamma(S)^+ \vdash \Gamma(P)$ | Dictum de Omni. ¹³ |
| (2) $\vdash TaT$ | Law of identity. |

¹⁰Aristotle allowed for the inference $Sa\bar{P} \vdash SeP$, but not for $SeP \vdash Sa\bar{P}$ (cf. Kneale & Kneale, p. 57). The reason seems to be that 'Socrates is not black' seems to have a 'wider' meaning than 'Socrates is non-black'. Similarly, many traditional logicians didn't allow for the inference $SoP \vdash Si\bar{P}$.

¹¹The soundness of these rules in the extended syllogistic is proved very elaborately by MacIntosh (1982). But as shown by Sánchez (1997), their soundness follows from a much more general relation between monotonicity and positive and negative occurrences of predicates in Lyndon (1959).

¹²It is of course also possible to get rid of SiP by defining it as $\neg(Sa\bar{P})$. In that case, we might use instead of the Dictum just Barbara as axiom, but now also use Celarent as axiom. The axiomatic system that seems to be most standardly assumed was taking Barbara and Celarent as axioms, together with the reduction rule, conversion, and subalternation.

¹³Actually, Barbara is enough, because Darii can be derived from the rest.

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|---|--|
| (3) $\vdash T \equiv \overline{\overline{T}}$ ¹⁴ | Double negation. |
| (4) $Sa\overline{P} \vdash Pa\overline{S}$ | Contraposition (or conversion, $SeP \vdash PeS$). ¹⁵ |
| (5) $\Gamma, \neg\phi \vdash \psi, \neg\psi$ ¹⁶ $\Rightarrow \Gamma \vdash \phi$ | Reductio per impossibile. |
| (6) $\vdash \neg(Ta\overline{T})$ (i.e. $\vdash TiT$) | Existential Import. |

The first five rules of **SYL** are obviously semantically valid. Axiom (6) is really an assumption: we only make use of non-empty terms. One can also easily check by hand that the above set of rules accounts for all valid syllogisms. In fact, from the above set of axioms and rules we can also account for derivations that are intuitively valid (e.g., come out true if we make use of Venn diagrams together with a given discourse domain), but do not correspond to a valid syllogism. This is the case, for instance, for $Ma\overline{P}, Mi\overline{S} \vdash \overline{SiP}$ simply because it is not of the required form: the conclusion \overline{SiP} can not be expressed without negative terms using only the Aristotelian ‘a’, ‘i’, ‘e’, and ‘o’. Having lifted one arbitrary restriction Aristotle put on valid syllogistic reasoning, we might as well slightly extend syllogistic reasoning in another way as well.¹⁷ We could do so by adding a distinguished ‘transcendental’ term ‘ \top ’ to our language, standing for something like ‘entity’. Obviously, the sentence $Sa\top$ should always come out true for each term S . To reflect this, we will add this sentence as an axiom to **SYL**. But adding \top as an arbitrary term to our language gives rise to a complication once we accept existential import for *all* terms, including negative ones: for negative term $\overline{\top}$ existential import is unacceptable. One way to get rid of this problem is to restrict existential import to *positive* categorical terms only. Most conservatively, we could say that all categorical terms are positive except for \perp . More generally, we could say that \top (but not \perp) is a primitive term, and then define a categorical term T to be positive iff it either is a primitive term, or it is one that occurs under an even number of term-negations. Either way, ‘ \top ’ is a positive categorical term, but ‘ $\overline{\top}$ ’, or ‘ \perp ’, is not.

Most traditional logicians treated *singular* propositions as universal ones. One can show that this indeed gives rise to the correct inferences if one limits one-selves to the traditional fragment.¹⁸ But there is still a problem with this proposal. Normally, a and e propositions are contrary to each other: they can’t both be true, but they *can* both be false. Thus, from $\neg(SaP)$ one *cannot* derive SeP . However, this inference is valid with singular terms: if ‘Socrates is wise’ is not true, it follows that ‘Socrates is not wise’ is true. In terms of the square of opposition this means that contrary and contradictory negation come down to the same. Sommers (1982) pointed out that to solve this problem, Leibniz (1966c) proposed that for singular propositions, a and i propositions coincide, just like e and o propositions.

How is it that opposition is valid in the case of singular propositions [...] since elsewhere a universal affirmative and a particular negative are opposed. Should we say that a singular proposition is equivalent to a particular and to a universal

¹⁴By this I really mean $\vdash Ta\overline{\overline{T}}$ and $\vdash \overline{\overline{TaT}}$.

¹⁵Contraposition and conversion can both be derived from the rest. For conversion, assume SeP . Now suppose that PeS is false, i.e., that PiS is true. By Ferio (which follows from DDO), it follows that PoP holds, which is contradictory to the axiom PaP . Thus, by reductio we conclude that PiS is false, which means that PeS is true. For contraposition, assume that $Sa\overline{P}$ is true and $Pa\overline{S}$ is false. Thus that both $Sa\overline{P}$ and $Po\overline{S}$, or PiS are true. By Darii (which follows from the DDO) it follows that $Pi\overline{P}$, which is PoP , which contradicts the axiom PaP . Thus by reductio we conclude that $Po\overline{S}$ is false, which means that $Pa\overline{S}$ is true.

¹⁶By which I mean that both ψ and $\neg\psi$ can be derived.

¹⁷Church (1965, p. 420) notes that Ockham allowed for empty subject terms and counted in such a case an affirmative proposition false and the negative true. A reviewer of this paper noted that Avicenna already adopted the same position.

¹⁸According to Thom (1981, p. 79), however, this proposal goes wrong in case we extend the formalism with relations

proposition? Yes, we should. So also when it is objected that a singular proposition is equivalent to a particular proposition, since the conclusion in the third figure must be particular, and can nevertheless be singular; e.g., ‘Every writer is a man, some writer is the Apostle Peter, therefore the Apostle Peter is a man’, I reply that there also the conclusion is really particular and it is as if we had drawn the conclusion ‘Some Apostle Peter is a man’. For ‘Some Apostle Peter’ and ‘Every Apostle Peter’ coincide, since the term is singular. (Leibniz, 1966c, p. 115)

This is a very natural idea, and adopted also by 20th century logicians as Quine and Montague. To account for it proof-theoretically, we need a special rule for singular terms.

We will denote the system consisting of (1), (2), (3), (4), (5) together with the following four rules by **SYL**⁺.

- (6') $\vdash CiC$, for all positive categorical terms C .
- (7) $\vdash Ta\top$, for all terms T .
- (8) $\overline{Sa}S \vdash \overline{Sa}P$,¹⁹ for all terms S and P .
- (9) for all singular terms I and terms T : $IiT \dashv\vdash IaT$.

Rules (7) and (8) tell us what to do with empty and transcendental terms. The use of rule (8) seems to be very limited: $\overline{Sa}S$ can be true only if S denotes (extensively) the universe of discourse, and \overline{S} thus the empty set. But, then, the conclusion of (8) follows already from (7), and adding (8) to our system doesn't do much harm either. We will make real use of (7) and (8) only in the following section, where we will think of other sentential connectives as well.

3 Propositional logic as part of syllogistic

Aristotelian logic doesn't contain propositional logic. But that doesn't mean that we cannot think of propositional logic in traditional syllogistic terms.²⁰ In order to do so, we have to treat sentences as terms. Until now we assumed that all terms were 1-ary predicates. To think of propositional logic in traditional terms, we have to allow for 0-ary predicates as well (see for instance Sommers; 1970, 1982). But once we do so, it is very natural to say that if S and P are 0-ary predicates, the conditional ‘If S , then P ’ should be represented by the categorical sentence SaP , and the conjunction ‘ S and P ’ by SiP . Indeed, this was a quite standard position among traditional logicians and it was explicitly defended by Gassendi (1658), Wallis (1687), and Leibniz (1966a, p. 66 and p. 78). How should 0-ary predicates be interpreted? Adopting an extensional semantics, we could interpret a 0-ary predicate S as a *set of possible worlds*. The conditional SaP is thus true iff all S -worlds are also P -worlds. This semantics is certainly reasonable (it interprets the conditional as a strict implication), but it doesn't result in the truth-conditional propositional logic that we are used to. To get to the standard logic, we have to adopt Frege's idea: 0-ary predicates denote either *the truth* or *the falsity*.²¹ Starting from this idea, there are at least two ways to get propositional logic. A first way makes crucial use of conjunctive terms (or non-traditional two-place connectives),

¹⁹See Shepherdson (1956) and reference in here for a motivation for this axiom.

²⁰Of course, traditional logicians thought of propositional logic, but not always in Aristotelian terms.

²¹According to Lenzen (1990), Leibniz didn't adopt this idea and analyzed conditionals as strict implication. Castañeda (1990), on the other hand, argues that in Leibniz (1966a) a material implication analysis of conditionals is proposed. The reason for this disagreement is that Leibniz (1966a) uses the words ‘impossible’ and ‘inconsistent’ that only Castañeda (1990) interprets (in this paper) as ‘false’ and ‘inconsistent with what is true’.

and was in fact proposed by Leibniz. The other method of obtaining propositional logic does not make use of conjunctive terms or new connectives, and is therefore perhaps more in accordance with tradition. I will discuss both ways in what follows, focussing mostly on the second method.

But let us start sketching the first way. Until the previous section we have assumed that if S and P are terms, \overline{S} and \overline{P} are terms as well. But it is possible, of course, to allow for more types of terms: we can assume that if S and P are terms, SP is a (conjunctive) term as well (of the same arity). Conjunctive terms are interpreted, naturally, by set-theoretic intersection. Obviously, once we have negative and conjunctive terms, we have disjunctive terms as well. To reason with such complex terms, we could add the following new rules to our proof system.

- Distributivity: $SaP, SaQ \vdash SaPQ$ and $SaPQ \vdash SaP$ and $SaPQ \vdash SaQ$.

It is easy to see that with the help of these axioms we can derive the following tautologies: *identity*, $\vdash TaTT$ (by $\vdash TaT$ and Distributivity) and $\vdash TT'aT$ (by $\vdash TT'aTT'$ and Distributivity); *commutativity*, $\vdash TT'aT'$ (by $\vdash TT'aTT'$ and Distributivity in both directions), and *associativity*, $\vdash T(T'T'') \equiv (TT')T''$ (by $\vdash T(T'T'')a(TT')T''$ and two times Distributivity in both directions). In fact, if we add the distributivity law to our syllogistic system we come close to derive all axioms of Boolean algebra: idempotence, commutativity and associativity for conjunctive terms, for instance, is (almost) given; the De Morgan laws follow immediately once we define the disjunctive term consisting of T and T' as $\overline{(\overline{T})(\overline{T'})}$; the complement laws follow if we assume that $\vdash T\overline{T} \equiv P\overline{P}$ and identify \top with $\overline{\overline{T}}$, and the laws $T\perp = \perp$ and $T\top = \top$ follow easily from *identity* and our rules (7) and (8). To prove the other laws is sometimes more tricky, and it seems we need in fact an extra rule like $SiP \vdash SiSP$ to derive distributivity.²² But this is enough to derive all axioms of Boolean algebra. Once we have enough to generate a Boolean algebra, it is clear that we have propositional logic as well.

In the rest of this section we will discuss how to account for propositional logic without allowing for a new set of conjunctive terms. We say that just as SaP and SiP are sentences if S and P are 1-ary terms, they are also sentences if S and P are 0-ary terms. Moreover, if S , P , Q and R are 1-ary predicates, $SaP, Ri\overline{Q}$, but also things like $(Si\overline{P})a(RaQ)$ will be 0-ary predicates as well. As mentioned earlier, we will assume that a 0-ary term will denote (from an extensional point of view) either *the* truth, or *the* falsity. To state things in a metaphysically less committing way, let us start again with a domain of individuals D . Now we assume that the denotation of any n -ary term will be a subset of D^n , where D^2 , for instance, is $\{\langle d_1, d_2 \rangle : d_1, d_2 \in D\}$. Thus, just like any 1-ary term will denote a subset of $D^1 = \{\langle d \rangle : d \in D\} = D$, any 0-ary terms will now denote a subset of $D^0 = \{\langle \rangle\}$. It is clear that D^0 has exactly two subsets: $\{\langle \rangle\}$ and \emptyset .²³ We say that if a sentence denotes $\{\langle \rangle\}$ it is true, and false otherwise. We can assume that for primitive 0-ary terms, or sentences, like ‘It is raining’ the denotation is given by the interpretation function. For complex propositional terms, its denotation is determined as expected:

- $V_M(SaP) = \{\langle \rangle : V_M(S) \subseteq V_M(P)\}$.
- $V_M(SiP) = \{\langle \rangle : V_M(S) \cap V_M(P) \neq \emptyset\}$.
- $V_M(SeP) = \{\langle \rangle : V_M(S) \cap V_M(P) = \emptyset\}$.
- $V_M(SoP) = \{\langle \rangle : V_M(S) - V_M(P) \neq \emptyset\}$.

²²Thanks to Elbert Booij to observe this.

²³At least, if D is not empty.

Notice that this interpretation works not only if ‘ S ’ and ‘ P ’ are 0-ary terms, but works if they both are monadic terms as well.

If we now think of ϕ (or $[\phi]$) and ψ as arbitrary 0-ary terms, we can form sentences (or 0-ary terms) like $[\phi]a[\psi]$, $[\phi]i[\psi]$, $[\phi]e[\psi]$ and $[\phi]o[\psi]$ and so on recursively. Let us see what some of them mean, if we don’t make the assumption of existential import (I used ‘1’ and ‘0’ as abbreviations for ‘ $\{\langle \rangle\}$ ’ and ‘ \emptyset ’, respectively):

| $[\phi]$ | $[\psi]$ | $[\phi]a[\psi]$ | $[\phi]i[\psi]$ | $[\phi]e[\psi]$ | $[\phi]o[\psi]$ | $[\phi]e[\phi]$ | $[[\phi]e[\phi]]a[\psi]$ |
|-----------------------|-----------------------|-----------------|-----------------|-----------------|-----------------|-----------------|--------------------------|
| $\{\langle \rangle\}$ | $\{\langle \rangle\}$ | 1 | 1 | 0 | 0 | 0 | 1 |
| $\{\langle \rangle\}$ | \emptyset | 0 | 0 | 1 | 1 | 0 | 1 |
| \emptyset | $\{\langle \rangle\}$ | 1 | 0 | 1 | 0 | 1 | 1 |
| \emptyset | \emptyset | 1 | 0 | 1 | 0 | 1 | 0 |

We see that $[\phi]a[\psi] \equiv \phi \rightarrow \psi$, $[\phi]i[\psi] \equiv \phi \wedge \psi$, $[\phi]e[\phi] \equiv \neg\phi$,²⁴ and $[[\phi]e[\phi]]a[\psi] \equiv \phi \vee \psi$. If we abbreviate ‘ SaS ’ by \top we can write the negation of ϕ more simply as ‘ $[\top]e[\phi]$ ’. Notice that because for 0-ary relation ϕ it holds that $V_M(\phi) = \{\langle \rangle\}$ iff $V_M([\top]a[\phi]) = \{\langle \rangle\}$, we can write ‘ $[\phi]$ ’ also as ‘ $[\top]a[\phi]$ ’.²⁵ Notice also that because $[\top]$ always denotes $\{\langle \rangle\}$, in our extensional fragment it holds for every ϕ that $V_M([\top]a[\phi]) = V_M([\top]i[\phi])$. (In the following I abbreviate ‘ $[\top]e[\phi]$ ’ sometimes by $\overline{[\phi]}$.)

Let us see now how things work from a proof-theoretic point of view. To implement the above suggestions, we will add to **SYL**⁺ the following three ideas: (i) in contrast to others terms, 0-ary terms don’t allow for existential import, (ii) \top^0 is a singular term, and (iii) P^0 is equal to $\top^0 i P^0$. The first idea is implemented with the help of axiom (6’) by stipulating that 0-ary terms are not categorical.

(10) 0-ary terms are not categorical terms and \top^0 is a singular term.

(11) $P^0 \dashv\vdash \top^0 i P^0$.

We will denote the system **SYLS**⁺ together with (10) and (11) by **SYL**⁺**PL**. This system can indeed account for all inferences in propositional logic. To see why, let us first look at the power of the *Dictum de Omni*.²⁶ We have seen that this rule played a crucial role in traditional logic. It turns out that this rule is very powerful, and immediately accounts for quite a number of inferences in propositional logic (see also Sommers, 1982). As we will see below, it immediately accounts for modus ponens, modus tollens, and the hypothetical and the disjunctive syllogism:²⁷

- Modus Ponens: $\phi, \phi \rightarrow \psi \vdash \psi$ $[\top]a[\phi], [\phi]a[\psi] \vdash_{DDO} [\top]a[\psi]$

²⁴Once we think of 0-ary predicates as terms, sentence negation is just 0-term negation. Thus $\overline{(SaP)}$ is by our earlier definition of ‘ \neg ’ identified with (SoP) , for instance.

²⁵If we require that all sentences are of subject-predicate structure, a primitive 0-ary term by itself doesn’t constitute a sentence. But this is a problem, because then it cannot be used in syllogistic reasoning. To nevertheless reason with such sentences after all, Wallis (1687) (and others) proposed that ‘it is raining’ really means something like ‘Now, it is raining’. To account for this we would have to add terms to the language that stand for points of time or place, and add singular terms like ‘now’, N , and ‘here’ that refers to the present point in time or place. A sentence like ‘It is raining’ can then be represented by something like NaR . A simpler route, which is more in accordance with what Frege did, would be to represent the sentence by $\top i R$ instead.

²⁶Scholastic and post-Scholastic logicians were well aware of its general applicability. Leibniz (1973) states in one sentence the following: “without loss of truth, the predicate can be put in place of the subject of a universal affirmative proposition, or the consequent in place of the antecedent of an affirmative proposition, in another proposition where the subject of the former proposition is the predicate or where the antecedent of the former is the consequent.”

²⁷Although Wallis (1687) does not seem to think of a conditional as a material implication, he explicitly argues (in chapter 16-18 of the third part) that these rules should be thought of as ordinary valid syllogisms of the form Barbara, Camestres, Barbara, and Camestres, respectively.

- Modus Tollens: $\neg\psi, \phi \rightarrow \psi \vdash \neg\phi$ $[\top]e[\psi] \vdash_{Contrap} [\psi]a[\perp], [\phi]a[\psi] \vdash_{DDO} [\phi]a[\perp]$
- Hypothetical Syllogism: $\phi \rightarrow \psi, \psi \rightarrow \chi \vdash \phi \rightarrow \chi$: $[\phi]a[\psi], [\psi]a[\chi] \vdash_{DDO} [\phi]a[\chi]$
- Disjunctive Syllogism ($\phi \vee \psi, \neg\phi \vdash \psi$): $([\top]e[\phi])a[\psi], [\top]a([\top]e[\phi]) \vdash_{DDO} [\top]a[\psi]$.

Also other ‘monotonicity’ inferences like ‘ $p \vdash q \rightarrow p$ ’, ‘ $p \vdash q \vee p$ ’, and ‘ $p \wedge q \vdash p$ ’ can be immediately accounted for if for each term P^0 , (i) we take P^0 to be equivalent with $\top^0 a P^0$ (as we argued above), and (ii) assume the validity of $P^n a \top^n$ as an axiom in our system **SYLS**⁺. Notice that $\top a P, Q a \top \vdash_{DDO} Q a P$, which accounts for ‘ $p \vdash q \rightarrow p$ ’. Second, $\top a P, \overline{Q} a \top (axiom) \vdash_{DDO} \overline{Q} a P$, which means that ‘ $p \vdash q \vee p$ ’. Finally, we derive $P i Q \vdash \top i P$ by means of the Reductio-rule and axiom (7), which means that ‘ $p \wedge q \vdash p$ ’. Suppose $P i Q$. By conversion this is $Q i P$. Assume now the negation of $\top i P$: $\top a \overline{P}$. Because $Q a \top$ is an axiom, we can use it to derive via the dictum $Q a \overline{P}$. Thus, we have reached a contradiction with the derived $Q i P$, and we conclude via the reductio-rule that $\top i P$.

It thus seems that the syntactic deduction system for propositional logic can make use of the syllogistic rules of inference. But is **SYL**⁺**PL** really enough? Can we already prove things like ‘ $p, q \vdash p \wedge q$ ’, ‘ $p \vdash p \wedge p$ ’, ‘ $p \vee p \vdash p$ ’, ‘ $p \vee q \vdash q \vee p$ ’, or ‘ $p \rightarrow q \vdash (r \rightarrow p) \rightarrow (r \rightarrow q)$ ’? It turns out that we can.²⁸

Let us first consider conjunction introduction. Recall that $p \equiv \top i p$. Because also $q \equiv \top i q$, it follows from the fact that \top is a singular term that $\top a q$. But from $\top i p$ and $\top a q$ it follows by the Dictum at once that $p i q$, thus $p, q \vdash p i q$. Next, we will make use of Aristotle’s reductio-rule to show that ‘ $p \vdash p \wedge p$ ’. So, assume p and $\neg(p i p)$. The latter is equivalent to $p a \overline{p}$. But from this together with p we derive via the Dictum that \overline{p} . Thus, assuming $\neg(p i p)$ we have reached a contradiction, from which we conclude via the reductio-rule that $p \vdash p i p$.

For the formulas involving disjunction it is important to realize that we represent $p \vee q$ by $\overline{p} a q$. For disjunctive elimination, we make use of rule (8): $\overline{S} a S \vdash \overline{S} a P$. First, notice that because we translate $p \vee p$ by $\overline{p} a p$, we can conclude by (8) to $\overline{p} a \perp$. Via contraposition and double negation we derive $\top a p$. Because \top is a singular term (rule 10)) it follows by (9) that $\top i p$, and thus via (11) that p . So we have validated $p \vee p \vdash p$. It is easier to validate ‘ $p \vee q \vdash q \vee p$ ’: it immediately follows by contraposition and double negation.

Notice that it would be very easy to account for ‘ $p \rightarrow q \vdash (r \rightarrow p) \rightarrow (r \rightarrow q)$ ’ if we could make use of the *deduction theorem*: $\Gamma, P \vdash Q \Rightarrow \Gamma \vdash P a Q$ (for this to make sense, P and Q have to be 0-ary terms, obviously). But this deduction theorem follows from **SYL**⁺**PL**: Assume $\Gamma, P \vdash Q$ and assume towards contradiction that $\neg(P a Q)$. This latter formula is equivalent to $P i \overline{Q}$. We have seen above that $\top i P$ can be derived from $P i \overline{Q}$ in **SYL**⁺. Because $\neg(P a Q) \vdash P$, it follows from the assumption $\Gamma, P \vdash Q$ that $\Gamma, \neg(P a Q) \vdash Q$. From this we derive $\top a Q$, and together with the validity of $P a \top$ we derive via the Dictum that $P a Q$. Thus, from Γ and assuming $\neg(P a Q)$ we derive a contradiction: $\Gamma, \neg(P a Q) \vdash \neg(P a Q), P a Q$. By Aristotle’s reductio-rule we conclude that $\Gamma \vdash P a Q$.

Now that we have shown that modus ponens follows from the Dictum de Omni, and that ‘ $p \vdash p \vee p$ ’, ‘ $p \vee p \vdash p$ ’, ‘ $p \vee q \vdash q \vee p$ ’, and ‘ $p \rightarrow q \vdash (r \rightarrow p) \rightarrow (r \rightarrow q)$ ’, we have in fact shown that we can derive all valid propositional formulas from our system **SYL**⁺**PL**! The reason is that we can axiomatize propositional logic by these four rules, together with modus ponens and the substitution rule of identicals.²⁹ Because we have seen in section 2 that the substitution rule follows from the Dictum de Omni, we can conclude that propositional logic *follows* from syllogistic logic if we (i) make the natural assumption that propositions are 0-ary terms, (ii) adopt Frege’s idea that the 0-ary term \top^0 is a singular term, and (iii) treat singular terms as proposed by Leibniz.

²⁸The De Morgan laws follow almost immediately by our definition of negation in section 2.

²⁹See Goodstein, 1963, chapter 4.

4 Relations

It is generally assumed that in traditional syllogistic logic there is no scope for relations. Thus — or so the Frege-Russell argument goes — it can be used neither to formalize natural language, nor to formalize mathematics. What we need, — or so Frege and Russell argued — is a whole new logic. But the Frege-Russell argument is only partly valid: instead of inventing a whole new logic, we might as well just extend the traditional fragment. Traditional logicians were well aware of an important limitation of syllogistic reasoning. In fact, already Aristotle recognized that the so-called ‘oblique’ terms (i.e. ones expressed in a grammatical case other than the nominative, and not having ‘widest scope’) gives rise to inferences that cannot be expressed in the ordinary categorical syllogistic. An example used by Aristotle is ‘Wisdom is knowledge, Wisdom is of the good, thus, Knowledge is of the good’ (Aristotle, Book 1, 36). This is intuitively a valid inference, but it, or its re-wording, is not syllogistically valid: ‘All wisdom is knowledge, Every good thing is object of some wisdom, thus, Every good thing is object of some knowledge’ (cf. Thom, 1981, 20). The re-wording shows that we are dealing with a binary relation here: ‘is object of’. Aristotle didn’t know how to deal with such inferences, but he noted that *if* there is a syllogism containing oblique terms, there must be a corresponding syllogism in which the term is put back into the nominative case.

As far as semantics is concerned, it is well-known how to work with relations. The main challenge, however, is to embed relations into the (generalized) syllogistic theory, and to extend the inference rules such that also proofs can be handled that crucially involve relations. As it turns out, part of this work has already been done by medieval logicians (Ockham, 1962; Buridan, 1976), and also by people like Leibniz and De Morgan when they were extending syllogistic reasoning such that it could account for inferences involving oblique terms, or relations.

To account for relations, we first have to extend the language. There are various ways to do so, but we will just combine relations with monadic terms by means of the ‘connectives’ $a, i, e,$ and o to generate new terms. This will just be a generalization of what we did before: When we combine a monadic term P with a monadic term S (and connective ‘ a ’, for instance), what results is a new 0-ary term like SaP . The generalization is now straightforward: if we combine an n -ary term/relation R with a monadic term S (and connective ‘ a ’, for instance), what results is a new $n - 1$ -ary term $(S^1aR^n)^{n-1}$. The semantics should now determine what such new terms denote. The $n - 1$ ary terms $(S^1aR^n)^{n-1}$ and $(S^1iR^n)^{n-1}$, for instance, would denote $\{\langle d_1, \dots, d_{n-1} \rangle : V_M(S) \subseteq \{d_n \in D : \langle d_1, \dots, d_n \rangle \in V_M(R^n)\}\}$ and $\{\langle d_1, \dots, d_{n-1} \rangle : V_M(S) \cap \{d_n \in D : \langle d_1, \dots, d_n \rangle \in V_M(R^n) \neq \emptyset\},$ respectively.³⁰

Now we can represent the natural reading of a sentence like ‘Every man loves a woman’ as $Ma(WiL^2)$. In traditional terminology it is said that ‘woman’ is an oblique term (in this sentence). The meaning of this formula is calculated as follows:

$$V_M(Ma(WiL^2)) = \{\langle \rangle : I(M) \subseteq \{d \in D : \langle d \rangle \in V_M(WiL^2)\}, \\ \text{with } V_M(WiL^2) = \{\langle d_1 \rangle : I(W) \cap \{d_2 \in D : \langle d_1, d_2 \rangle \in I(L^2)\} \neq \emptyset\}.$$

To represent the sentence ‘There is a woman who is loved by every man’ we will follow medieval practice and make use of the *passive form* of ‘love’: *being loved by*. For every binary relation R , we represent its passive form by R^\cup , interpreted as indicated above: $V_M(R^\cup) = \{\langle d_2, d_1 \rangle : \langle d_1, d_2 \rangle \in I_M(R)\}.$ ³² Now we represent ‘There is a woman who is

³⁰This by itself is not general enough for express important mathematical relations. For this we need to be able to combine relations with other non-monic terms as well. Now we could express *symmetry* and *asymmetry*, for instance, simply by RaR^\cup and RoR^\cup respectively. To express *transitivity* we also need the operation of *composition* ‘ \circ ’ with semantics $V_M(R \circ R') = \{\langle d, d'' \rangle : \exists d' : \langle d, d' \rangle \in V_M(R) \wedge \langle d', d'' \rangle \in V_M(R')\}$. Of course, this is all well known from work in relation algebra.

³¹As usual, $\langle d \rangle$ is identified with d .

³²Of course, the active-passive transformation only works for binary relations. For more-ary relations it

loved by every man’ as follows: $Wi(MaL^U)$. Also in this sentence ‘woman’ is an oblique term. This sentence is true iff:

$$V_M(Wi(MaL^U)) = \{\langle \rangle : I(W) \cap \{d \in D : \langle d \rangle \in V_M(MaL^U)\} \neq \emptyset\},$$

$$\text{with } V_M(MaL^U) = \{\langle d_1 \rangle : I(W) \subseteq \{d_2 \in D : \langle d_1, d_2 \rangle \in V_M(L^U)\}\}.$$

Both truth conditions are intuitively correct, and correspond with those of the two first-order formulas $\forall x[M(x) \rightarrow \exists y[W(y) \wedge L(x, y)]]$ and $\exists y[W(y) \wedge \forall x[M(x) \rightarrow \wedge L(x, y)]]$, respectively.³³

What we want to know, however, is how we can reason with sentences that involve relations. Let us first look at the re-wording of Aristotle’s example: ‘All wisdom is knowledge, Every good thing is object of some wisdom, thus, Every good thing is object of some knowledge’. If we translate this into our language this becomes $WaK, Ga(WiR) \vdash Ga(KiR)$, with ‘ R ’ standing for ‘is object of’. But now observe that we immediately predict that this inference is valid by means of the Dictum de Omni, if we can assume that ‘ W ’ occurs positively in ‘ $Ga(WiR)$ ’! We can mechanically determine that this is indeed the case.³⁴ First, we say that if a sentence occurs out of context, the sentence occurs positively. From this, we determine the positive and negative occurrences of other terms as follows:

$$\bar{P} \text{ occurs positively in } \Gamma \text{ iff } P \text{ occurs negatively in } \Gamma.$$

$$\text{If } (SaR) \text{ occurs positively in } \Gamma, \text{ then } S^-aR^+, \text{ otherwise } S^+aR^-.$$

$$\text{If } (SiR) \text{ occurs positively in } \Gamma, \text{ then } S^+iR^+, \text{ otherwise } S^-iR^-.$$

Thus, first we assume that ‘ $Ga(WiR)$ ’ occurs positively. From this it follows that the term ‘ WiR ’ occurs positively, from which it follows in turn that ‘ W ’ occurs positively. Assuming that ‘ WaK ’ is true, the Dictum allows us to substitute K for W in $Ga(WiR)$, resulting in the desired conclusion: $Ga(KiR)$. Notice that in a very similar way we could account for the inference ‘Every horse is an animal, A man sees every animal, thus A man sees every horse’, by using the Dictum de Nullo. Similarly with the inference from ‘Every man gives all flowers to some girl’ and ‘All roses are flowers’ to ‘Every man gives all roses to some girl’.

These are nice results, but we want more. Here is one classical example discussed by Jungius (1957) and Leibniz (1966e): ‘Every thing which is a painting is an art (or shorter, painting is an art), thus everyone who learns a thing which is a painting learns a thing which is an art’ (or shorter: everyone who learns painting learns an art). Formally: $PaA \vdash (PiL^2)a(AiL^2)$.³⁵ Semantically it is immediately clear that the conclusion follows from the premiss. But the challenge for traditional logic was to account for this inference in a proof-theoretic way. As Leibniz already observed, we can account for this inference in syllogistic logic if we add the extra (and tautological) premiss ‘Everybody who learns a thing which is a painting learns a thing which is a painting’, i.e. $(PiL^2)a(PiL^2)$. Now $(PiL^2)a(AiL^2)$ follows from PaA and $(PiL^2)a(PiL^2)$ by means of the *Dictum the Omni*, because by our above rules the second occurrence of ‘ P ’ in $(PiL^2)a(PiL^2)$ occurs in a monotone increasing position.³⁶ Notice that Leibniz’ example exactly parallels a famous example discussed by De Morgan (1847): ‘Every horse is an animal, thus Every owner of a horse is an owner of

fails. Fortunately, we can do something similar here, making use of some functions introduced by Quine in his proof that variables are not essential for first-order predicate logic. We won’t go into this here.

³³If we forget about existential import.

³⁴Cf. Sommers (1982) and van Benthem (1986).

³⁵One of De Morgan’s (1847, p.114) examples is similar: ‘Every man is an animal, thus He who kills a man kills an animal’.

³⁶In terms of our framework, Leibniz assumed that all terms being part of the predicate within sentences of the form SaP and SiP occur positively. But this is not necessarily the case once we allow for all types of complex terms: P doesn’t occur positively in $Sa(PaR)$, for instance. On Leibniz’s assumption, some invalid inferences can be derived (cf. Sánchez, 1991). These invalid inferences are blocked by our more fine-grained calculation of monotonicity marking.

an animal' also follows immediately with the Dictum if we assume the extra tautological premiss that Every owner of a horse is the owner of a horse, represented by $(HiO^2)a(HiO^2)$.

To account for other inferences we need to assume more than just a tautological premiss. For instance, we cannot yet account for many inferences Ockham (1962) already could account for. Ockham states a whole lists of oblique syllogisms to be valid, shows how they relate to standard syllogisms of the various figures, and remarks that oblique terms other than the first can be deduced to the first figure (cf. Thom, 1977). Some types of syllogisms listed by Ockham we have already discussed (e.g. the following oblique variants of syllogisms of the first figure: $Ma(PiR), SaM \vdash Sa(PiR)$ and $MaP, Sa(MiR) \vdash Sa(PiR)$). Other examples are more interesting and we need additional tools. The inference from 'No man is such that an ass sees him' $Me(AiS^U)$ and 'Everything that laughs sees a man' $La(MiS)$ to 'Nothing that laughs is an ass' LeA is intuitively valid, but cannot yet be accounted for. Something similar holds for the inference from 'Every boy loves some girl' $Ba(GiL)$ and 'No girl is such that some widow loves her' $Ge(WiL^U)$ to 'No boy is a widow' BeW . Ockham sees those inferences as oblique variants of the valid syllogism of the first figure Celarent. To account for these inferences we need to enrich our system **SYL+PL** with the rule of *oblique conversion* (12), and the passive rule (13) (for binary relations R , and predicates S and O):

(12) Oblique Conversion: $Si(OiR) \equiv Oi(SiR^U)$.

from 'A man loves a woman' we infer that 'A woman is loved by a man'.
and the converse of this.³⁷

(13) Double passive: $R^{UU} \equiv R$.

In standard predicate logic one can easily prove the equivalence of $\exists x[Mx \wedge \exists y[Wy \wedge Rxy]]$ with $\exists xyWy \wedge \exists x[Wx \wedge Rxy]$. But in contrast to predicate logic, our system demands that the sequence of arguments of a relational term is in accordance with the scope order of the associated terms. Because of this, we have to state how sentences with 'reverse' relations correspond with ordinary relational sentences. It is important to realize that the following conversion rule, (12') $Se(OiR) \equiv Oe(SiR^U)$, immediately follows from (12).

To account for the first of the above inferences, notice that by (12'), $Me(AiS^U)$ is equivalent to $Ae(MiS)$. But now we see that in this premiss and the premiss $La(MiS)$ the same term 'MiS' occurs, and we can immediately infer (by DDN) to AeL . By E -conversion the conclusion LeA follows. Ockham, instead would first convert the first premiss by ordinary E -conversion to $(MiS)eA$, and then derive the conclusion as a ordinary Celarent syllogism. Similarly for the second of the above inferences. This time we infer from $Ge(WiL^U)$ by (12') to $We(GiL)$. Together with the second premiss $Ba(GiL)$ it follows by the DDN that WeB and with E -conversion we derive BeW . Other inferences counted as valid by Ockham could be accounted for in a similar way. Ockham uses oblique conversion also to reduce valid oblique syllogisms in figures other than the first to valid oblique syllogism of the first figure. As observed by Thom (1977), Ockham (1962, Part IIa, ch. 9, 11. 1-13) says that some oblique syllogisms in the first figure are governed by the Dictum de Omni et Nullo. Thom (1977) argues, however, that this doesn't hold for the following valid oblique syllogism of the first figure (what he calls Barbara XSS): $MaP, Sa(MiR) \vdash Sa(PiR)$, because this is not a special case of Barbara. This might be true, but it is clear that it immediately follows by the DDO nevertheless, because of our general formulation of this rule in section 2.

The formulation of this rule in section 2 is not general enough, however, to account for the inference from 'There is a woman who is loved by every man' to 'Every man loves a woman'. We need the more general formulation of the Dictum in (1')³⁸ and the following

³⁷In fact, we only required $Si(OiR)aOi(SiR^U)$ once we have also (13). Thom (1977) introduces 7 more conversion rules and a rule of 'Ecthesis' that I don't use.

³⁸This Dictum is more general than the so-called Barbara XSS and Barbara XPP mentioned by (Thom, 1981, p. 256). The rule is semantically sound.

variant of oblique conversion:

(1') Dictum de Omni: $\Gamma(MaR)^+, \Theta(M)^+ \vdash \Gamma(\Theta(R))$.

(12'') Oblique Conversion: $Sa(OaR) \equiv Oa(SaR^U)$.³⁹
 from ‘every man loves every woman’ we infer that ‘every woman is loved by every man’ and the converse of this.

Let us see how we can account for the inference from $Wi(MaL^U)$ to $Ma(WiL)$:

1. $Wi(MaL^U)$ premiss.
2. $(MaL^U)a(MaL^U)$ a tautology (everyone loved by every man is loved by every man).
3. $Ma((MaL^U)aL^{UU})$ from 2 and (12'') ($S = (MaL^U)$ and $S' = M$).
4. $Ma((MaL^U)aL)$ by 3 and (13), substitution of L for L^{UU} .
5. $Ma(WiL)$ by 1 and 4, by the Dictum de Omni (1').⁴⁰

This type of validity can now be used to account for other inferences. For instance, in terms of it we can now infer from ‘Some man loves every woman’ ($Mi(WaL)$) and ‘Some teacher loves a woman’ ($Ti(WiL)$) that Some teacher loves someone that is loved by some man ($Ti((MiL^U)iL)$). The reason is that from the first premiss it follows that $Wa(MiL^U)$ holds, which together with the second premiss and the Dictum de Omni allows us to infer the conclusion.

5 Conclusion

In this paper I have argued with Sommers (1982) and others that interesting parts of standard logic could have, and perhaps even have been, developed naturally out of traditional syllogistic. Singular propositions straightforwardly fit into the system, and the syllogistic can naturally be extended to account for full propositional reasoning, and even for reasoning with relational terms. This is interesting, for one thing because syllogistic logic seems much closer to natural language than first-order predicate logic. In contrast to the latter, the former analyses sentences as being of subject-predicate structure. Indeed, the idea that natural language is misleading because it systematically analyses sentences in a wrong way is foreign to syllogistic logic. Syllogistic logic is also of great interest for empirical linguistics because it makes essential use of the *Dictum de Omni (et Nullo)*, distributivity, or monotonicity. Though Geach (1962) famously argued that the traditional theory of distribution was hopelessly confused, developments in modern Generalized Quantifier Theory shows that the traditional notion of monotonicity is of great value for linguistic and logical purposes. Moreover, Geurts & van der Silk (2005) showed experimentally that inferences based on monotonicity are ‘easy’ in an interesting sense.

Though we used neither a distinguished relation of identity, nor make use of variables to allow for binding, we have seen that we could nevertheless adequately express many types of sentences for which these tools are normally used in predicate logic. This doesn't mean that our extended syllogistic is as expressive as standard first-order logic. One easy

³⁹This rule can be derived from the earlier one. Assume $Sa(OaR)$. This is equivalent with $Se(\overline{OaR}) \equiv Se(OoR) \equiv Se(Oi\overline{R}) \equiv (Oe(Si\overline{R}^U) \equiv Oa(\overline{Si\overline{R}^U}) \equiv Oa(Se\overline{R}^U) \equiv Oa(SaR^U)$.

⁴⁰With $\Gamma = Ma, \Theta = Wi, R = L$, and $M = MaL^U$.

extension we have already suggested is the use of conjunctive terms to represent sentences like ‘Every tiger is a striped animal’.⁴¹ More seriously, what we cannot (yet) represent are sentences which *crucially* involve variables/pronouns and/or identity. Some examples for which these tools are crucial are the following: ‘Every/some man loves himself’, ‘All parents love their children’, ‘Everybody loves somebody else’, ‘There is a unique king of France’, and ‘At least 3 men are sick’. As it turns out, we can extend our language with *numerical quantifiers* (cf. Murphee, 1997) and Quinean predicate functors (Quine, 1976) to solve these problems, but these extensions have their price. Pratt-Hartmann (2009) shows that syllogistic systems with numerical quantifiers cannot be axiomatized (but see van Eijck (2007) for a related more positive result), and adding Quinean predicate functors forces the fragment to jump over the decidability border. In the formal system we have so far, the sequence of arguments of a relational term will always be in accordance with the scope order of the associated terms. Thinking of this system as a fragment of FOL means that this logic has a very interesting property. Following an earlier suggestion of Quine, Purdy (1998) shows that the limits of decidability are indeed close to the limits of what can be expressed in (our fragment of) traditional formal logic.⁴² This suggests that the extended syllogistic system discussed in this paper — the part of logic where we don’t essentially need variables or predicate functors — is indeed a very interesting part of logic. Indeed, a small contingent of modern logicians (e.g. Suppes, Sommers, van Benthem, van Eijck, Sánchez, Purdy, Pratt-Hartmann, Moss) seeks to develop a system of *natural logic* which is very close to what we have done in this paper in that it is essentially variable-free.

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⁴¹Alternatively, and more in accordance with tradition, we could add the following new connectives: if S and P are n -ary terms, then Sa^*P and Si^*P are also n -ary terms (of course, one of them is only required). These new complex terms will be interpreted as follows: $V_M(Sa^*P) = V_M(\bar{S}) \cup V_M(P)$ and $V_M(Si^*P) = V_M(S) \cap V_M(P)$. Notice that semantically it holds that SaP and SiP are true iff $\top^n a(Sa^*P)$ and $\top^n i(Si^*P)$ are true. To implement this, we could add these equivalences as axioms to the proof theory in the spirit of Leibniz (With the substitutions as suggested by Castañeda, 1990):

A proposition itself can be conceived as a term: thus, ‘Some A is B ’, i.e., ‘ AB is a true term’ is a term – namely, ‘ AB true’. Again, we have ‘Every A is B ’, i.e. ‘ A not- B is false’, i.e. ‘ A not- B false’ is a new term; and again ‘No A is B ’, i.e. ‘ AB is false’, i.e. ‘ AB false’ is a new term. (Leibniz, 1966a)

⁴²As noted by Jan van Eijck (p.c.), the extensions of syllogistic explicitly discussed in this paper are more easily seen to be decidable: they are all in the two-variable fragment of first-order logic, which was proved to have the finite model property by Mortimer (1975), and hence has decidable satisfiability. However, an extension of the formal system with ternary relations along the same lines, for instance, is natural, and still proved to be decidable by the results of Purdy (1998).

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