

Exam

Functional Analysis Bachelor wiskunde jaar 3

Tentamen

Date: 26 maart, 2019

Time: 9.00-12.00

Number of pages: 2 (including front page)

Number of questions: 6

Maximum number of points: 30

For each question is indicated how many points it is worth.

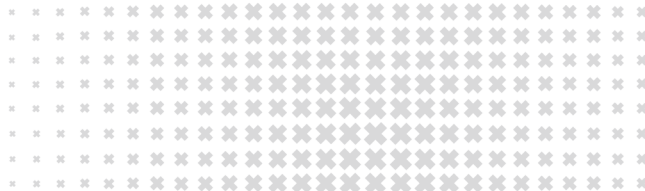
BEFORE YOU START

- Check if your version of the exam is complete.
- Write down **your name, student ID number**, and if applicable the **version number** on **each sheet** that you hand in. Also **number the pages**.
- Your **mobile phone** has to be switched off and be put in your coat or bag. Your **coat and bag** should be on the ground.
- **Tools allowed:** kladpapier. Other tools are not allowed.

PRACTICAL MATTERS

- The first 30 minutes you are not allowed to leave the room, not even to visit the toilet.
- 15 minutes before the end, you will be warned that the time to hand in is approaching.
- If applicable, fill out the evaluation form at the end of the exam.
- You are obliged to identify yourself at the request of the examiner (or his representative) with a proof of your registration and a valid ID.
- During the examination it is not permitted to visit the toilet, unless the invigilator gives permission to do so.

Good luck!



1. (4pt) Prove that for a non-empty subset E of a Hilbert space H , it holds that $\overline{\text{Sp}} E = H$ if and only if $E^\perp = \{0\}$.
2. (4pt) Recall that for a non-empty set S , and a normed space X , $F_b(S, X)$ is the normed linear space of bounded functions $f: S \rightarrow X$ equipped with $\|f\|_b = \sup\{\|f(s)\|: s \in S\}$. Show that $F_b(S, X)$ is a Banach space when X is a Banach space.
3. (6pt) Let X be a complex inner product space, and $T: X \rightarrow X$ be a linear operator.
 - (a) Show that for any $u, w \in X$,
$$4\langle Tu, w \rangle = \langle T(u+w), u+w \rangle - \langle T(u-w), u-w \rangle + i\left(\langle T(u+iw), u+iw \rangle - \langle T(u-iw), u-iw \rangle\right).$$
 - (b) Show that $\langle Tu, u \rangle = 0$ for all $u \in X$, implies that $T = 0$.
 - (c) Give an example that demonstrates that (3b) is generally not true in a real inner product space.
4. (6pt) Given $\{y_i\} \subset \mathbb{C}$, for $N \in \mathbb{N}$, let $L_N: \{x_i\} \mapsto \sum_{i=1}^N x_i y_i$.
 - (a) Show that $L_N \in c'_0$ with $\|L_N\|_{c'_0} = \sum_{i=1}^N |y_i|$.
 - (b) Suppose that for all $\{x_i\} \in c_0$, it holds that $\sum_{i \geq 1} x_i y_i$ is convergent. Prove that $\{y_i\} \in \ell^1$.
5. (4pt) For a Banach space X , let $T \in B(X)$. Assume that for any $x \in X$, $\sum_{n \geq 0} T^n x$ is convergent. Show that $I - T$ is boundedly invertible.
6. (6pt) Let X be an inner product space.
 - (a) Show that for any $z \in X$, $f_z(x) := \langle x, z \rangle$ defines an $f_z \in X'$ with $\|f_z\|_{X'} = \|z\|_X$.
 - (b) Show that $z \mapsto f_z: X \rightarrow X'$ is surjective if and only if X is a Hilbert space.