

Problem Set Five and a Half – More Curvature Tools – February 20, 2015

Question 1

Using the index symmetries of the Riemann tensor,

1. Show that the Ricci Tensor is symmetric;
2. Show that $R_{\alpha\beta\gamma}{}^{\delta} = R^{\delta}{}_{\gamma\beta\alpha}$;
3. Prove the First Bianchi Identity ($R_{[\alpha\beta\gamma]}{}^{\delta} = 0$);
4. Prove the Second Bianchi Identity ($\nabla_{[\mu}R_{\nu\alpha]\beta}{}^{\rho} = R^{\rho}{}_{\beta[\alpha\nu;\mu]} = 0$).

Question 2

Show that $R_{\mu\nu} = 0$ (Ricci flatness of spacetime) does not imply $R^{\rho}{}_{\beta\sigma\alpha} = 0$ (absence of spacetime curvature).

Question 3

1. Compute the components of the Einstein Tensor $G_{\mu\nu}$ in an arbitrary spacetime, in terms of
 - (a) The affine connection Γ and its derivatives;
 - (b) The metric $g_{\mu\nu}$ and its derivatives.
2. Show that these expressions simplify dramatically in local inertial frames.