# Conjunctive interpretation of disjunction[*]
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**Abstract**  In this extended commentary I discuss the problem of how to account for 'conjunctive' readings of some sentences with embedded disjunctions for globalist analyses of conversational implicatures. Following Franke (2009, to appear), I propose that earlier proposals failed, because they did not take into account the interactive reasoning of what else the *speaker* could have said, and how else the *hearer* could have interpreted the (alternative) sentence(s). I show how Franke's idea relates to more traditional pragmatic interpretation strategies.

## 1  Introduction

Neo-Gricean explanations of what is meant but not explicitly said are very appealing. They start with what is explicitly expressed by an utterance, and then seek to account for what is meant in a *global* way by comparing what the speaker actually said with what he could have said. Recently, some researchers (e.g. Levinson (2000), Chierchia (2006), Fox (2007)) have argued that it is wrong to start with what is explicitly expressed by an utterance. Instead – or so it is argued –, implicatures should be calculated *locally* at linguistic clauses. For what it is worth, I find the traditional globalist analysis of implicatures more appealing, and all other things equal, I prefer the global analysis to a localist one. But, of course, not all things are equal. Localists provided two types of arguments in favor of their view: experimental evidence, and linguistic data. I believe that the ultimate 'decision' on which line to take should, in the end, depend only on experimental evidence. I have not much

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to say about this, but I admit to be happy with experimental results as reported by Chemla (2009) and Geurts & Pouscoulous (2009b) which mostly seem to favor a neo-Gricean explanation.

But localists provided linguistic examples as well, examples that according to them could not be explained by standard 'globalist' analyses. Impossibility proofs in pragmatics, however, are hard to give. Many examples involve triggers of scalar implicatures like 'or' or 'some' embedded under other operators. Some early examples include $\phi \vee (\psi \vee \chi)$ and $\Box(\phi \vee \psi)$. Localist theories of implicatures were originally developed to account for examples like this. As for the first type of example, globalists soon pointed out that these are actually unproblematic to account for. As for the second type, Geurts & Pouscoulous (2009b) provide experimental evidence that implicature triggers like 'or' and 'some' used under the scope of an operator like 'believe' or 'want' do not necessarily give rise to local implicatures. That is, many more participants of their experiments infer the implicature (1-b) from (1-a), than infer (3-b) and (2-b) from (2-a) and (3-a), respectively. Moreover, they show that there is little evidence that people in fact infer (3-b) from (3-a).

(1)     a.     Anna ate some of the cookies.
        b.     Anna didn't eat all of the cookies.

(2)     a.     Bob believes that Anna ate some of the cookies.
        b.     Bob believes that Anna did not eat all of the cookies.

(3)     a.     Bob wants Anna to hear some of the Verdi operas.
        b.     Bob wants Anna not to hear all of the Verdi operas.

These data are surprising for localist theories of implicatures according to which scalar inferences occur systematically and freely in embedded positions. The same data are accounted for rather easily, however, on a global analysis.[1] Thus, Geurts and Pouscoulous (2009a) argue that localist theories of embedded implicatures tend to over-generate, and that global neo-Gricean theories predict much better.

It is well-known, however, that globalist theories have serious problems as well with other examples involving triggers used in embedded contexts. Problematic examples include conditionals with disjunctive antecedents like $(\phi \vee \psi) > \chi$ and free choice permissions like $\Diamond(\phi \vee \psi)$. Both examples seem to give rise to 'conjunctive' interpretations: from $\Diamond(\phi \vee \psi)$, for example, we infer $\Diamond\phi \wedge \Diamond\psi$. Standard neo-Gricean analyses like those of Sauerland (2004) and van Rooij & Schulz (2004), however, do not predict this. Fox (2007) has shown that this conjunctive interpretation follows once we make use of recursive exhaustification, and Chemla (2009) has defined a new operator that can be applied globally to the formula $\Diamond(\phi \vee \psi)$,

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1 See Geurts & Pouscoulous (2009b) and Geurts & Pouscoulous (2009a) for discussion, and footnote 18.

and still gives rise to the desired conjunctive reading. This is certainly appealing, but it is not so clear that Chemla's analysis is truly neo-Gricean. In the words of Geurts & Pouscoulous (2009a), "Defining an operator is one thing; providing a principled pragmatic explanation is quite another". Franke (2009a,b) provided such a principled pragmatic explanation of these data making use of game theory.[2] The purpose of this paper is to show how this analysis relates to more traditional pragmatic interpretation strategies. As we will see, this reformulation also involves multiple uses of exhaustive interpretation. I will explain how the analysis still differs from the analysis of Fox (2007), and suggest that it is more Gricean in spirit.

The experimental data of Chemla (2009) are mostly problematic for localist analyses of implicatures. He found, for instance, that sentences of the form $\forall x (Px \lor Qx)$ do not routinely give rise to the expected 'local' implicature that $\forall x \neg (Px \land Qx)$.[3] Still, there is at least one type of experimental result that, he claims, favors a localist analysis. Chemla (2009) found that just as sentences of the form $\Diamond(\phi \lor \psi)$, also sentences of the form $\forall x \Diamond (Px \lor Qx)$ give rise to a 'conjunctive' interpretation: it licenses the inference to $\forall x \Diamond Px \land \forall x \Diamond Qx$. Chemla claims that this inference is predicted by a localist analysis, but not by a globalist one. In section 4.3 we will come back to this issue.

## 2 In need of pragmatic explanation

### 2.1 Conditionals with disjunctive antecedents

It seems reasonable that any adequate theory of conditionals must account for the fact that at least most of the time instantiations of the following formula (Simplification of Disjunctive Antecedents, SDA) are true:

(4)    (SDA)    $[(\phi \lor \psi) > \chi] \rightarrow [\phi > \chi) \land (\psi > \chi)]$

For instance, intuitively we infer from (5-a) that both (5-b) and (5-c) are true:

(5)    a.    If Spain had fought on either the Allied side or the Nazi side, it would have made Spain bankrupt.
       b.    If Spain had fought on the Allied side, it would have made Spain bankrupt.
       c.    If Spain had fought on the Nazi side, it would have made Spain bankrupt.

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2 For a rather different pragmatic explanation of these data, see Chemla (2010).

3 Geurts & Pouscoulous (2009b) found something similar, and claim that on the basis of their data one should conclude that this inference simply never takes place. I am not sure, though, whether they also tested that the inference also does not take place in case a sentence like 'Everybody likes bananas or apples' is given as answer to the explicit question 'What does everybody like?'.

Of course, if the conditional is analyzed as material or strict implication, this comes out immediately. Many researchers, however, don't think these analyses are appropriate, and many prefer an analysis along the lines of Lewis and Stalnaker. Adopting the limit assumption,[4] one can formulate their analyses in terms of a selection function, $f$ that selects for each world $w$ and sentence/proposition $\phi$ the closest $\phi$-worlds to $w$. A conditional represented as $\phi > \chi$ is now true in $w$ iff $f_w(\phi) \subseteq [\![\chi]\!]$. This analysis, however, does not make (SDA) valid. The problem is that if we would make this principle valid, e.g. by saying that $f_w(\phi \vee \psi) = f_w(\phi) \cup f_w(\psi)$, then the theory would loose one of its most central features, its non-monotonicity. The principle of monotonicity,

(6)     (MON)    $[\phi > \chi] \rightarrow [(\phi \wedge \psi) > \chi]$,

becomes valid. That is, by accepting SDA, we can derive MON on the assumption that the connectives are interpreted in a Boolean way,[5] and we end up with a strict conditional account. We have seen already that the strict conditional account (or the material conditional account) predicts SDA, but perhaps for the wrong reasons. The Lewis/Stalnaker account does not validate MON because SDA is not a theorem of their logic. Although there are well-known counterexamples to (SDA),[6] we would still like to explain why it holds in 'normal' contexts. A simple 'explanation' would be to say that a conditional of the form $(\phi \vee \psi) > \chi$ can only be used *appropriately* in case the best $\phi$-worlds and the best $\psi$-worlds are equally similar to the actual world. Though this suggestion gives the correct predictions, it is rather ad hoc. We would like to have a 'deeper' explanation of this desired result in terms of a general theory of pragmatic interpretation.

## 2.2   Free choice

The free choice problem is a problem about permission sentences. Intuitively, from the (stated) permission 'You may take an apple or a pear' one can conclude that you can take an apple *and* that you can take a pear (though perhaps not both). This intuition is hard to account for, however, on any standard analysis of permissions sentences. There is still no general agreement of how to interpret such sentences. In standard deontic logic (e.g. Kanger (1981), though basically due to Leibniz (1930)) it is assumed that permission sentences denote propositions that are true or false in a world, and that deontic operators (like *ought* and *permit*) apply to propositions.

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4 The assumption that for any world there is at least one closest $\phi$-world for any consistent $\phi$ - see Lewis (1973) for classic discussion.

5 From $\phi > \chi$ and the assumption that connectives are interpreted in a Boolean way, we can derive $((\phi \wedge \psi) \vee (\phi \wedge \neg\psi)) > \chi$. By SDA we can then derive $(\phi \wedge \psi) > \chi$.

6 See Fine (1975).

The permission $\Diamond\phi$ is considered to be true in $w$ just in case there exists a world deontically accessible from $w$ in which $\phi$ is true. Obviously, such an analysis predicts that $\Diamond\phi \models \Diamond(\phi \vee \psi)$.[7] This analysis does not predict, however, that $\Diamond(\phi \vee \psi) \models \Diamond\phi \wedge \Diamond\psi$. According to other traditions (e.g. Wright (1957); Lewis (1979)), we should look at permission sentences from a more *dynamic* perspective. But there are still (at least) two ways of doing this. According to the *performative analysis* (cf. Lewis (1979)), the main point of making a permission is to *change* a prior permissibility set to a posterior one. This analysis might still be consistent with the deontic logic approach in that it assumes that what is permitted denotes a proposition. Another tradition (going back to Wright (1957)) is based on the assumption that deontic concepts are usually applied to *actions* rather than propositions. Although permissions are now said to apply to actions, a permission sentence by itself is still taken to denote a proposition, and is true or false in a world.[8]

### 2.2.1 A conditional analysis with dynamic logic

Let us first look at the latter approach according to which deontic operators are construed as action modalities. Dynamic logic (Harel (1984)) makes a distinction between *actions* (and action-expressions) and *propositions*. Propositions *hold* at states of affairs, whereas actions produce a *change* of state. Actions may be nondeterministic, having different ways in which they can be executed. The primary logical construct of standard dynamic logic is the modality $\langle\alpha\rangle\phi$, expressing that $\phi$ holds *after* $\alpha$ is performed. This modality operates on an action $\alpha$ and a proposition $\phi$, and is true in world $w$ if some execution of the action $\alpha$ in $w$ results in a state/world satisfying the proposition $\phi$.

Dynamic logic starts with two disjoint sets; one denoting atomic propositions, the other denoting atomic actions. The set of *action expressions* is then defined to be the smallest set $\mathbf{A}$ containing the atomic actions such that if $\alpha, \beta \in \mathbf{A}$, then $\alpha \vee \beta \in \mathbf{A}$ and $\alpha;\beta \in \mathbf{A}$.[9] The set of *propositions* is defined as usual, with the addition that it is assumed that if $\alpha$ is an action expression and $\phi$ a proposition, then $\langle\alpha\rangle\phi$ is a proposition as well. To account for permission sentences we will assume that in that case also $Per(\alpha)$ is a proposition.

Propositions are just true or false in a world. To interpret the action expressions, it is easiest to let them denote *pairs* of worlds. The mapping $\tau$ gives the interpretation

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7 In the philosophical literature, this is sometimes called the paradox of free choice permission, because it is taken to be problematic.

8 There is yet another way to go, which recently became popular as well (e.g. Portner (2007)): assume that permission applies to an action, but assume that a permission statement also changes what is permitted. I won't go into this story here.

9 I will ignore iteration here.

of atomic actions. The mapping $\tau$ is extended to give interpretations to all action expressions by $\tau(\alpha;\beta) = \tau(\alpha); \tau(\beta)$ and $\tau(\alpha \vee \beta) = \tau(\alpha) \cup \tau(\beta)$. The action $\alpha;\beta$ consists of executing first $\alpha$, and then $\beta$. The action $\alpha \vee \beta$ can be performed by executing either $\alpha$ or $\beta$. We write $\tau_w(\alpha)$ for the set $\{v \in W : \langle w,v \rangle \in \tau_w(\alpha)\}$. Thus, $\tau_w(\alpha)$ is the set of all worlds you might end up in after performing $\alpha$ in $w$. We will say that $Per(\alpha)$ is true in $w$, $w \models Per(\alpha)$,[10] just in case $\tau_w(\alpha) \subseteq P_w$, where $P_w$ is the set of permissible worlds in $w$. Notice that this way of interpreting permissions gives them a conditional flavor: $Per(\alpha)$ really means that it is acceptable to perform $\alpha$.[11] Given the interpretation of disjunctive actions, it immediately follows that we can account for free choice permission: from the truth of $Per(\alpha \vee \beta)$ we can infer the truth of $Per(\alpha)$ and $Per(\beta)$.[12]

Although free choice permission follows, one wonders whether it should be built into the semantics: if I allow you to do $\alpha$ this doesn't mean that I allow you to do $\alpha$ in *any* way you want. I only allow you to do $\alpha$ *in the best way*. To account for this latter rider, we can add to our models a selection function, $f$, that picks out the best elements of any set of possible worlds $X$ for every world $w$. Then we say that $\Diamond \alpha$ is true in $w$ iff $f_w(\tau_w(\alpha)) \subseteq P_w$. But even if $f_w(X \cup Y) \subseteq f_w(X) \cup f_w(Y)$, it is still not guaranteed that $f_w(X \cup Y) = f_w(X) \cup f_w(Y)$, and thus the free choice permission inference isn't either. Of course, the inference follows in case $f_w(\alpha \vee \beta) = f_w(\tau_w(\alpha)) \cup f_w(\tau_w(\beta))$, but we would like to have a pragmatic explanation of why this should be the case if an assertion of the form $\Diamond(\alpha \vee \beta)$ is given.

### 2.2.2 A performative analysis

Lewis (1979), Kamp (1973) and Kamp (1979) have proposed a performative analysis of command and permission sentences involving a master and his slave. On their analysis, such sentences are not primarily used to make true assertions about the world, but rather to *change* that what the slave is obliged/permitted to do.[13] But how will permission sentences govern the change from the prior permissibility set, $\Pi$, to the posterior one, $\Pi'$? Kamp (1979) proposes that this change depends on a reprehensibility ordering, $\leq$, on possible worlds. The effect of allowing $\phi$ is that the best $\phi$-worlds are added to the old permissibility set to figure as the new permissibility set. This set will be denoted as $\Pi_\phi^*$ is and defined in terms of the

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10 Strictly speaking the definition of $\models$ should be relativized to a model, but the model remains implicit here as throughout the paper.

11 See Asher & Bonevac (2005) for a conditional analysis of permissions sentences.

12 Notice also that another paradox of standard deontic logic is avoided now: from the permission of $\alpha$, $Per(\alpha)$, the permission of $\alpha \vee \beta$, $Per(\alpha \vee \beta)$ doesn't follow.

13 For further discussion of this model, see e.g, van Rooij (2000).

relation $\leq$ as follows:

$$\Pi_\phi^* \quad \overset{\text{def}}{=} \quad \{u \in [\![\phi]\!] | \, \forall v \in [\![\phi]\!] : u \leq v\}$$

Thus, the change induced by the permission 'You may do $\phi$' is that the new permission set, $\Pi'$, is just $\Pi \cup \Pi_\phi^*$. Note that according to this performative account it does not follow that for a permission sentence of the form 'You may do $\phi$ or $\psi$' the slave can infer that according to the new permissibility set he is allowed to do any of the disjuncts. Still, in terms of Kamp's analysis we can give a pragmatic explanation of why disjuncts are normally interpreted in this 'free-choice' way. To explain this, let me first define a deontic preference relation between propositions, $\prec$, in terms of our reprehensibility relation between worlds, $<$. We can say that although both $\phi$ and $\psi$ are incompatible with the set of ideal worlds, $\phi$ is still preferred to $\psi$, $\phi \prec \psi$, iff the best $\phi$-worlds are better than the best $\psi$-worlds, $\exists v \in [\![\phi]\!]$ and $\forall u \in [\![\psi]\!] : \, v < u$. Then we can say that with respect to $\prec$, $\phi$ and $\psi$ are equally reprehensible, $\phi \approx \psi$, iff $\phi \preceq \psi$ and $\psi \preceq \phi$. It is easily seen that $\Pi_{\phi \vee \psi}^* = \Pi_\phi^* \cup \Pi_\psi^*$ iff $\phi \approx \psi$. How can we now explain the free choice effect? According to a straightforward suggestion, a disjunctive permission can only be made appropriately in case the disjuncts are equally reprehensible.[14] This suggestion, of course, exactly parallels the earlier suggestions of when conditionals with disjunctive antecedent can be used appropriately, or disjunctive permissions according to the dynamic logic approach. Like these earlier suggestions, however, this new suggestion by itself is rather *ad hoc*, and one would like to provide a 'deeper' explanation in terms of more general principles of pragmatic reasoning.

## 3 Pragmatic interpretation

### 3.1 The standard received view

Implicatures come in many varieties, but scalar implicatures have received most attention by linguists. A standard way to account for the scalar implicatures of '$\phi$' is to assume that $\phi$ is associated with a set of alternatives, $A(\phi)$, and that the assertion of $\phi$ implicates that all its stronger alternatives are false.

(7)  $Prag(\phi) = \{w \in [\![\phi]\!] | \neg \exists \psi \in A(\phi) : w \in [\![\psi]\!] \, \& \, [\![\psi]\!] \subset [\![\phi]\!]\}.$

If the alternative of '*Some* of the students passed' is '*All* of the students passed', the desired scalar implicature is indeed accounted for. McCawley (1993) noticed, however, that if one scalar item is embedded under another one — as in (8)[15] — an

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14 For an alternative proposal using this framework, see van Rooij (2006).
15 Landman (2000) and Chierchia (2004) discuss structurally similar examples like 'Mary is either working at her paper *or* seeing *some* of her students.'

interpretation rule like *Prag* does not give rise to the desired prediction that only one student passed if the alternatives are defined in the traditional way.

(8)    Alice passed *or* (Bob passed *or* Cindy passed).

This observation can be straightforwardly accounted for if we adopt a different pragmatic interpretation rule and a different way to determine alternatives. First, we assume that the set of alternatives includes {*Alice passed, Bob passed, Cindy passed*} (which should perhaps be closed under conjunction and disjunction). According to the new pragmatic interpretation rule *Exh*, $w$ is compatible with the pragmatic interpretation of $\phi$ iff (i) $\phi$ is true in $w$, and (ii) there is no other world $v$ in which $\phi$ is true where less alternatives in $A(\phi)$ are true than are true in $w$, see (9). In the following, we abbreviate the condition $\forall \psi \in A(\phi) : v \in [\![\psi]\!] \Rightarrow w \in [\![\psi]\!]$ by $v \leq_{A(\phi)} w$, and define $v <_{A(\phi)} w$ in terms of this in the usual way.

(9)    $Exh(\phi) = \{w \in [\![\phi]\!] | \neg \exists v \in [\![\phi]\!] : v <_{A(\phi)} w\}.$

The pragmatic interpretation rule *Exh* correctly predicts that from (8) we can pragmatically infer that only one of Alice, Bob, and Cindy passed. In fact, this pragmatic interpretation rule is better known as the *exhaustive interpretation* of a sentence (e.g. Groenendijk & Stokhof (1984); van Rooij & Schulz (2004); Schulz & van Rooij (2006); Spector (2003); Spector (2006)). By interpreting sentences exhaustively one can account for many conversational implicatures. But from a purely Gricean point of view, the rule is too strong. All that the Gricean maxims seem to allow us to conclude from a sentence like '*Some* of the students passed' is that the speaker does *not know* that '*All* of the students passed' is true; not the stronger proposition that the latter sentence *is* false. To account for this intuition, the following weaker interpretation rule, *Grice*, can be stated, which talks about knowledge rather than facts (where $\mathbf{K}\phi$ means that the speaker knows $\phi$):[16]

(10)    $Grice(\phi) = \{w \in [\![\mathbf{K}\phi]\!] | \forall v \in [\![\mathbf{K}\phi]\!], \forall \psi \in A(\phi) : w \models \mathbf{K}\psi \rightarrow v \models \mathbf{K}\psi\}$

As shown by Spector (2003) and van Rooij & Schulz (2004), exhaustive interpretation follows from this, if we assume that the speaker is as competent as possible insofar as this is compatible with *Grice*.

## 3.2   The problem

Although these interpretation rules account for many conversational implicatures, they give rise to the wrong predictions for more complex statements involving disjunction. Two prime examples are (i) free choice permissions of the form $\Diamond(\phi \vee$

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16 A similar weaker interpretation is given by Sauerland (2004).

$\psi$), and (ii) conditionals with disjunctive antecedents like $(\phi \vee \psi) > \chi$. It is widely held that the alternatives of these sentences are respectively $\Diamond\phi$, $\Diamond\psi$ and $\Diamond(\phi \wedge \psi)$, and $\phi > \chi$, $\psi > \chi$ and $(\phi \wedge \psi) > \chi$. Before we can discuss the possible pragmatic interpretations, let us first note that according to standard deontic logic $\Diamond(\phi \vee \psi) \models \Diamond\phi \vee \Diamond\psi$,[17] and that adopting the Lewis/Stalnaker analysis of conditionals, it holds that $(\phi \vee \psi) > \chi \models (\phi > \chi) \vee (\psi > \chi)$.

Let us first look at the standard pragmatic interpretation rule *Prag*. Given that $\Diamond\phi$ and $\Diamond\psi$ express stronger propositions than $\Diamond(\phi \vee \psi)$, it immediately follows that $Prag(\Diamond(\phi \vee \psi)) = \emptyset$, which is obviously wrong. Let's turn then to exhaustive interpretation. We take the only relevantly different worlds in which $\Diamond(\phi \vee \psi)$ are true to be $\{u, v, w\}$, where $\Diamond\phi$ is true in $u$ and $w$, and $\Diamond\psi$ is true in $v$ and $w$. Recall that $Exh(\phi)$ holds in worlds in which as few as possible alternatives to $\phi$ are true. But this means that $Exh(\Diamond(\phi \vee \psi)) = \{u, v\}$, from which we can wrongly conclude that only one of the permissions is true. The desired conclusion that both permissions are true is incompatible with this pragmatic interpretation. A similar story holds for conditionals with disjunctive antecedents.

Let us turn now to the weaker Gricean interpretation *Grice*. This weaker Gricean rule indeed predicts *an* interpretation that the sentences in fact have. For the disjucntive permission $\Diamond(\phi \vee \psi)$ it is predicted that neither $\Diamond\phi$ nor $\Diamond\psi$ are known to be true, but that they both are possibly true, perhaps even together. This prediction is appealing, but strengthening this by assuming our earlier form of competence doesn't give rise to the desired conclusion: the resulting exhaustive interpretation gives rise to the wrong prediction.

Perhaps this just means that the set of alternatives is chosen wrongly, or that the competence assumption is formalized in the wrong way. Indeed, this was proposed by Schulz (2003) and Schulz (2005) to account for free choice permissions. As for the latter, she took a speaker to be competent in case she knows of each alternative *whether* it is true. Second, she took the set of alternatives of $\Diamond\phi$ to be the set $\{\Box\psi : \psi \in A(\phi)\} \cup \{\Box\neg\psi : \psi \in A(\phi)\}$.[18] First, notice that by applying *Grice* to a sentence of the form '$\Diamond(\phi \vee \psi)$' it immediately follows that the speaker knows neither $\Box\neg\phi$ nor $\Box\neg\psi$, in formulas, $\neg\mathbf{K}\Box\neg\phi$ and $\neg\mathbf{K}\neg\Box\psi$. What we would like is that from here we derive the free choice reading: $\Diamond\phi$ and $\Diamond\psi$, which would follow from $\mathbf{K}\neg\Box\neg\phi$ and $\mathbf{K}\neg\Box\neg\psi$. Of course, this doesn't follow yet, because it might be that the speaker does not know what the agent may or must do.[19] But now assume

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17 Similarly, $f_w(\tau_w(\alpha \vee \beta)) \subseteq f_w(\tau_w(\alpha)) \cup f_w(\tau_w(\beta))$ and $\Pi^*_{\phi \vee \psi} \subseteq \Pi^*_{\phi} \cup \Pi^*_{\psi}$.

18 Taking $\Box\phi$ as an alternative is natural to infer from $\Diamond\phi$ to the falsity of this necessity statement.

19 Notice, though, that this inference *does* follow if '$\Box$' and '$\Diamond$' stand for epistemic *must* and epistemic *might*. This is so, because for the epistemic case we can safely assume that the speaker knows what he believes, which can be modeled by taking the epistemic accessibility relation to be fully introspective. This gives the correct predictions, because from 'Katrin might be at home or at work', it intuitively

that the speaker is *competent* on this in Schulz' sense. Intuitively, this means that $\mathbf{P}\Box\phi \equiv \mathbf{K}\Box\phi$ and $\mathbf{P}\Diamond\phi \equiv \mathbf{K}\Diamond\phi$. Remember that after applying *Grice*, it is predicted that neither $\mathbf{K}\Box\neg\phi$ nor $\mathbf{K}\Box\neg\psi$ holds, which means that $\mathbf{P}\neg\Box\neg\phi$ and $\mathbf{P}\neg\Box\neg\phi$ have to be true. The latter, in turn, are equivalent to $\mathbf{P}\Diamond\phi$ and $\mathbf{P}\Diamond\psi$. By competence we can now immediately conclude to $\mathbf{K}\Diamond\phi$ and $\mathbf{K}\Diamond\psi$, from which we can derive $\Diamond\phi$ *and* $\Diamond\psi$ as desired, because knowledge implies truth.[20]

Although I find this analysis appealing, it is controversial, mainly because of her choice of alternatives. This also holds for other proposed pragmatic analyses to account for free choice permissions, such as, for example, that of Kratzer & Shimoyama (2002). In section 4 I will discuss some other possible analyses that explain the desired free choice inference that assume that the alternatives of $(\phi \lor \psi) > \chi$ and $\Diamond(\phi \lor \psi)$ are $\phi > \chi$ and $\psi > \chi$, and $\Diamond\phi$ and $\Diamond\psi$, respectively.

## 4 Taking both directions into account

### 4.1 The intuition[21]

Suppose we adopt a Stalnaker/Lewis style analysis of conditional sentences. In that case we have to assume a selection function $f$, to evaluate the truth-value of the sentence. Take now a set of worlds in which $[\![(\phi \lor \psi) > \chi]\!] = \{u, v, w\}$ such that (i) $f_u(\phi \lor \psi) = f_u(\phi) \subseteq [\![\chi]\!]$ and $f_u(\psi) \not\subseteq [\![\chi]\!]$, (ii) $f_v(\phi) \not\subseteq [\![\chi]\!]$ and $f_v(\phi \lor \psi) = f_v(\psi) \subseteq [\![\chi]\!]$, and (iii) $f_w(\phi \lor \psi) = f_w(\phi) \cup f_w(\psi) \subseteq [\![\chi]\!]$.[22] We would like to conclude via pragmatic reasoning that the speaker who asserted $(\phi \lor \psi) > \chi$

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follows that, according to the speaker, Katrin might be at home, and that she might be at work (cf. Zimmermann (2000).

20 Notice that it is also Schulz' reasoning and notion of competence for 'Anna ate all of the cookies' that is used to explain why from (2-a) we conclude to (2-b).

21 The intuition of the following solution I owe to Michael Franke (see Franke (2009b)). One way of working out this intuition will be somewhat different, though, from what Franke proposed. This way makes use of bidirectional optimality theory. Earlier accounts making use of Bi-OT include Saebo (2004) and Aloni (2007). What I always found problematic about such earlier Bi-OT solutions (I was a co-author of an earlier version of Aloni (2007)) is that *complexity* of alternative expressions was taken to play a crucial role. But explanations based on complexity are not always equally convincing. Following Franke (2009b), I believe that making use of complexity is not required. At a 2009-conference in Leuven where I presented Bi-OT and game-theoretic 'solutions' of the problem of free-choice inferences, Bart Geurts presented a solution that was based on a similar intuition (I am not sure in how far complexity played a crucial role here, or not), to be presented in Geurts (to appear). I believe that also Edgar Onea suggested a solution very much in the same spirit. Perhaps this should be taken as an indication how *natural* a solution in this spirit is.

22 It might seem that I wrongly assume that $\phi > \chi \models (\phi \lor \psi) > \chi$ and $\psi > \chi \models (\phi \lor \psi) > \chi$. This is not, and should not, be the case. It might well be, for instance, that $\phi > \chi$ is true in $w$, but $(\phi \lor \psi) > \chi$ is not. However, our reasoning will not depend on such worlds, because we will only consider worlds in which $(\phi \lor \psi) > \chi$ is true.

implicated that we are in world $w$. In that case both $\phi > \chi$ and $\psi > \chi$ are true as well, and we derived the 'conjunctive' interpretation of the conditional with a disjunctive antecedent. The reasoning will go as follows. First, we are going to assume that the speaker is competent: she knows in which world she is. It seems unreasonable that she is in $u$, because otherwise the speaker could have used an alternative expression, $\phi > \chi$, which (limiting ourselves to worlds in which $(\phi \vee \psi) > \chi$ is true) more accurately singles out $\{u\}$ than $(\phi \vee \psi) > \chi$ does. For the same reason we can conclude that the speaker is not in world $v$. In the only other case, $w$, $f_w(\phi \vee \psi) = f_w(\phi) \cup f_w(\psi) \subseteq [\![\chi]\!]$, and thus both $\phi > \chi$ *and* $\psi > \chi$ are true. Of course, one might wonder whether also this state cannot be expressed more economically by an alternative expression. But the answer to this will be negative, because we have already assumed that $(\phi > \chi) \wedge (\phi > \chi)$ is *not* an alternative to $(\phi \vee \psi) > \chi$. Thus, $(\phi \vee \psi) > \chi$ pragmatically entails $\phi > \chi$ and $\psi > \chi$, because if not, the speaker could have used an alternative expression which more accurately singled out the actual world.

Intuitive solutions are ok, but to test them, we have to make them precise. In the following I will suggest two ways to implement the above intuition. Both implementations are based on the idea that to account for the desired 'conjunctive' inferences of the disjunctive sentences, alternative expressions *and* alternative worlds/interpretations must play a very similar role in pragmatic interpretation. Thinking of it in somewhat different terms, we should take seriously both the speaker's and the hearer's perspective. Fortunately, there are two well-known theories on the market that look at pragmatic interpretation from such a point of view: *Bi-directional Optimality Theory* (e.g. Blutner (2000)), and *Game Theory* (e.g. Benz, Jaeger & van Rooij (2005)). In the following I will discuss two possible ways to proceed, but they have something crucial in common: both ways make use of different *levels* of interpretation. The first proposal is game theoretic in nature, and due to Franke (2009b), Franke (2009a). The second suggestion is a less radical departure from the 'received view' in pragmatics, and is more in the spirit of Bi-OT. It makes crucial use of exhaustive interpretation and of different levels of interpretation, but like in Bi-OT, alternative worlds and expressions that initially played a role in interpretation need not play a role anymore at higher levels.[23]

## 4.2 Franke's game theoretic solution

Game theoretic and optimality theoretic analyses of conversational implicatures seek to account in one systematic way for both scalar implicatures *and* for implicatures involving marked and unmarked meanings/interpretations, inspired by Horn's

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23 For the exact relation between Bi-OT and the game theoretical best response dynamics Franke makes use of, see Franke (2009b).

| $P(w\|[\![\phi]\!])$ | $u$ | $v$ | $w$ | | $\sum_w P(w\|[\![\phi]\!])$ |
|---|---|---|---|---|---|
| 'Some' | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $=$ | 1 |
| 'Most' | 0 | $\frac{1}{2}$ | $\frac{1}{2}$ | $=$ | 1 |
| 'All' | 0 | 0 | 1 | $=$ | 1 |

**Figure 1**    $P(w\|[\![\phi]\!])$ for standard scale

division of pragmatic labor. In order to do so, they associate with an expression not just a semantic meaning, but assign also probabilities. According to the most straightforward proposal, the probability of $w$ given $[\![\phi]\!]$, $P(w\|[\![\phi]\!]) = \frac{1}{card([\![\phi]\!])}$ if $w \in [\![\phi]\!]$, 0 otherwise. Recall that according to one standard approach pragmatic interpretation works as follows:

(7)    $Prag(\phi) = \{w \in [\![\phi]\!] | \neg \exists \psi \in A(\phi) : w \in [\![\psi]\!] \ \& \ [\![\psi]\!] \subset [\![\phi]\!]\}.$

On the assumption that all worlds are equally likely, here is a straightforward way to reformulate (7) making use of probabilities:

(11)    $Prag'(\phi) = \{w \in [\![\phi]\!] | \neg \exists \psi \in A(\phi) : P(w\|[\![\psi]\!]) > P(w\|[\![\phi]\!])\}$

Look now at a standard example with $[\![All]\!] = \{w\} \subset [\![Most]\!] = \{v, w\} \subset [\![Some]\!] = \{u, v, w\}$.[24] From the assertion 'Some' the desired implicature immediately follows, as can be seen from figure 1.

The idea is that, for instance, 'Most' is pragmatically interpreted as $\{v\}$, because (i) there is no world in which 'Most' gets a higher value, and (ii) in $v$ it is best to utter 'Most', because for all alternatives $\psi$, $P(v\|[\![\psi]\!]) < P(v\|'Most')$.

Let us now do the same for the sentence $(\phi \lor \psi) > \chi$, together with its alternatives. Suppose that if we have in the columns the alternative worlds (with $u_{\phi \prec \psi}$ standing for the world where the best $\phi$-worlds are closer to $u$ than any $\psi$-world), and that we assume that $\chi$ is true in the most similar worlds (but not in others). In that case we get figure 2 (where $f$ is an arbitrary form, or expression).

A number of things are worth remarking. First of all, all sentences are true in $w_{\phi \approx \psi}$. As a result of this, a (naive) hearer will interpret, for instance, $\phi > \chi$ as equally likely true in $u_{\phi \prec \psi}$ as in $w_{\phi \approx \psi}$. Now take the speaker's perspective. Which statement would, or should, she make given that she is in a particular situation, or world? Naturally, that statement that gives her the highest chance that the (naive) hearer will interpret the message correctly. Thus, she should utter that sentence

24 With '*All*' abbreviating 'All Ps are Qs'.

| $P(w\|\llbracket f\rrbracket)$ | $u_{\phi \prec \psi}$ | $v_{\psi \prec \phi}$ | $w_{\phi \approx \psi}$ | $\sum_w P(w\|\llbracket f\rrbracket)$ | |
|:---:|:---:|:---:|:---:|:---:|:---:|
| $\phi > \chi$ | $\frac{1}{2}$ | $0$ | $\frac{1}{2}$ | $=$ | $1$ |
| $\psi > \chi$ | $0$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $=$ | $1$ |
| $(\phi \vee \psi) > \chi$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $=$ | $1$ |

**Figure 2**    $P(w|\llbracket f\rrbracket)$ for counterfactual

which gives the highest number in the column. But this means that in $u_{\phi \prec \psi}$ she should (and rationally would) utter $\phi > \psi$, in $v_{\psi \prec \phi}$ she should utter $\psi > \chi$, and in $w_{\phi \approx \psi}$ it doesn't matter what she utters, both are equally good. The boxed entries model this speaker's choice. The important thing to note is that according to this reasoning, no speaker (a speaker in no world) would ever utter $(\phi \vee \psi) > \chi$. Still, this is exactly the message that was uttered and should be interpreted, so we obviously missed something.

Franke (2009b) proposes that our reasoning didn't go far enough. We should now take the hearer's perspective again, *taking into account the optimal speaker's message-choice given a naive semantic interpretation of the hearer.*[25] This can best be represented by modeling the probabilities of the messages sent according to the previous reasoning, given the situation/world that the speaker is in.[26] How should the hearer now interpret the messages? Well, because the speaker would always send $\phi > \chi$ in $u_{\phi \prec \psi}$, while the chance that she sends $\phi > \chi$ in $w_{\phi \approx \psi}$ is lower (and taking the a priori probabilities of the worlds to be equal), there is a higher chance that the speaker of $\phi > \chi$ is in world $u_{\phi \prec \psi}$ than in $w_{\phi \approx \psi}$, and thus the hearer will choose accordingly. This is represented by the boxed entry in figure 3 (in which $P(f|w)$ stands for the probability with which the speaker would say $f$ if she were in $w$). Something similar holds for $\psi > \chi$. As for $(\phi \vee \psi) > \chi$, it is clear that all worlds are equally likely now, given that a previous speaker would not make this utterance in any of those worlds.

Having specified how such a more sophisticated hearer would interpret the alternative utterances, we turn back to the speaker, but also one that takes such a more sophisticated hearer into account. First we fill in the probabilities of the worlds, given the previous reasoning. Notice that these probabilities are crucially different from the earlier $P(w|\llbracket f\rrbracket)$. The speaker now chooses optimally given these probabilities: i.e. the speaker chooses (one of) the highest rows in the columns.

25 Jaeger & Ebert (2009) make a similar move. Both models are instances of Iterated Best Response (IBR) models.

26 For a more precise description, the reader should consult Franke (2009b), obviously.

| $P(f\|w)$ | $u_{\phi \prec \psi}$ | $v_{\psi \prec \phi}$ | $w_{\phi \approx \psi}$ |
|---|---|---|---|
| $\phi > \chi$ | 1 | 0 | $\frac{1}{2}$ |
| $\psi > \chi$ | 0 | 1 | $\frac{1}{2}$ |
| $(\phi \vee \psi) > \chi$ | 0 | 0 | 0 |
| $\sum_f P(f\|w)$ | 1 | 1 | 1 |

**Figure 3**    $P(f\|w)$ for $1^{st}$-level hearer

| $P(w\|f)$ | $u_{\phi \prec \psi}$ | $v_{\psi \prec \phi}$ | $w_{\phi \approx \psi}$ | | $\sum_w P(w\|f)$ |
|---|---|---|---|---|---|
| $\phi > \chi$ | 1 | 0 | 0 | = | 1 |
| $\psi > \chi$ | 0 | 1 | 0 | = | 1 |
| $(\phi \vee \psi) > \chi$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | = | 1 |

**Figure 4**    $P(w\|f)$ for $1^{st}$-level speaker

In $u_{\phi \prec \psi}$ and $v_{\psi \prec \phi}$ she would choose as before, but in $w_{\phi \approx \psi}$ she now chooses $(\phi \vee \psi) > \chi$ instead of either of the others. This is again represented by boxed entries in figure 4.

If we take the hearer's perspective again, the iteration finally reaches a fixed point. As illustrated by figure 5, $(\phi \vee \psi) > \chi$ is now interpreted by the even more sophisticated hearer in the desired way. From the truth of $(\phi \vee \psi) > \chi$, both $\phi > \chi$ and $\psi > \chi$ pragmatically follow.

| $P(f\|w)$ | $u_{\phi \prec \psi}$ | $v_{\psi \prec \phi}$ | $w_{\phi \approx \psi}$ |
|---|---|---|---|
| $\phi > \chi$ | 1 | 0 | 0 |
| $\psi > \chi$ | 0 | 1 | 0 |
| $(\phi \vee \psi) > \chi$ | 0 | 0 | 1 |
| $\sum_f P(f\|w)$ | 1 | 1 | 1 |

**Figure 5**    $P(f\|w)$ for $2^{nd}$-level hearer

Franke (2009b), Franke (2009a) shows that by exactly the same reasoning free choice permissions are accounted for as well.[27] What is more, using exactly the same machinery he can even explain (by making use of *global* reasoning) why we infer from $(\phi \lor \psi) > \chi$ and $\Diamond(\phi \lor \psi)$ that the alternatives $(\phi \land \psi) > \chi$ and $\Diamond(\phi \land \psi)$ are *not* true, inferences that are sometimes taken to point to a *local* analysis of implicature-calculation.

## 4.3   A Bidirectional-like solution

Is there any relation between the above game-theoretic reasoning and the 'received' analysis making use of pragmatic interpretation rule (7) or that of exhaustive interpretation, (9)? I will suggest that a 'bidirectional' received view is at least very similar to Franke's proposal sketched above, and does the desired work as well.

In the above explanation, we started with looking at the semantic interpretation from the *hearer*'s point of view. This way of starting things was motivated by pragmatic interpretation rule (7):

(7)     $Prag(\phi) = \{w \in [\![\phi]\!] \mid \neg \exists \psi \in A(\phi) : w \in [\![\psi]\!] \ \& \ [\![\psi]\!] \subset [\![\phi]\!]\}.$

But we could have started with the pragmatic interpretation rule (9) as well.

(9)     $Exh(\phi) \ = \ \{w \in [\![\phi]\!] \mid \neg \exists v \in [\![\phi]\!] : v <_{A(\phi)} w\}.$

In that case we wouldn't have started from the hearer's, but rather from the *speaker*'s point of view. Also this would have given rise to a reformulation and a table, but now the probability function, $P(\psi|w)$, gives the probabilities with which the speaker would have used the alternative expression $\psi$ given the world $w$ she is in. The naive assumption now is that $P(\psi|w)$ is simply $\frac{1}{card(\{\chi \in A(\phi):w \models \chi\})}$, if $w \models \psi$, and 0 otherwise. The reformulation now looks as follows:

(12)     $Exh'(\phi) \ = \ \{w \in [\![\phi]\!] \mid \neg \exists v : P(\phi|v) > P(\phi|w)\}.$

For the simple scalar implicature, the table to start with from a naive speaker's point of view is given in figure 6:

Though the way of choosing would be different (it is the hearer now who chooses the column with the highest number), the result would be exactly the same. What would be the beginning table for our problematic sentence $\Diamond(\phi \lor \psi)$? It is given in figure 7.

Just as we derived using the rule of exhaustive interpretation, the first prediction

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27 Franke uses standard deontic logic, but that doesn't seem essential. Starting with one of the two more dynamic approaches, he could explain the free choice inference as well using a very similar reasoning.

| $P(\phi|w)$ | $u$ | $v$ | $w$ |
|:---:|:---:|:---:|:---:|
| 'Some' | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ |
| 'Most' | 0 | $\frac{1}{2}$ | $\frac{1}{3}$ |
| 'All' | 0 | 0 | $\frac{1}{3}$ |
| $\sum_f P(f|w)$ | 1 | 1 | 1 |

**Figure 6**    $P(\phi|w)$ for standard scale

| $P(f|w)$ | $u$ | $v$ | $w$ |
|:---:|:---:|:---:|:---:|
| $\Diamond\phi$ | $\frac{1}{2}$ | 0 | $\frac{1}{3}$ |
| $\Diamond\psi$ | 0 | $\frac{1}{2}$ | $\frac{1}{3}$ |
| $\Diamond(\phi\vee\psi)$ | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{3}$ |
| $\sum_f P(f|w)$ | 1 | 1 | 1 |

**Figure 7**    $P(f|w)$ for $0^{th}$-level speaker

would be that $\Diamond(\phi \vee \psi)$ is interpreted as $\{u,v\}$. To improve things, we have to look again at the hearer's perspective. And, in fact, this could be done in Franke's framework, and we end up with exactly the same desired solution. What this suggests is two things: (i) adopting speaker's and hearer's point of view closely corresponds with pragmatic interpretation rules (9) and (7), respectively; (ii) to correctly predict the pragmatic interpretation of $\Diamond(\phi \vee \psi)$ we have to take both types of interpretation rules into account.

Recall the intuition as expressed in the previous subsection. That reasoning corresponded very closely to the following pragmatic interpretation rule: $Prag^*(\phi) = \{w \in Exh(\phi)| \neg \exists \psi \in A(\phi): w \in [\![\psi]\!]^{[\![\phi]\!]} \wedge [\![\psi]\!]^{[\![\phi]\!]} \subset [\![\phi]\!]\}$, where $[\![\psi]\!]^{[\![\phi]\!]}$ denotes $[\![\psi]\!] \cap [\![\phi]\!]$ and $\psi$ is taken not to be an element of $A(\phi)$.[28] Notice that this rule is close to interpretation rule (7), with the important difference that exhaustive interpretation (the speaker's point of view) plays an important role. Unfortunately, just as the earlier $Prag$, also this rule wrongly predicts that a sentence like $\Diamond(\phi \vee \psi)$ doesn't have a pragmatic interpretation ($Prag^*(\Diamond(\phi \vee \psi)) = \emptyset$). For this reason we have to iterate,[29] although the intuition behind this new rule will remain the same: $\Diamond(\phi \vee \psi)$ pragmatically entails $\Diamond\phi$ and $\Diamond\psi$, because if not, the speaker could have used an alternative expression which more accurately singled out the actual world.

In the following we will abbreviate the condition that $\forall \psi \in A(\phi): v \in [\![\psi]\!] \Rightarrow w \in [\![\psi]\!]$ by $v \leq_{A(\phi)} w$ as before. If $K$ is the set of worlds in which the sentence under consideration is true, I will also abbreviate $[\![\psi]\!] \cap K$ by $[\![\psi]\!]^K$. Moreover, $\psi \prec^n \phi$ will be an abbreviation for the condition '$[\![\psi]\!]^{K_n} \subset [\![\phi]\!]^{K_n}$, if $n = 0$, and $[\![\psi]\!]^{K_n} \subseteq [\![\phi]\!]^{K_n}$, otherwise'. Intuitively, $\psi \prec^n \phi$ expresses the fact that at least some worlds of the $n$th-level interpretation of $\phi$ could be expressed more precisely by alternative expression $\psi$. We will make use of the following definitions:[30]

(13) $\quad Exh_n^{K_n}(\phi) \overset{def}{=} \{w \in [\![\phi]\!]^{K_n} | \neg \exists v \in [\![\phi]\!]^{K_n}: v <_{A_n(\phi)} w\}.$

(14) $\quad Prag_n^{K_n}(\phi) \overset{def}{=} \{w \in Exh_n^{K_n}(\phi) | \neg \exists \psi \in A_n(\phi), w \in [\![\psi]\!]^{K_n} \,\&\, \psi \prec^n \phi\}.$

(15) $\quad K_{n+1} \overset{def}{=} \{w \in [\![\phi]\!]^{K_n} | w \notin Exh_n^{K_n}(\phi)\}.$

(16) $\quad A_{n+1}(\phi) \overset{def}{=} \{\psi \in A_n(\phi) | \neg \exists w \in Exh_n^{K_n}(\phi), w \in [\![\psi]\!]^{K_n} \,\&\, \psi \prec^n \phi\}.$

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28 Notice that if $w \in Prag^*(\phi)$, one can think of the pair $\langle \phi, w \rangle$ as – using bidirectional OT-terminology – a *strong* optimal form-meaning pair.

29 In OT-terminology, we have to look at the notion of *weak* optimality.

30 I won't try to prove this here, but I believe that the analysis would be almost equivalent to Franke's game theoretic approach, if we redefined the definitions of the orderings '$v <_{A_n(\phi)} w$' and '$\psi \prec^n \phi$' in *quantitative* rather than *qualitative* terms as follows: $v \leq_{A_n(\phi)} w$ iff$_{def}$ $card(\{\psi \in A(\phi): v \in [\![\psi]\!]\}) \leq card(\{\psi \in A(\phi): v \in [\![\psi]\!]\})$, and $\psi \prec^n \phi$ iff$_{def}$ $card([\![\psi]\!]^{K_0}) < card([\![\phi]\!]^{K_n})$, if $n = 0$, and $card([\![\psi]\!]^{K_n}) \leq card([\![\phi]\!]^{K_n})$, otherwise.

The pragmatic interpretation of $\phi$ with respect to set of worlds $K$ and alternative expressions $A(\phi)$, $Prag^K(\phi)$, will now be $Prag_n^{K_n}(\phi)$ for the first $n$ such that $Prag_n^{K_n}(\phi) \neq \emptyset$. If there is no such $n$, $Prag^K(\phi) = Exh_0^{K_0}(\phi)$, where $K_0 = K$ and $A_0(\phi) = A(\phi)$.

Notice that (13) and (14) are just the straightforward generalizations with respect to a set of worlds $K$ of standard exhaustive interpretation rule (9) and pragmatic interpretation rule (7) respectively.

(9)    $Exh^K(\phi)$  $=$  $\{w \in [\![\phi]\!]^K | \neg \exists v \in [\![\phi]\!]^K : v <_{A(\phi)} w\}$.

(7)    $Prag^K(\phi)$  $=$  $\{w \in [\![\phi]\!]^K | \neg \exists \psi \in A(\phi), w \in [\![\psi]\!]^K \ \& \ [\![\psi]\!]^K \subset [\![\phi]\!]^K\}$

The only difference between (13) and (9) is that the relevant set of worlds and the relevant set of alternatives might depend on earlier stages in the interpretation. If we limit ourselves to the first interpretation (i.e., level 0), the two interpretation rules are identical. Similarly for the difference between (14) and (7): the relevant alternatives depend on earlier stages, and the set of worlds with respect to which the entailment relation between $\psi$ and $\phi$ must be determined depends on earlier stages as well. Indeed, if we look at the first interpretation, the only important difference is that (14) takes as input the exhaustive interpretation of $\phi$, while this is not the case for (7). This difference implements the view that speaker's and hearer's perspective are both required.

The definitions (15) and (16) determine which worlds and alternative expressions are relevant for the interpretation at the $n + 1$th level of interpretation. We start with interpretation 0 (the first interpretation). Notice first that level 1 is only reached in case $Prag_0^{K_0}(\phi) = \emptyset$, i.e., in case for each world $v$ in the exhaustive interpretation of $\phi$ there is an alternative expression $\psi$ that is true in $v$ and which is stronger than $\phi$. Thus, in that case there is no world $v \in Exh(\phi)$ such that $\phi$ is at least as specific as any other alternative that is true in $v$. For the interpretation $\phi$ at level 1 we will not consider worlds in the 0th-level exhaustive interpretation of $\phi$ anymore. This is what (15) implements. The new set of alternatives determined by (16) are those elements of the original set of alternatives $A_0$ that did not help to eliminate worlds in $Exh(\phi)$ at the 0th level of interpretation.

Let us see how things work out for some particular examples. Let us first look at $\Diamond(\phi \vee \psi)$ with $A(\Diamond(\phi \vee \psi)) = \{\Diamond\phi, \Diamond\psi, \Diamond(\phi \wedge \psi)\}$, and assume that $K = \{u, v, w, x\}$, $[\![\Diamond(\phi \vee \psi)]\!] = \{u, v, w, x\}$, $[\![\Diamond\phi]\!] = \{u, w, x\}$, $[\![\Diamond\psi]\!] = \{v, w, x\}$, and $[\![\Diamond(\phi \wedge \psi)]\!] = \{x\}$. Observe that $Exh_0^{K_0}(\Diamond(\phi \vee \psi)) = \{u, v\}$. But neither $u$ nor $v$ can be an element of $Prag_0^{K_0}(\Diamond(\phi \vee \psi))$, because $[\![\Diamond\phi]\!]^{K_0} \subset [\![\Diamond(\phi \vee \psi)]\!]$ and $[\![\Diamond\phi]\!]^{K_0} \subset [\![\Diamond(\phi \vee \psi)]\!]$. It follows that $Prag_0^{K_0}(\Diamond(\phi \vee \psi)) = \emptyset$. We continue, and calculate $K_1$ and $A_1(\Diamond(\phi \vee \psi))$. The new set of worlds we have to consider, $K_1$, is just $K - Exh_0^{K_0}(\Diamond(\phi \vee \psi)) =$

$\{w,x\}$. The new set of alternatives, $A_1(\Diamond(\phi \vee \psi))$, is just $\{\Diamond(\phi \wedge \psi)\}$. Now, we have to determine $Exh_1^{K_1}(\Diamond(\phi \vee \psi))$ and $Prag_1^{K_1}(\Diamond(\phi \vee \psi))$. Because $K_1 = \{w,x\}$ and $\Diamond(\phi \wedge \psi)$ is only true in $x$, both will be $\{w\}$. But this means that also $Prag^K(\Diamond(\phi \vee \psi)) = \{w\}$, and thus that we can pragmatically infer both $\Diamond\phi$ and $\Diamond\psi$ from the assertion that $\Diamond(\phi \vee \psi)$, as desired. A very similar calculation shows that we can pragmatically infer both $\phi > \chi$ and $\psi > \chi$ from the assertion that $(\phi \vee \psi) > \chi$. What's more, we have even explained why we can pragmatically infer from $\Diamond(\phi \vee \psi)$ that the alternative $\Diamond(\phi \wedge \psi)$ is not true, just as Franke (2009b) could.

These predications are exactly as desired, but how does our machinery work for more simple examples, like $\phi \vee \psi$? Fortunately, it predicts correctly here as well. First, assume that $[\![\phi \vee \psi]\!] = \{u,v,w\} = K, [\![\phi]\!] = \{u,w\}, [\![\psi]\!] = \{v,w\}$, and $[\![\phi \wedge \psi]\!] = \{w\}$. Observe that $Exh_0^{K_0}(\phi \vee \psi) = \{u,v\}$. On the basis of these facts, we can conclude that $Prag_0^{K_0}(\phi \vee \psi) = \emptyset$. This is just the same reasoning as before. The difference shows up when we go to the next level and determine $Prag_1^{K_1}(\phi \vee \psi)$, because now there will be an alternative left over which plays a crucial role. But first calculate $K_1$ and $A_1(\phi \vee \psi)$: $K_1 = \{w\}$ and $A_1(\phi \vee \psi) = \{\phi \wedge \psi\}$. Obviously, $Exh_1^{K_1}(\phi \vee \psi) = \{w\}$, but because $w \in [\![\phi \wedge \psi]\!]$, it follows that $(\phi \wedge \psi) \prec^1 (\phi \vee \psi)$, and thus $Prag_1^{K_1}(\phi \vee \psi) = \emptyset$. It follows that $K_2 = \emptyset$, from which we can conclude that $Prag^K(\phi \vee \psi) = Exh_0^{K_0}(\phi \vee \psi) = \{u,v\}$, as desired.

Let us now see what happens if we look at multiple occurrences of disjunctions: examples like $\phi \vee \psi \vee \chi$, $\Diamond(\phi \vee \psi \vee \chi)$, and $(\phi \vee \psi \vee \chi) > \xi$. First look at $\phi \vee \psi \vee \chi$ and assume that $[\![\phi]\!] = \{w_1, w_4, w_5, w_7\}, [\![\psi]\!] = \{w_2, w_4, w_6, w_7\}$, and $[\![\chi]\!] = \{w_3, w_5, w_6, w_7\}$. Observe that $Exh_0^{K_0}(\phi \vee \psi \vee \chi) = \{w_1, w_2, w_3\}$. On the basis of these facts, we can conclude that $Prag_0^{K_0}(\phi \vee \psi \vee \chi) = \emptyset$. If only the separate disjuncts were alternatives of $\phi \vee \psi \vee \chi$, it would result that $K_1 = \{w_4, w_5, w_6, w_7\}$, which would then also be the inferred pragmatic interpretation. We have to conclude that thus we need other alternatives as well. It is only natural to assume that also $\phi \wedge \psi, \phi \wedge \chi, \psi \wedge \chi$, and $\phi \wedge \psi \wedge \chi$ are alternatives. In that case $K_1$ is still $\{w_4, w_5, w_6, w_7\}$, but now the new set of alternatives is $\{\phi \wedge \psi, \phi \wedge \chi, \psi \wedge \chi, \phi \wedge \psi \wedge \chi\}$, and the resulting pragmatic meaning will be different. In particular, $Exh_1^{K_1}(\phi \vee \psi \vee \chi) = \{w_4, w_5, w_6\}$. However, none of these worlds remains in $Prag_1^{K_1}(\phi \vee \psi \vee \chi)$, because $w_4 \in [\![\phi \wedge \psi]\!]$ which is a stronger expression than $\phi \vee \psi \vee \chi$, and similarly for $w_5$ and $w_6$. This means we have to go to the next level where $K_2 = \{w_7\}$. But $w_7$ won't be in $Prag_2^{K_2}(\phi \vee \psi \vee \chi)$, because $w_7 \in [\![\phi \wedge \psi \wedge \chi]\!] = \{w_7\}$. As a result, $Prag^K(\phi \vee \psi \vee \chi) = Exh_0^{K_0}(\phi \vee \psi \vee \chi) = \{w_1, w_2, w_3\}$, just as desired.

What about $\Diamond(\phi \vee \psi \vee \chi)$, for instance? Once again we have to make a closure assumption concerning the alternatives. As it turns out, the correct way to go is also the most natural one: first, $A(\Diamond\phi) = \{\Diamond\psi : \psi \in A(\phi)\}$, and second, $A(\phi \vee \psi \vee \chi) =$

$\{\phi, \psi, \chi, \phi \wedge \psi, \phi \wedge \chi, \psi \wedge \chi, \phi \wedge \psi \wedge \chi, \phi \vee \psi, \phi \vee \chi, \psi \vee \chi\}$. Thus, at the 'local' level, the alternatives are closed under disjunction as well. Let us now assume that $[\![\Diamond\phi]\!] = \{w_1, w_4, w_5, w_7\}, [\![\Diamond\psi]\!] = \{w_2, w_4, w_6, w_7\}$, and $[\![\Diamond\chi]\!] = \{w_3, w_5, w_6, w_7\}$. Let's assume for simplicity that in none of these worlds any conjunctive permission like $\Diamond(\phi \wedge \psi)$ is true. Observe that $Exh_0^{K_0}(\Diamond(\phi \vee \psi \vee \chi)) = \{w_1, w_2, w_3\}$. It follows that $K_1 = \{w_4, w_5, w_6, w_7\}$ and the new set of alternatives is the earlier set minus $\{\Diamond\phi, \Diamond\psi, \Diamond\chi\}$. The new exhaustive interpretation will be $Exh_1^{K_1}(\Diamond(\phi \vee \psi \vee \chi)) = \{w_4, w_5, w_6\}$, but all these worlds are ruled out for $Prag_1^{K_1}(\Diamond(\phi \vee \psi \vee \chi))$ because of our disjunctive alternatives. This means that we have to go to the next level. At level 2, the new set of worlds is just $\{w_8\}$, which is thus also $Exh_2^{K_2}(\Diamond(\phi \vee \psi \vee \chi))$. World $w_8$ cannot be eliminated by a more precise alternative, which means that also $Prag_2^{K_2}(\Diamond(\phi \vee \psi \vee \chi)) = \{w_8\}$, which is what $Prag^K(\Diamond(\phi \vee \psi \vee \chi))$ will then denote as well. Notice that in $w_8$ it holds that all of $\Diamond\phi$, $\Diamond\psi$, and $\Diamond\chi$ are true: the desired free choice inference. A similar reasoning applies to $(\phi \vee \psi \vee \chi) > \xi$.

These calculations have made clear that to account for free choice permission, we have to make use of exhaustive interpretation several times. In this sense it is similar to the analysis proposed by Fox (2007). Still, there are some important differences. One major difference is that Fox (2007) exhaustifies not only the sentence that is asserted, but also the relevant alternatives. Moreover, Fox uses exhaustification to turn alternatives into other alternatives, thereby 'syntacticising' the process. We don't do anything like this, and therefore feel that what we do is more in line with the Gricean approach. Exhaustification always means looking at 'minimal' worlds: we don't change the alternatives. The worst that can happen to them is that they are declared not to be relevant anymore to determine the pragmatic interpretation.

Notice that our analysis also immediately explains why it is appropriate to use 'any' under $\Diamond$, but not under $\Box$: whereas $\Diamond(\phi \vee \psi \vee \chi)$ pragmatically entails $\Diamond(\phi \vee \chi)$, $\Box(\phi \vee \psi \vee \chi)$ does not pragmatically entail $\Box(\phi \vee \chi)$. It is easy to see that our analysis can account for the 'free choice'-inference of the existential sentence as well: that from 'Several of my cousins had cherries or strawberries' we naturally infer that some of the cousins had cherries and some had strawberries.[31] In formulas, from $\exists x[Px \wedge (Qx \vee Rx)]$ we can pragmatically infer that both $\exists x[Px \wedge Qx]$ and $\exists x[Px \wedge Rx]$ are true. But this shows that yet another 'paradoxical' conjunctive reading of disjunctive sentences can be accounted for as well.[32] If we analyze comparatives as proposed by Larson (1988), for instance, it is predicted that 'John is taller than Mary or Sue' should be represented as something like $\exists \mathbf{d}[\mathbf{d}(T)(j) \wedge (\neg\mathbf{d}(T)(m) \vee \neg\mathbf{d}(T)(s))]$, with $\mathbf{d}$ a measure function from (denotations of) adjectives

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31 I believe that Nathan Klinedinst and Regine Eckhardt were the first to observe that these inferences should go through. Perhaps it should be pointed out that Schulz (2003) could straightforwardly account for these inferences as well.

32 This observation is due to Krasikova (2007), though she uses Fox's analysis of free-choice inferences.

to sets of individuals. Pragmatically we can infer from this that John is taller than Mary *and* that John is taller than Sue.

Chemla (2009) argued that sentences of the form $\forall x \Diamond (Px \vee Qx)$ give rise to inferences that are more problematic to account for by globalist approaches towards conversational implicatures than by localist approaches. He found that people inferred from 'Everybody is allowed to take Algebra or Literature' that everybody can choose which of the two they will take. This suggests that in general we infer from $\forall x \Diamond (Px \vee Qx)$ both $\forall x \Diamond Px$ and $\forall x \Diamond Qx$. In their commentary article, Geurts and Poulouscous (200b) suggested that the observed 'conjunctive' inference might very well depend on the particular construction being used, however, and thus less general than predicted by a localist approach. Moreover, they suggest that universal permission sentences are just summaries of permissions of the form $\Diamond (\phi \vee \psi)$ made to multiple addressees, in which case the data can be explained by any global analysis that can explain standard free choice permissions. I don't know what is the appropriate analysis of these inferences. I can point out, however, what we would have to add to our analysis to account for the conjunctive interpretation. If this conjunctive interpretation really depends on the particular construction being used (as suggested by Geurts and Poulouscous), then it would be wise not to make use of this extra addition.

As it turns out, our approach predicts the conjunctive interpretation if we include $\exists x \Diamond (Px \vee Qx)$ among the alternatives, and we exchange in the definition of $\psi \prec \phi^n$ the notion $[\![\psi]\!]^{K_n}$ by the pragmatic interpretation of $\psi$, $Prag^K(\psi)$.[33] The crucial step in this case is the one in which a minimal world where $\exists x \Diamond Px$ and $\exists x \Diamond Qx$ are true but both $\forall x \Diamond Px$ and $\forall x \Diamond Qx$ false is eliminated, because such a world could be more accurately expressed (given the truth of $\forall x \Diamond (Px \vee Qx)$) by the alternative $\exists x \Diamond (Px \vee Qx)$. While the inclusion of $\exists x \Diamond (Px \vee Qx)$ among the alternatives of $\forall x \Diamond (Px \vee Qx)$ is not a significant change to our framework, it has to be admitted that the exchange of the notion $[\![\psi]\!]^{K_n}$ by the pragmatic interpretation of $\psi$ does. From an intuitive point of view, the effect of this exchange would be that we do not only look at the exhaustive interpretation of $\phi$, the sentence asserted, but also at the exhaustive interpretations of the alternatives. As a result, our analysis would become much closer to the proposal of Fox (2007). But, as mentioned above, if we were to adopt the suggestion of Geurts & Pouscoulous (2009a), this would, in fact, not be the way to go.

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33 Thus, $\psi \prec^n \phi$ will be an abbreviation for the condition '$[\![\psi]\!]^{K_n} \subset [\![\phi]\!]^{K_n}$, if $n = 0$, and $Prag^K(\psi) \subseteq [\![\phi]\!]^{K_n}$, otherwise'. where $Prag^K(\psi)$ is, as before, $Prag_n^{K_n}(\psi)$ for the first $n$ such that $Prag_n^{K_n}(\psi) \neq \emptyset$

## 5 Conclusion

The papers of Geurts & Pouscoulous (2009b) and Chemla (2009) provide strong empirical evidence that sentences in which a trigger of a scalar implicature occurs under a universal does not in general give rise to an embedded implicature. This evidence favors a globalist analysis of conversational implicatures over its localist alternative. As far as I know, it is uncontroversial that triggers occurring under an existential *do* give rise to implicatures. In this paper, and following Franke (2009a,b), I discussed some ways in which these challenging examples for a 'globalist' analysis of conversional implicatures could be given a principled global pragmatic explanation after all. I suggested how potentially problematic examples for our global pragmatic analysis of the form '$\forall x \Diamond (Px \vee Qx)$', as discussed by Chemla (2009), could be treated as well. At least two things have to be admitted, though. First, our global analysis still demands that the alternatives are calculated locally. I don't think this is a major concession to localists. Second, according to Zimmermann (2000), even a disjunctive permission of the form 'You may do $\phi$ or you may do $\psi$' gives rise to the free choice inference, and according to Merin (1992) a conjunctive permission of the form 'You may do $\phi$ and $\psi$' allows the addressee to perform only $\phi$. I have no idea how to pragmatically account for those intuitions without reinterpreting the semantics of conjunction as well as disjunction. If our analysis is acceptable, it points to the direction in which richer pragmatic theories have to go: (i) we have to take both the speaker's and the hearer's perspective into account, and (ii) one-step inferences (or strong Bi-OT) are not enough, more reasoning has to be taken into account (i.e., weak Bi-OT, or iteration). These are what I take to be the main messages of this paper.

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