- **Page 300, Solution 7.11 (c):** For the definition of a Hilbert-Schmidt operator to make sense we need to assume that H is separable, and so has an orthonormal basis (the rest of the exercise does not require this). The given solution then works if the orthonormal sequence $\{e_n\}$ is a basis for H. If not, let $Y = \overline{\mathrm{Sp}}\{e_n\}$ and let $\{g_n\}$ be an orthonormal basis for $Y^\perp$. The union $\{e_n\} \cup \{g_n\}$ is then a basis for H (see Exercise 3.26) and it is clear from the definitions that $Tg_n = 0$ for all the vectors in $\{g_n\}$. Hence, we can use the basis $\{e_n\} \cup \{g_n\}$, together with the previous calculation, to show that T is Hilbert-Schmidt.