Problem Set Eight – Gravitational Waves – March 6, 2015

Question 1

Although the energy of a GW cannot be localised inside a wavelength, it is possible to give an effective stress-energy, smeared out over a macroscopic region several wavelengths' size. In a (nearly) inertial frame of linearized theory ($g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$) and averaged over several wavelengths, we have

$$T_{\mu\nu}^{(\mathbf{GW})} = \frac{1}{32\pi G} \left\langle h^{TT}_{\alpha\beta,\mu} \, h^{\alpha\beta}_{TT,\nu} \right\rangle$$

1. Show that this effective stress-energy is divergence-free.

   (Hint: remember that the background is a flat vacuum)

2. Find the components of this effective stress-energy tensor for the GWs described by

   $$ds^2 = -\mathrm{d}t^2 + [1 + h_+ \cos(\omega(t - z))]\mathrm{d}x^2 + [1 - h_+ \cos(\omega(t - z))]\mathrm{d}y^2 + \mathrm{d}z^2 \quad (1)$$

   then check your answers against Carroll pp. 310-311.

3. Argue that this quantity is not *really* a tensor, both (i) from the perspective of the Equivalence Principle, and (ii) from that of differential geometry.